

Book of Abstracts

Dynamics Days Europe

Bremen, July 29 – August 2, 2024

Plenaries

Forcing of teleconnections among tipping elements in the climate system

Jürgen Kurths (PIK Potsdam Institute for Climate Impact Research)

Monday, July 29, 08:30–09:30, SCC Conference Hall

Abstract: Tipping elements are components of the Earth system that may shift abruptly and irreversibly from one state to another at specific thresholds. An important aspect is how different tipping points are interrelated and how the corresponding teleconnections are changing due to external forcing. Here, we propose a climate network approach to analyse the global impacts of a prominent tipping element, the Amazon Rainforest Area (ARA). We find that the ARA exhibits strong correlations with regions such as the Tibetan Plateau (TP) and the West Antarctic ice sheet. Models show that the teleconnection propagation path between the ARA and the TP is rather robust under climate change. We further uncover that various climate extremes are synchronized between the ARA and the TP. Then we analyze how different ENSO regimes influence this specific teleconnection and find that there is indeed a substantial dependence on the ENSO state. Hence, this interaction of ARA and TP can only be understood as a non-autonomous system. The implications of this basic feature will be discussed in detail.

On some Impact-like Hamiltonian systems

Vered Rom-Kedar (The Weizmann Institute of Science)

Monday, July 29, 09:30–10:30, SCC Conference Hall

Abstract: The dynamics associated with mechanical Hamiltonian systems with smooth potentials that include sharp fronts is traditionally modeled by Hamiltonian impact systems: a class of generalized billiards by which the dynamics in the domain's interior are governed by smooth potentials and at the domain's boundaries by elastic reflections. I will first discuss the properties of this singular limit, culminating in the recent work with D. Turaev in which we established the non-ergodicity of smooth N repelling particles in a box at arbitrarily high energy. Then, I will introduce the class of quasi-integrable Hamiltonian impact systems, where the motion on some level sets is conjugated to a directed motion on a translation surface of a genus larger than one. We propose mechanical realizations of such systems, analyze ergodic properties and quantum properties of classes of such systems, and study their behavior under perturbations (in joint works with L. Becker, S. Elliott, B. Firester, S. Gonen Cohen, I. Pazi, M. Pnueli, K. Fraczek, O. Yaniv and A. Zobova).

Dynamics on homogeneous spaces: a quantitative viewpoint

Amir Mohammadi (University of California, San Diego)

Monday, July 29, 16:00–17:00, SCC Conference Hall

Abstract: Rigidity phenomena in homogeneous spaces have been extensively studied over the past few decades with several striking results and applications. We will give an overview of activities pertaining to the quantitative aspect of the analysis in this context with an emphasis on recent developments.

Probing and Controlling Many-Body Quantum Chaos

Klaus Richter (University of Regensburg)

Tuesday, July 30, 08:30–09:30, SCC Conference Hall

Abstract: The notions of chaos and order are central to understanding the statistical physics of many-body systems. Thermalization and the spread of quantum information in many-body dynamics is presently attracting a lot of attention across various fields, ranging from statistical physics via cold atom physics to quantum gravity. Starting from the concept of a “quantum butterfly effect”, this includes questions of how many-body quantum interference and scarring affect equilibration, more generally non-classicality in many-particle quantum physics. Vice versa, it is long known how to harness exponential sensitivity to changes in initial conditions for control purposes in classically chaotic systems. We will generalize this concept, using chaos as a resource for steering many-body quantum dynamics. We will address the above phenomena using semiclassical methods based on interfering Feynman paths, thereby bridging the classical and quantum chaotic many-body world.

Banach spaces and Bernoulli Billiards in Bremen

Viviane Baladi (CNRS)

Tuesday, July 30, 09:30–10:30, SCC Conference Hall

Abstract: It has been a quarter century now that Banach spaces of anisotropic distributions have been introduced to study statistical properties of chaotic dynamical systems via Ruelle transfer operators. This approach gives both new proofs of classical results and new results. The first successes were obtained for smooth hyperbolic dynamics. However, some natural dynamical systems, such as dispersive (Sinai) billiards are not smooth. The singularities cause challenging technical difficulties. We shall survey new results on dispersive (discrete or continuous time) dispersive billiards obtained in the past five years using anisotropic Banach spaces, ending with a very recent construction of the measure of maximal entropy for billiard flows satisfying a condition of sparse recurrence to singularities. (In this joint work with Carrand and Demers, we obtain Bernoullicity, but no control of the speed of mixing.)

Turbulent-laminar patterns

Laurette Tuckerman (CNRS PMMH)

Wednesday, July 31, 08:30–09:30, SCC Conference Hall

Abstract: Experiments and numerical simulations have shown that turbulence in transitional wall-bounded shear flows such as plane Couette and Poiseuille flow takes the form of long oblique bands if the domains are sufficiently large to accommodate them. At their upper Reynolds-number threshold, laminar regions carve out gaps in otherwise uniform turbulence, thereby forming regular oblique turbulent-laminar patterns with a large spatial wavelength. At the lower threshold, isolated turbulent bands sparsely populate otherwise laminar domains and complete laminarization takes place via their disappearance characterized by the 2D directed percolation scenario.

Quantum theory for modeling classical dynamics

Dimitrios Giannakis (Dartmouth College)

Wednesday, July 31, 09:30–10:30, SCC Conference Hall

Abstract: In recent years, a problem that has been receiving increasing attention is simulation of classical dynamical systems by quantum systems – a primary motivation being the prospect of practical quantum computing and the advances in information processing capabilities that it promises to deliver. In broad terms, techniques for quantum simulation of classical dynamics are based on mappings of classical states and observables into states and observables of a quantum system, together with a corresponding mapping of the classical dynamical evolution maps into an evolution of quantum states. Here, a major challenge stems from the fact classical systems of interest are typically nonlinear, whereas quantum dynamics proceeds by unitary (linear) transformations of the quantum state.

In this talk, we survey quantum mechanical approaches for modeling classical dynamics based on operator methods from ergodic theory. We begin by considering the case of measure-preserving ergodic flows with pure point spectra of the associated Koopman and transfer operators. In this setting, the highly structured nature of the dynamics allows an efficient factorization of the transfer operator that acts on tensor product Hilbert spaces associated with quantum computational systems, along with a representation of classical observables by multiplication operators. Next, we discuss methods for representing more general (potentially mixing) measure-preserving systems based on approximations of the generator with discrete spectra. The final part of the talk explores ideas from many-body quantum theory as a framework for modeling classical dynamics.

Life in a warmer world

Thomas Jung (Alfred Wegener Institute)

Wednesday, July 31, 20:00–21:00, SCC Conference Hall

Abstract: While the global mean temperature indicates that human activities are altering the climate, the local implications of these changes are much less explored. This presentation aims to bridge that gap by discussing recent developments in climate modelling, including storylines, that provide a more tangible understanding of climate change impacts at the local level. Through these insights, we aim to enhance our comprehension of the real-world consequences of a warming planet, thereby supporting adaptation and mitigation.

Infinite Ergodic Theory and Irregularities in Uniform Distribution

Omri Sarig (The Weizmann Institute of Science)

Thursday, August 1, 08:30–09:30, SCC Conference Hall

Abstract: Suppose a is an irrational number. Weyl proved that $na \bmod 1$ is uniformly distributed on the unit interval. i.e., the frequency of visits of $na \bmod 1$ to a subinterval of $[0,1]$ tends to the length of the subinterval.

It has long been known that the error term in this limit theorem can exhibit strong bias, reflecting an “irregularity” in uniform distribution in the higher-order term. This bias depends on the fine number theoretic properties of a . For example, the square roots of two and three do not behave the same way (!)

I will explain how infinite ergodic theory can shed light on this phenomenon, and describe some recent joint work with Dmitry Dolgopyat on the equidistribution of the error term. There are amusing connections to the geometry of translation surfaces with infinite genus, and to local limit theorems of inhomogeneous Markov chains.

Top Lyapunov exponent for advection-diffusion

Martin Hairer (EPFL)

Thursday, August 1, 09:30–10:30, SCC Conference Hall

Abstract: We show that, for a class of random velocity fields that cover solutions to the stochastic Navier-Stokes equations, the Eulerian Lyapunov exponent for passive scalar advection/diffusion is finite. The proof relies on constructive bounds on the projective process that allow us to keep track of its dependence on the diffusion coefficient.

On anomalous diffusion

Vlad Vicol (Courant Institute of Mathematical Sciences, New York University)

Friday, August 2, 08:30–09:30, SCC Conference Hall

Abstract: Anomalous diffusion is the fundamental ansatz of phenomenological theories of passive scalar turbulence. As with the anomalous dissipation of kinetic energy in a turbulent fluid, the anomalous dissipation of passive scalar variance in a turbulent flow, as the Reynolds and Peclet numbers diverge, has been confirmed numerically and experimentally to an extraordinary extent. A satisfactory theoretical explanation

of this phenomenon is however not available. In this talk, I will discuss a joint work with Scott Armstrong in which we construct a class of incompressible vector fields that have many of the properties observed in a fully turbulent velocity field, and for which the associated scalar advection-diffusion equation generically displays anomalous diffusion. We also propose an analytical framework in which to study anomalous diffusion, via a backward cascade of renormalized eddy viscosities. Our proof is by "fractal" homogenization, that is, we perform a cascade of homogenizations across arbitrarily many length scales.

Learning in Living Systems

Viola Priesemann (Max Planck Institute for Dynamics and Self-Organization)

Friday, August 2, 09:30–10:30, SCC Conference Hall

Abstract: Living systems hinge on self-organization and learning. We will show basic principles of self-stabilization and learning, using two complementary systems — Living Neural Networks that have been optimized for emergent information processing, and Societal Networks, which might not always be optimal. We will examine the self-organization in these systems, their emergent dynamics, how they (fail to) self-stabilize with homeostatic mechanisms, and how they can learn in a self-organized manner. Thereby, we shed light on how self-organization helps these living systems to navigate their complex environment across scales.

Minisymposia

Minisymposium on Adaptive dynamical networks

The impact of group interactions on opinion formation

Jan Mölter (Technische Universität München)

Thursday, August 1, 13:30–14:00, Research II Lecture Hall

Abstract: Opinions are rarely formed in isolation but rather through interactions with other individuals in society. As such, they are a product of each individual's social neighbourhood, but, at the same time, they also shape this neighbourhood. Classical models of opinion formation, such as the adaptive voter model, demonstrate how the interplay between the propagation of opinions and the adaptation of the social neighbourhood can ultimately, if the latter dominates, lead to a fragmentation of society and the formation of echo chambers. Focussing exclusively on pairwise interactions, these models leave out forces like the striving for conformity that manifests itself, for instance, in peer pressure. Taking these into account requires considering group interactions, and in this talk, I will discuss their impact on opinion formation. To this end, I will also introduce a recent extension of the adaptive voter model that involves considering the analogous processes of propagation and adaptation from the classical model on an underlying hypergraph instead of a graph.

Interplay between self-induced stochastic resonance, heterogeneity, and breathing chimera in coupled oscillators

Marius Yamakou (Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU))

Thursday, August 1, 14:00–14:30, Research II Lecture Hall

Abstract: Our analysis challenges previous findings by revealing that, unlike synchronization or coherence resonance phenomena, there is no optimal network heterogeneity that can enhance SISR; instead, it always acts antagonistically toward SISR. In the second part of the talk, we introduce a novel form of chimera called self-induced-stochastic-resonance breathing chimera (SISR-BC). We show how SISR-BC combines the mechanisms of SISR with rotational coupling symmetry breaking between oscillators' slow and fast subsystems, along with the characteristics of breathing chimeras—marked by non-stationary periodic dynamics of coherent-incoherent patterns. Unlike other types of chimeras, SISR-BC exhibits impressive resilience to stochastic perturbations over a broad range and persists even when the purely excitable network is far from

the Hopf bifurcation threshold, thanks to SISR's mechanisms, and globally attract random initial conditions. The last part of the talk briefly discusses with the audience how adaptivity in the network can affect SISR-BC. This is joint work with Els Heinsalu, Marco Patriarca, Stefano Scialla, and Jinjie Zhu.

Modeling tumor disease and sepsis in physiological networks

Eckehard Schöll (TU Berlin)

Thursday, August 1, 14:30–15:00, Research II Lecture Hall

Abstract: In this study, we provide a dynamical systems perspective to the modelling of pathological states induced by tumors or infection. A unified disease model is established using the innate immune system as the reference point. We propose a two-layer network model for carcinogenesis and sepsis based upon the interaction of parenchymal cells (organ tissue) and immune cells via cytokines, and the co-evolutionary dynamics of parenchymal, immune cells, and cytokines [1,2]. Our aim is to show that the complex cellular cooperation between parenchyma and stroma (immune layer) in the physiological and pathological case can be functionally described by a simple paradigmatic model of phase oscillators. By this, we explain carcinogenesis, tumor progression, and sepsis by destabilization of the healthy state (frequency synchronized), and emergence of a pathological state (multifrequency cluster). The coupled dynamics of parenchymal cells (metabolism) and nonspecific immune cells (reaction of innate immune system) are represented by nodes of a duplex layer. The cytokine interaction is modeled by adaptive coupling weights.

[1] Sawicki, J., Berner, R., Löser, T., and Schöll, E., *Frontiers Netw. Physiology* 1, 730385 (2022)

[2] Berner, R., Sawicki, J., Thiele, M., Löser, T., and Schöll, E.: *Frontiers Netw. Physiology* 2, 904480 (2022)

Dynamics of adaptive networks with Hebbian and Anti-Hebbian plasticity

David Fox, Andreas Amann (University College Cork)

Thursday, August 1, 15:00–15:30, Research II Lecture Hall

Abstract: We study a simple neural model with tanh nonlinearity. The plasticity is modeled using variants of the Hebbian learning rule where the synaptic connections between two neurons evolves based on their correlation in the recent past. Based on the complete understanding of the simple case of two neurons, we investigate the dynamics of a network with larger number of neurons, which shows both stable and transient frequency cluster synchronization.

Dimensional reductions for the adaptive Kuramoto model

Erik Andreas Martens (Lund University), Rok Cestnik (Lund University), Christian Bick (Vrije Univ. Amsterdam)

Thursday, August 1, 15:30–16:00, Research II Lecture Hall

Abstract: A growing interest has arisen for networks with adaptive coupling, where the interaction strength between the oscillators co-evolves with the nodal dynamics – inspired by neural interactions which change slowly in time. From a modelling perspective, this gives rise to richer dynamics and new oscillatory states which have not yet been described. Large networks are abundant in nature and therefore of particular interest. However, since the number of connections grows quadratically with the number of nodes, a large number of network nodes is prohibitive for numerical simulations. Here, we study the Kuramoto model with adaptive coupling to address this issue. Using an approximate dimensional reduction and considering the continuum limit, we derive a self-consistency equation which we can analyze obtain the bifurcation diagram for the adaptive Kuramoto oscillator model. Moreover, we discover regions with new states and regions with dense multistability. Finally, we discuss the dynamics of the adaptive coupling, constrained to sub-manifolds, and under which conditions the constrained dynamics naturally align to symmetries underlying the system.

Oscillations and avalanches in an Ising-like class of adaptive neural networks

Fabrizio Lombardi (University of Padova), Selver Pepić (Institute of Science and Technology (IST) Austria),

Oren Shriki (Ben-Gurion Univ. of the Negev), Gašper Tkačik (Institute of Science and Technology (IST) Austria), Daniele De Martino (Biofisika Institute (CSIC, UPV-EHU) and Ikerbasque Foundation)
Friday, August 2, 11:00–11:30, Research II Lecture Hall

Abstract: Neurons in the brain are wired into adaptive networks that exhibit a range of collective dynamics. Oscillations, for example, are paradigmatic synchronous patterns of neural activity with a defined temporal scale. Neuronal avalanches, in contrast, do not show characteristic spatial and temporal scales, and are often considered as evidence of brain tuning to quasi-criticality. While models have been developed to account for oscillations or neuronal avalanches separately, they typically do not explain both phenomena, are too complex to analyze analytically, or intractable to infer from data rigorously. Here we propose a non-equilibrium feedback-driven Ising-like class of neural networks that simultaneously and quantitatively captures scale-free neuronal avalanches and scale-specific oscillations. In the most simple yet fully microscopic model version we can analytically compute the phase diagram and make direct contact with human brain resting-state activity recordings via tractable inference of the model's two essential parameters. The inferred model quantitatively captures the dynamics over a broad range of scales, from single sensor oscillations and collective behaviors of nearly-synchronous extreme events on multiple sensors, to neuronal avalanches unfolding over multiple sensors across multiple time bins. Importantly, the inferred parameters correlate with model-independent signatures of "closeness to criticality", indicating that the coexistence of scale-specific (neural oscillations) and scale-free (neuronal avalanches) dynamics in brain activity occurs close to a non-equilibrium critical point at the onset of self-sustained oscillations.

Critical Drift in Adaptive Networks

Thilo Gross (Helmholtz Institute for Functional Marine Biodiversity at the University of Oldenburg (HIFMB))

Friday, August 2, 11:30–12:00, Research II Lecture Hall

Abstract: Self-organized criticality (SOC) has been extensively studied in the dynamical systems community. However, we typically think of SOC as a process in which a single parameter is tuned to the critical point. Recently, SOC has received much attention in neuroscience in the context of the critical brain hypothesis, which claims that the brain operates on or near a critical state. If so, the brain might achieve and maintain criticality through the self-tuning of synapses between neurons, so-called synaptic plasticity. Hence criticality is not achieved by adjusting a single parameter but the combined action of countless individual parameters that interact in a complex network. The same might be said about other classic SOC systems, such as forest fire models, but they have rarely been regarded in this way. In a high-dimensional parameter-space the critical points form a manifold of codimension-1. Hence criticality is not confined to a specific point but forms, but is found in a space that is itself large and high-dimensional. Here we demonstrate that this leads to drift of the system on the critical manifold, which can greatly contribute to the robustness of SOC in high-dimensional systems. Moreover, it opens the door to further phenomena such as adaptive multi-criticality in which a system self-organizes to a point in which a system is critical in several—and possibly many—ways at the same time.

A universal adaptive network formulation of power grid dynamics

Anna Büttner (Potsdam Institute for Climate Impact Research (PIK))

Friday, August 2, 13:30–14:00, Research II Lecture Hall

Abstract: The ongoing energy transition reshapes the dynamics of power grids by introducing new categories of actors. An important example are grid-forming inverters (GFIs) that are employed to enhance grid stability.

This talk delves into the urgent and complex task of understanding the collective behavior and stability of future grids that are characterized by a heterogeneous mix of dynamics.

Recent advancements have significantly improved our ability to describe modern power grid dynamics. Firstly, the development of the normal form for grid-forming actors offers a technology-neutral framework to describe the dynamics of GFIs. Secondly, the concept of complex frequency facilitates a seamless depiction

of the impact of the nodal dynamics on power flows within the grid.

This presentation's primary focus is on demonstrating the synergy of the normal form and complex frequency in unraveling the inherent adaptive nature of power grids. We unveil a simple yet universal equation governing the collective dynamics. Remarkably, this equation is expressed solely through a matrix of complex couplings and is devoid of the network topology. These complex couplings enable a novel adaptive network formulation of power grids.

Finally, we present recent validation results of the normal form through system identification and show its accuracy in modeling a broad range of GFIs. These validations encompass laboratory measurements and simulation data. This success underscores the success of our adaptive approach in power grid modeling.

Fixation of fast oscillations to slow variables: Adaptation and Differentiation in Epigenetic Dynamics

Kunihiko Kaneko (Copenhagen University)

Friday, August 2, 14:00–14:30, Research II Lecture Hall

Abstract: Biological systems consist of different time scales: Genetic changes through evolution are slow, while epigenetic modifications are somewhat faster, protein expression dynamics have much faster time scales, and metabolic processes involve even faster time scales. In neuroscience, synaptic changes are much slower than the dynamics of neural activity. Here, these processes with different timescales interfere with each other, and often faster changes are embedded in slower variables. Biological adaptation, development, memory, and evolution are expected to take advantage of such multi-timescale dynamics. Indeed, to understand robust developmental processes, Waddington proposed the metaphorical epigenetic landscape, where the valleys characterizing the attractors of cell states are supposed to be branched by slower epigenetic changes. However, how the interplay between faster dynamics (e.g., protein expression dynamics) and faster epigenetic modification leads to such a robust differentiation process remains elusive. Here, by introducing a theoretical model with positive feedback between the faster and slower dynamics, we demonstrate that the robust differentiation process (homeorhesis) generally results as an interplay between fast oscillatory gene expression and slow epigenetic modification. Using this model, we also demonstrate "cell reprogramming": differentiated cells can regain pluripotency, i.e. the potential to differentiate, by overexpressing only four genes. We showed that global reversion to the initial unstable pluripotent state is indeed possible by overexpressing only a few genes. The generality of this mechanism was confirmed by dynamical systems analysis and simulations with random and experimentally extracted gene regulatory networks. Robust differentiation and cellular reprogramming are thus universally explained. Last, we show that fast chaotic dynamics with slow fixation can allow for adaptation to a variety of environment.

- [1] Homeorhesis in Waddington's landscape by epigenetic feedback regulation, Y Matsushita, K Kaneko, Physical Review Research 2 (2020), 023083
- [2] Dynamical systems theory of cellular reprogramming, Y Matsushita, TS Hatakeyama, K Kaneko, Physical Review Research 4 (2022), L022008
- [3] Generic optimization by fast chaotic exploration and slow feedback fixation, Y. Matsushita, K. Kaneko Physical Review Research, 5(2023), 023017.

Recurrent chaotic clustering and slow chaos in adaptive networks

Serhiy Yanchuk (University College Cork), Matheus Rolim Sales, Jürgen Kurths

Friday, August 2, 14:30–15:00, Research II Lecture Hall

Abstract: Adaptive dynamical networks are network systems in which the structure co-evolves and interacts with the dynamical state of the nodes. We study an adaptive dynamical network in which the structure changes on a slower time scale relative to the fast dynamics of the nodes. We identify a phenomenon we refer to as recurrent adaptive chaotic clustering, in which chaos is observed on a slow time scale, while the fast time scale exhibits regular dynamics. Such slow chaos is further characterized by long (relative to the fast time scale) regimes of frequency clusters or frequency-synchronized dynamics, interrupted by fast jumps between these regimes. We also determine parameter values where the time intervals between jumps are chaotic and show that such a state is robust to changes in parameters and initial conditions.

Minisymposium on Analysis and numerics of nonlinear dynamical systems

Computing Self-similar solutions to NLS equations: A computational/bifurcation analysis approach

Efstathios Charalampidis (California Polytechnic State University)

Wednesday, July 31, 16:00–16:30, Research I Lecture Hall

Abstract: In this talk, we will focus on the self-similar collapse of the (1+1)-dimensional NLS equation with general nonlinearity exponent σ . Upon performing a dynamic rescaling on the NLS, we will present a general method that is capable of identifying self-similar waveforms as steady-state solutions in the so-called "co-exploding frame" for the NLS. Then, we will bring forth bifurcation analysis techniques as well as computational methods associated with them in order to perform a spectral stability analysis of the pertinent waveforms as a function of the nonlinear exponent σ . Most importantly, conclusions will be drawn about how the spectral picture in the co-exploding/self-similar frame connects with the one in the original frame. If time permits, connections with the identification of rogue waves as self-similar patterns will be discussed as well as recent advances on the complex Ginzburg-Landau equation.

Numerical bifurcation analysis for geometric PDEs

Hannes Uecker (Universität Oldenburg), Alexander Meiners (Universität Oldenburg)

Wednesday, July 31, 16:30–17:00, Research I Lecture Hall

Abstract: We describe some differential geometric bifurcation problems and their treatment in the Matlab continuation and bifurcation toolbox pde2path. The continuation steps consist in solving the PDEs for the normal displacement of an immersed surface in \mathbb{R}^3 , with bifurcation detection and possible subsequent branch switching. The examples include minimal surfaces such as Enneper's surface and a Schwarz-P-family, some non-zero constant mean curvature surfaces such as liquid bridges, and some 4th order biomembrane models. In all of these we find interesting symmetry-breaking bifurcations. A few of these are (semi)analytically known and hence used as benchmarks.

Variational formalism for the Klein-Gordon oscillon

Igor Barashenkov (University of Cape Town)

Wednesday, July 31, 17:00–17:30, Research I Lecture Hall

Abstract: The variational method employing the amplitude and width as collective coordinates of the Klein-Gordon oscillon leads to a dynamical system with unstable periodic orbits that blow up when perturbed. We propose a multiscale variational approach free from the blow-up singularities. An essential feature of the proposed trial function is the inclusion of the third collective variable: a correction for the nonuniform phase growth. In addition to determining the parameters of the oscillon, our approach detects the onset of its instability.

Time discretization makes integrable systems chaotic

Sergej Flach (Institute for Basic Science (IBS))

Wednesday, July 31, 17:30–18:00, Research I Lecture Hall

Abstract: Numerical simulation of integrable Hamiltonian dynamics using time discretization destroys integrability and induces dynamical chaos. We use various symplectic integrators and the Toda chain as a testing ground. The largest Lyapunov exponent is nonzero and depends on the chosen time step [1]. We extend the analysis to the classical and integrable BCS model of superconductivity. The Lyapunov spectrum scaling with the time step is used to characterize the universality class of the thermalization slowing down upon reducing the step value. We briefly discuss the consequences for quantum computing algorithms using quantum hardware platforms and the concept of trotterization.

- [1] Carlo Danieli, Emil A. Yuzbashyan, Boris L. Altshuler, Aniket Patra and Sergej Flach, Dynamical chaos in the integrable Toda chain induced by time discretization, Chaos 34 033107 (2024)

Collisions of solitons in non-integrable scalar theories

Yakov Shnir (HWK Delmenhorst)

Thursday, August 1, 11:00–11:30, Research I Lecture Hall

Abstract: We study kink-antikink collisions in a model which interpolates smoothly between the completely integrable sine-Gordon theory, the ϕ^4 model, and a ϕ^6 -like model with three degenerate vacua. We find a rich variety of behaviours, including integrability breaking, resonance windows with increasingly irregular patterns, and new types of windows near the ϕ^6 -like regime. False vacua, extra kink modes and kink fragmentation play important roles in the explanations of these phenomena. Our numerical studies are backed up by detailed analytical considerations.

Peakon formation in a Stochastic Camassa-Holm equation

Darryl Holm (Imperial College London)

Thursday, August 1, 11:30–12:00, Research I Lecture Hall

Abstract: A famous feature of the Camassa–Holm equation is its admission of peaked soliton solutions known as peakons. We investigate this equation under the influence of stochastic transport. Noting that peakons are weak solutions of the equation, we present a finite-element discretization for it, which we use to explore the formation of peakons. Our simulations using this discretization reveal that peakons can still form in the presence of stochastic perturbations. Peakons can emerge both through wave breaking, as the slope turns vertical, and without wave breaking as the inflection points of the velocity profile rise to reach the summit. <https://doi.org/10.1098/rspa.2021.0224>

Joint work with Tom Bendall (UK Met Office) and Colin Cotter (Imperial Mathematics)

Hopfion Vortices in Anisotropic Heisenberg Magnetic Systems

Avadh Saxena (Los Alamos National Lab)

Thursday, August 1, 12:00–12:30, Research I Lecture Hall

Abstract: Knotted topological defects called hopfions have been recently observed in a variety of materials including chiral magnets, nematic liquid crystals and even in ferroelectrics as well as studied in other physical contexts such as Bose-Einstein condensates. These topological objects can be modeled using the relevant physical variable, e.g., magnetization, polarization or the director field. Here, we find exact soliton solutions for the unit spin vector field of an inhomogeneous, anisotropic three dimensional (3D) Heisenberg ferromagnet and calculate the corresponding Hopf invariant H analytically and obtain an integer, demonstrating that these solitons are indeed hopfion vortices [1]. The invariant H is a product of two integers, the first being the usual winding number of a skyrmion in two dimensions, while the second encodes the periodicity (and twist) in the third dimension. We also study the underlying geometry of H , by mapping the 3D unit vector field to tangent vectors of three appropriately defined space curves [2]. Our analysis shows that a certain intrinsic twist is necessary to yield a nontrivial topological invariant (linking number). Finally, we focus on the formation energy and stability of hopfion vortices to study their properties for potential applications.

[1] R. Balakrishnan, R. Dandoloff, and A. Saxena, Phys. Lett. A 480 128975 (2023).

[2] R. Balakrishnan, R. Dandoloff, and A. Saxena, Phys. Lett. A 493 129261 (2024).

One-dimensional hydrodynamic PDE model of turbulent flow with the enstrophy cascade

Takashi Sakajo (Kyoto University)

Thursday, August 1, 13:30–14:00, Research I Lecture Hall

Abstract: We propose a one-dimensional partial differential equation model generating turbulent flows. This model is based on the generalized Constantin-Lax-Majda-DeGregorio (gCLMG) equation with viscous dissipation subject to large-scale forcing. We have established the existence of a unique solution locally for the inviscid case and globally for the viscous case. We here consider two kinds of forcing functions. First, we choose a stochastic random forcing. Second, we regard the forcing function as a random variable defined

on a certain probability space. For the stochastic forcing case, we show numerically that the model exhibits the cascade of enstrophy, which is an inviscid conserved quantity. We also find the broad energy spectrum similar to the dimensional-analysis prediction and self-similarity in the dynamical system structure. For the random variable case, we also confirm the same scaling laws as those in the stochastic case, while we can observe structure functions in detail. The key to understanding the statistical properties is a steady solution with singular peaks of the equation under a deterministic forcing. The equation can be utilized as a good candidate for one-dimensional turbulent models that is amenable to mathematical and numerical analysis.

Stability of solitons in the parametrically driven and damped nonlinear Dirac equation

Niurka R Quintero (University of Seville)

Thursday, August 1, 14:00–14:30, Research I Lecture Hall

Abstract: Title: Stability of solitons in the parametrically driven and damped nonlinear Dirac equation Authors: Bernardo Sánchez-Rey, David Mellado-Alcedo, Renato Alvarez-Nodarse, and Niurka R. Quintero(*) Abstract: The stabilization of damped nonlinear Dirac (NLD) solitons is addressed by considering a time-periodic parametric force that supplies the required balance between energy losses and gains. We obtain two exact stationary soliton solutions and derive the sufficient conditions for their existence. Their stability is studied by linearizing the NLD equation around these solutions and by solving the corresponding eigenvalue problem. The prediction on stability of stationary waves are fully confirmed by numerical simulations of the parametrically driven, damped NLD equation.

A very complicated route for wavelength change in vertical convection

Zheng Zheng (École Polytechnique Fédérale de Lausanne (EPFL)), **Laurette Tuckerman** (PMMH (CNRS, ESPCI - PSL, Sorbonne)), Tobias Schneider (École Polytechnique Fédérale de Lausanne (EPFL))

Thursday, August 1, 14:30–15:00, Research I Lecture Hall

Abstract: Convection in a vertical channel subjected to a horizontal temperature gradient is investigated numerically. Previous simulations (Gao, Sergent, Podvin, Xin, Le Quere, Tuckerman, Phys. Rev. E 88, 023010, 2013) reveal a variety of behaviors: steady, time-periodic and chaotic. We extend this work by constructing stable and unstable branches of the equilibria and periodic orbits of the underlying Oberbeck–Boussinesq equations by parametric continuation.

In a narrow domain of aspect ratio ten, the phenomenology is dominated by the competition between three and four co-rotating rolls via a mixed-mode or connector branch. This seemingly simple scenario turns out to be mediated by several bifurcations, which are in turn dictated by the symmetries of the primary three- and four-roll branches. D4 symmetry requires that eight mixed-mode branches bifurcate simultaneously from the four-roll branch, of two qualitatively different types. In the typical case of pattern formation on a square, these two types are associated with the aligned (vertical or horizontal) vs. diagonal orientations. In our case of wavenumber adjustment of a four-roll state, the two types of branches are associated with two routes to roll-number-reduction: merging of two rolls vs. disappearance of a roll. Another feature particular to our case is that each branch of the roll-merging type merges smoothly with a branch of the roll-disappearance type. This merging occurs at a transcritical bifurcation with the three-roll primary branch, a feature of the breaking of D3 symmetry.

We also observe other manifestations of the competition between three and four rolls, in which the symmetry in time or in the transverse direction is broken, leading to limit cycles or wavy rolls, respectively.

Numerical investigation of self-similar blow-up solutions in the generalized Korteweg de Vries equation

Michail Kavousanakis (National Technical University of Athens)

Thursday, August 1, 15:00–15:30, Research I Lecture Hall

Abstract: In this study, we revisit the generalized Korteweg-de Vries equation, focusing on its parametric behavior with respect to the nonlinearity exponent. Our aim is to investigate the occurrence of blow-up solutions, specifically as traveling waveforms lose stability beyond a critical parameter value. Moreover, we

systematically analyze the linearization spectrum not only for the traveling states but also for the emergent collapsing waveforms within the co-exploding frame, where these waveforms are observed/computed as stationary states. This spectrum reveals interesting patterns of negative eigenvalues, which we thoroughly characterize. Finally, we delve into the dynamics of unstable solitary waves beyond the critical parameter value within the co-exploding frame.

Some solutions and their stability in a nonlinear Dirac equation

Justin Cole

Friday, August 2, 11:00–11:30, Research I Lecture Hall

Abstract: Motivated by nonlinear Floquet topological insulators in photonic waveguide arrays, a nonlinear Dirac equation is obtained. This equation represents an averaged model for envelopes whose spectral values reside near the Dirac point. In the linear limit, spiral waves are observed and analytically described. When nonlinearity is strong, collapse can occur. When the effects of diffraction and self-focusing nonlinearity balance, solitons are found to exist. A family of band gap solitons are computed.

Nonlinear dynamics and control of ferrofluid interfaces

Ivan Christov (Purdue University), **Zongxin Yu** (Northwestern University)

Friday, August 2, 11:30–12:00, Research I Lecture Hall

Abstract: Magnetic materials' dynamics can be "remotely" controlled with external magnetic fields. This property makes magnetic fluids and soft solids a popular platform for soft and amorphous "robots" that can also be physiologically compatible and operate in conditions that traditional machines cannot. Inspired by such systems, we have developed a fundamental understanding of the nonlinear dynamics of ferrofluid interfaces. Specifically, we show that a suitably tailored magnetic field configuration, comprised of a radial and an azimuthal external magnetic field, can manipulate a confined ferrofluid droplet into a spinning "gear" via interfacial instability [1]. A weakly nonlinear theory provides insight into tuning the initial unstable growth and yields an accurate prediction of the rotation speed of the spinning gear. Next, via a center manifold reduction, we show the geometric equivalence between a two-harmonic-mode coupled system arising from the weakly nonlinear analysis of the ferrofluid interface and a Hopf bifurcation [2]. Inspired by the well-known bifurcation delay phenomenon of time-dependent Hopf bifurcations, we design a slowly time-varying magnetic field to control the spinning gear's timing and emergence. Finally, we derive a Landau (amplitude) equation as a simple yet predictive reduced model of the nonlinear dynamics of the ferrofluid interface.

This work is supported by the US National Science Foundation under grant CMMI-2029540.

[1] Yu & Christov, Physical Review E 103 (2021) 013103

[2] Yu & Christov, Physical Review E 107 (2023) 055102

Rogue waves in driven dissipative systems

Edgar Knobloch (University of California-Berkeley)

Friday, August 2, 12:00–12:30, Research I Lecture Hall

Abstract: Two examples of forced dissipative systems exhibiting rogue wave-like behavior will be exhibited. The first consists of a set of nonlinear oscillators on a ring with nearest neighbor coupling while the second arises in Meinhardt's model of sidebranching in systems undergoing ramification such as occurs in the growth of the vascular system or the lung. The former is an ODE system. When run for parameter values in which individual oscillators oscillate periodically the spatial coupling between them can generate an intermittent but very large spatially localized response at different locations on the ring. The latter is a PDE system that admits a parameter regime in which large amplitude spikes grow intermittently from an unstable homogeneous state before collapsing back to this state and regrowing elsewhere. Attempts at predicting an impending rogue wave, and the spatial and temporal distributions of the resulting events will be described. Numerical evidence that the rogue wave generation process is a memoryless Poisson process will be presented.

This is joint work with P. Subramanian (Auckland), P. Kevrekidis (U. Mass.) and A. Yochelis (Be'ersheva).

Bose-Einstein condensate in quasi-periodic potential. Multimode dynamics.

Vladimir Konotop (University of Lisbon), Henrique Prates (University of Lisbon)

Friday, August 2, 13:30–14:00, Research I Lecture Hall

Abstract: We consider a one-dimensional Bose-Einstein condensate in a quasi-periodic potential formed by the superposition of two optical lattices with incommensurate periods and comparable amplitudes. All atomic states with energies below the mobility edge become localized, leading to the emergence of unusual dynamical regimes. Such system enables the implementation of a Josephson junction with pairs, quartets, or more pairs of coupled modes (located in different spatial positions) [1], as well as oscillatory dynamics in the presence of an additional weak lattice tilt [2]. The latter dynamics represents Bloch-Landau-Zener oscillations occurring in a non-periodic system. The description is based on the periodic approximants of the quasi-periodic potential. Both linear and nonlinear dynamical regimes are considered.

[1] H. C. Prates, D. A. Zezyulin, and V. V. Konotop, Phys. Rev. Res. 4, 033219 (2022).

[2] H. C. Prates and V. V. Konotop, Phys. Rev. Res. (2024) (to appear)

Instabilities and bifurcations of static black holes

Jutta Kunz (University of Oldenburg), Burkhard Kleihaus (Universität Oldenburg), Jose Luis Blazquez-Salcedo (Universidad Complutense de Madrid (UCM))

Friday, August 2, 14:00–14:30, Research I Lecture Hall

Abstract: Schwarzschild black holes can become unstable with respect to growing scalar hair, when source terms in the Einstein-Klein-Gordon equation act like a tachyonic effective mass. These instabilities are associated with bifurcating branches of new black holes solutions, that carry nontrivial scalar fields. Here the pattern of these branches and their stability is addressed. Interestingly, it may happen, that scalarized branches of spherically symmetric solutions possess no radial instabilities but only higher multipole instabilities.

Time-dependent localized patterns in a predator-prey model

Fahad Al Saadi (Military Technological College), Edgar Knobloch, Hannes Uecker, Mark Nelson

Friday, August 2, 14:30–15:00, Research I Lecture Hall

Abstract: Numerical continuation is used to compute solution branches in a two-component reaction-diffusion model of Leslie–Gower type. Two regimes are studied in detail. In the first, the homogeneous state loses stability to supercritical spatially uniform oscillations, followed by a subcritical steady state bifurcation of Turing type. The latter leads to spatially localized states embedded in an oscillating background that bifurcate from snaking branches of localized steady states. Using two-parameter continuation we uncover a novel mechanism whereby disconnected segments of oscillatory states zip up into a continuous snaking branch of time-periodic localized states, some of which are stable. In the second, the homogeneous state loses stability to supercritical Turing patterns, but steady spatially localized states embedded either in the homogeneous state or in a small amplitude Turing state are nevertheless present. We show that such behavior is possible when sideband Turing states are strongly subcritical and explain why this is so in the present model. In both cases the observed behavior differs significantly from that expected on the basis of a supercritical primary bifurcation.

Minisymposium on Current topics in delay equations

Bifurcations of Limit Cycles in DDEs: from Theory to Software

Bram Lentjes (Hasselt University), Maikel M. Bosscheart (Hasselt University), Len Spek (University of Twente), Yuri A. Kuznetsov (Utrecht University)

Thursday, August 1, 11:00–11:30, IRC Seminar Room 2

Abstract: Explicit formulas to compute critical normal form coefficients for all codimension 1 bifurcations of limit cycles in finite-dimensional ODEs were derived by (Kuznetsov et al., 2005), and implemented in the well-known software continuation package MATCONT. These formulas are useful to detect codimension 2 points and allow us to distinguish between sub- and supercritical bifurcations. The question arises if this construction can be lifted to infinite-dimensional DDEs. In this talk, we generalize this construction towards DDEs in four steps. First, we prove the existence of a smooth periodic finite-dimensional center manifold near a nonhyperbolic cycle. Second, we prove the existence of periodic normal forms by generalizing the results from (Iooss, 1988). Third, we derive formulas for the periodic (adjoint) (generalized) eigenfunctions via the characteristic operator, a generalization of the well-known characteristic matrix (Kaashoek and Verduyn Lunel, 1992). Last, we derive the explicit formulas for the critical normal coefficients of all codimension 1 bifurcations of limit cycles in DDEs and provide numerical examples. This is joint work with Maikel M. Bosschaert, Len Spek and Yuri A. Kuznetsov.

Time-delayed stochastic processes in nonequilibrium statistical physics

Sarah A.M. Loos (Cambridge University)

Thursday, August 1, 11:30–12:00, IRC Seminar Room 2

Abstract: Many approaches in statistical physics are based on the Markov assumption, although real-world complex systems often exhibit non-negligible memory effects. This is particularly true for systems that operate far from thermal equilibrium, such as molecular machines, flocks of bird, robotic systems, or systems subject to external control. In this talk, I will discuss the origins and implications of time delays that arise in nonequilibrium systems from the perspective of statistical physics and stochastic thermodynamics [1-5]. As particular examples, I will discuss time delays appearing in feedback control on mesoscale objects, where we find that time delays are always associated with entropy production, as the presence of delay itself breaks time-reversal symmetry. As a remarkable consequence, delays can yield local cooling effects [2,3]. Furthermore, time delays arising in the communication between individuals can severely impact the collective behaviour of groups of animals. We further find that time delays need to be taken into account to match the predictions of typical mathematical models of swarming with the real-world data of swarms of midges [5].

- [1] Loos, Klapp: Fokker–Planck Equations for Time-Delayed Systems via Markovian Embedding, *J. Stat. Phys.* 177, 95–118 (2019).
- [2] Loos, Hermann, Klapp: Medium Entropy Reduction and Instability in Stochastic Systems with Distributed Delay, *Entropy* 23, 696 (2021).
- [3] Loos, Klapp: Heat flow due to time-delayed feedback, *Sci. Rep.* 9, 1-11 (2019).
- [4] Holubec, Ryabov, Loos, Kroy: Equilibrium stochastic delay processes, *New Journal of Physics* 24, 023021, (2022).
- [5] Holubec, Geiss, Loos, Kroy, Cichos: Finite-size scaling at the edge of disorder in a time-delay Vicsek model, *Phys. Rev. Lett.* 127, 258001 (2021)

Strong coupling yields abrupt synchronization transitions in delay coupled oscillators

Kyle Wedgwood (University of Exeter)

Thursday, August 1, 12:00–12:30, IRC Seminar Room 2

Abstract: Coupled oscillator networks often display transitions between qualitatively different phase-locked solutions—such as phase synchrony and rotating wave solutions—following perturbation or parameter variation. In the limit of weak coupling, these transitions can be understood in terms of commonly studied phase approximations. As the coupling strength increases, however, predicting the location and criticality of transition, whether continuous or discontinuous, from the phase dynamics may depend on the order of the phase approximation—or a phase description of the network dynamics that neglects amplitudes may become impossible altogether. Here we analyze synchronization transitions and their criticality systematically for varying coupling strength in theory and experiments with coupled electrochemical oscillators. First, we analyze bifurcations analysis of synchrony and splay states in an abstract phase model and discuss conditions under which synchronization transitions with different criticalities are possible. Second, we illustrate that transitions with different criticality indeed occur in experimental systems. Third, we highlight that the

amplitude dynamics observed in the experiments can be captured in a numerical bifurcation analysis of delay-coupled oscillators. Our results showcase that reduced order phase models may miss important features that one would expect in the dynamics of the full system.

Synchronization and mode hopping in pulse-coupled oscillator networks with stochastic delays

Otti D’Huys (Maastricht University), Vladimir Klinshov (Institute of Applied Physics of the Russian Academy of Sciences)

Friday, August 2, 11:00–11:30, IRC Seminar Room 2

Abstract: Systems with pulse-like interactions include populations of biological neurons, cardiac cells, wireless networks, chemical and electronic oscillators, and many others. A common approach to studying dynamics of pulse-coupled oscillatory systems is to describe the oscillator by its phase and the interaction by the so-called phase resetting curve, which describes the effect of an incoming pulse.

Here, we consider the dynamics of small motifs of such pulsed-coupled oscillators. The connections contain a temporal delay; due to this delay multiple stable spiking solutions coexist. We apply different types of perturbations: noise acting on the oscillator, stochastically varying connection delays, and variations of the pulse magnitude. We show that these different sources of noise all induce switching between stable coexisting states, but the system shows different scaling properties for different noise sources. For a single oscillator with feedback, we develop a linear model that provides qualitative explanations for the switching dynamics due to each of the noise sources. We extend these results to motifs, and show how connectivity and network size influence the mode hopping characteristics.

Localized structures in delay-differential equations with large delay

Matthias Wolfrum (WIAS Berlin)

Friday, August 2, 11:30–12:00, IRC Seminar Room 2

Abstract: We study the dynamics of localized structures such as temporal dissipative solitons or square waves in DDEs with large delay. We show how such solutions can be treated as homoclinic solutions of an equation with an advanced argument. Based on this, we use concepts of classical homoclinic bifurcation theory to study their bifurcations and instabilities. As examples we present pulse solutions in the Morris-Lecar model with delayed feedback and square wave solutions of a DDE model for the Kerr-Gires-Tournois interferometer.

Phase reduction with and without time delay

Babette de Wolff (Vrije Universiteit Amsterdam)

Friday, August 2, 13:30–14:00, IRC Seminar Room 2

Abstract: In many real-life networks systems, it takes a significant time for signals to travel from node to node, leading to time delays in the coupling. Experiments show that coupling delays have a crucial and often counterintuitive effect on collective phenomena, including the synchronisation behaviour of coupled oscillators.

In this talk, I will introduce a phase reduction technique for delay-coupled oscillators, which gives a systematic way to derive equations for the phases of coupled oscillators. The resulting phase model is lower dimensional than the original model (in fact, it is finite dimensional while the original model is infinite dimensional), which facilitates further analysis of synchronisation phenomena.

I will first discuss the mathematical approach to phase reduction in delay-coupled oscillators, including the approach to compute higher-order terms (i.e. terms of order at least 2) in the coupling parameter. By means of an illustrative example, I will then show how including these higher-order terms yields more accurate predictions of synchronisation behaviour.

Stochastically Perturbed Mackey–Glass Type Equations: Existence of Invariant Measures

Mark van den Bosch (Leiden University), Onno van Gaans (Leiden University), Sjoerd Verduyn Lunel

(University of Utrecht)

Friday, August 2, 14:00–14:30, IRC Seminar Room 2

Abstract: In this talk, we present recent results regarding the existence of invariant probability measures for delay equations with stochastic negative feedback. Applications include Wright's equation, Nicholson's blowflies equation, and the Mackey-Glass equations.

In general, additive noise typically destroys all dynamical properties of the underlying dynamical system. Therefore, we are motivated to study a class of stochastic perturbations that preserve some of the dynamical properties of the negative feedback systems we consider. We will prove, both analytically and numerically, the existence of a (nontrivial) invariant probability distribution in a setting relevant to the applications. Throughout this talk, we will use the Mackey-Glass equations to illustrate our main results and to identify the specific challenges that occur in our analysis.

This is joint work with Onno van Gaans and Sjoerd Verduyn Lunel.

Collocation methods for periodic boundary value problems of state-dependent delay differential equations

Alessia Andò (University of Udine), Jan Sieber (University of Exeter)

Friday, August 2, 14:30–15:00, IRC Seminar Room 2

Abstract: The convergence of piecewise collocation methods for computing periodic solutions of delay differential equations (DDEs) has recently been proved in the case of constant delays. Some of the necessary conditions for the convergence concern the regularity of the right-hand side. In the case of state-dependent DDEs, however, the right-hand side suffers from an inherent lack of regularity, and only satisfies a milder definition of differentiability. We show that this does not impede the convergence analysis of collocation methods based on piecewise polynomials, similarly to the constant delay case. We further substantiate the results by means of some illustrative numerical examples of state-dependent DDEs.

Minisymposium on Cyclic dynamical systems

Boundary Influx Prevents Fixation / Extinction in Spatially Extended Stochastic Population Dynamics with Cyclic and Hierarchical Competition

Uwe C. Täuber (Virginia Polytechnic Institute and State University (Virginia Tech))

Monday, July 29, 11:00–11:30, East Hall 8

Abstract: Stochastic population dynamics in finite systems tends to ultimately terminate in an absorbing state. However, in sufficiently large spatially extended models, the time to reach species fixation or extinction often exceeds relevant biological scales, effectively allowing species coexistence. Yet tuning certain control parameters, e.g., increasing the predation rate in predator-prey systems or enhancing asymmetries in cyclic dominance models, may render coexistence states in finite systems highly vulnerable against stochastic fluctuations. Intriguingly, though, such systems displaying a finite-size fixation or extinction threshold may be efficiently stabilized through continuous influx from the system's boundaries. For example, diffusively coupling the vulnerable ecology to an adjacent region with stable species coexistence generates planar immigration front waves across the interfaces. We discuss (semi-) quantitative criteria that delineate the conditions for this remarkable boundary flow stabilization of finite-size absorbing-state instabilities in stochastic population dynamics with cyclic and hierarchical competition.

This research was supported by the U.S. National Science Foundation, Division of Mathematical Sciences under Award No. NSF DMS-2128587.

Temporal Clusters Prefer to be Equally Distributed - an example from the Yeast Cell Cycle

Todd Young (Ohio University)

Monday, July 29, 11:30–12:00, East Hall 8

Abstract: Synchrony, the phenomenon where components of a system experience events in unison, seems to be common in many systems. Relatedly, Temporal Clustering or Phase Synchrony, is where sub-groups

(or cohorts) of components synchronize among themselves, but are out of phase with other cohorts, in a cyclic periodic orbit.

In bioreactor experiments on yeast metabolic oscillations we discovered a case where a culture of yeast exhibits temporal clustering in which two groups progress through their cell cycles in anti-phase. In these experiments the cell cycle clusters and oscillations in the metabolism are seen to be tightly linked.

The discovery raises a number of mathematical questions such as: 'What accounts for the difference between a system that synchronizes and one that forms clusters?', 'What determines the number of clusters that appear?', and 'How do individual cells distribute among clusters?'

In this talk we will focus on the last question using a biologically motivated non-linear model. We observe in numerical simulations of the model that the clusters tend very strongly toward having nearly equal numbers of cells. This is consistent with the available experimental data. We study the case of two unequal clusters analytically and conclude that solutions with two unequal clusters are locally asymptotically stable in the clustered subspace, but, local asymptotic stability of unbalanced clusters in the full phase space depends delicately on details of the model. However, global dynamics reveal a more robust picture. If clusters become unbalanced, the influence of the larger cluster can radically shift the 'basins of attraction' of individual clusters, making it more likely for an unclustered cell to join a smaller cluster. We propose that this points to a general principle: systems that form temporal clusters via non-local coupling tend to form nearly equal clusters.

Stability of oscillations in the spatially extended May-Leonard model

Idan Sorin (Technion -Israel Institute of Technology), Alexander Nepomnyashchy (Technion - Israel Institute of Technology), Vladimir Volpert (Northwestern University)

Monday, July 29, 13:30–14:00, East Hall 8

Abstract: A peculiar feature of the May-Leonard model of cyclic dynamics is the existence of a family of orbitally stable periodic solutions on the border between the stability region of the stationary coexistence state and that of the attracting heteroclinic cycle. We analyze the stability of those solutions in the framework of a spatially extended model, which contains the system of three coupled PDEs of the reaction-diffusion type. The long-wave spatial modulations are considered by means of a special kind of perturbation theory. The short-wave spatial modulations of time-periodic oscillations and oscillations corresponding to the attraction to a heteroclinic cycle are studied within the piecewise-constant approximation. Some generalizations of the model are considered.

Topological features in coupled oscillatory circuits

Hildegard Meyer-Ortmanns (Constructor University), Manoj Aravind (Constructor University)

Monday, July 29, 14:00–14:30, East Hall 8

Abstract: We consider networks of coupled circuits. Each of these circuits is composed of a positive and negative feedback loop in a motif that is frequently found in genetic and neural networks. We analyze different versions of repressive couplings between these circuits. For a broad range of parameters and arrangements, we observe topological features of edge and corner states, differing from bulk states by the shape of oscillating spikes. This means that the shape of the spike is primarily determined by the location of the spiking unit within a finite geometry rather than by the dynamics of the individual unit. The location at a corner, an edge or in the bulk determines the number, type and strength of interactions to neighboring units. The qualitative features are rather stable against parameter changes.

Deciphering the interface laws of Turing foams

Erwin Frey (LMU - Ludwig Maximilian University of Munich)

Monday, July 29, 14:30–15:00, East Hall 8

Abstract: Protein pattern formation is central to the spatiotemporal self-organization of both prokaryotic and eukaryotic cells. Moreover, it is employed as a key spatial control system in the design of artificial cells. However, it remains unclear how the properties of the macroscopic, highly nonlinear reaction-diffusion

patterns can be systematically linked to the underlying reaction network [1]. Here, we show that protein patterns are generically governed by an effective interfacial tension arising from cyclic steady-state currents of attachment and detachment at the interface. Angle laws for the non-equilibrium interface junctions follow, which resemble but deviate systematically from the Neumann law in equilibrium phase separation. We furthermore recover generalized Plateau and von Neumann laws for two-dimensional liquid foams in two-dimensional mesh patterns. This leads us to introduce “Turing foams,” which show generic behavior governed by the interplay of interfacial-tension-driven dynamics and interrupted coarsening, and that we observe experimentally in the in vitro Min protein system. Our theory offers a new ansatz to find principles of macroscopic self-organization in mass-conserving systems far from equilibrium.

- [1] Halatek, J., Brauns, F. & Frey, E. Self-organization principles of intracellular pattern formation. *Philos. Trans. R. Soc. B Biol. Sci.* 373, 20170107 (2018).

Minisymposium on Dimensional reduction of chaotic dynamics

Effective dimensionality of the Fokker-Planck operator in chaos

Domenico Lippolis (Jiangsu University)

Tuesday, July 30, 11:00–11:30, IRC Seminar Room 3

Abstract: The phase space (‘state space’) of a chaotic system often features a self-similar (fractal) structure of infinite resolution. However, every system experiences noise, coming from experimental uncertainties, neglected degrees of freedom, or roundoff errors, for example. No matter how weak, noise smooths out fractals, giving the system a finite resolution. The consequences are dramatic for the computation of long-time dynamical averages, such as diffusion coefficients or escape rates, since infinite-dimensional operators describing the evolution of the system (such as Fokker-Planck or Koopman) effectively become finite matrices. This talk presents an effective dimensionality reduction technique for the noisy evolution operator in a chaotic system, based on partitioning of the phase space by means of unstable periodic orbits. A constrained Fokker-Planck equation is solved in the neighborhood of each periodic orbit, in order to determine the non-wandering intervals that serve as building blocks for an optimal partition of the state space, and the consequent discretization of the evolution operator.

Dimensionality reduction of the pinning control problem for network synchronization

Francesco Sorrentino

Tuesday, July 30, 11:30–12:00, IRC Seminar Room 3

Abstract: In this talk I review the pinning control problem for synchronization of networks of coupled oscillators. I first consider the case of connections all of the same type. Then I consider the network pinning control problem in the presence of two different types of coupling: (i) node-to-node coupling among the network nodes and (ii) input-to-node coupling from the source node to the ‘pinned nodes’. For the latter problem, I show how the stability analysis of the target synchronous solution can be decoupled into subproblems of the lowest dimension by using the techniques of simultaneous block diagonalization (SBD) of matrices.

Koopman analysis and its application in chaotic dynamics

Yueheng Lan (Beijing University of Posts and Telecomm)

Tuesday, July 30, 12:00–12:30, IRC Seminar Room 3

Abstract: How to extract dynamical features of nonlinear systems from time series? How to determine the key modes of system dynamics? How to build a reduced model with identified characters of a given system? These are fundamental problems in the study of complex systems which call for an effective analyzing framework. The theory of Koopman operators directly starts from observables and investigate evolution of functions defined in the phase space, which converts a nonlinear problem to a linear one in an infinite-dimensional functional space. Henceforth, a high-dimensional linear system could be used to approximate the original nonlinear system in certain sense if a proper functional space is selected such that

the low-frequency spatiotemporal characters are captured. In this talk, we will start from the definition and basic properties of the Koopman operator and explain the numerical calculation of its spectrum, providing several interesting applications in typical nonlinear dynamical systems.

Neural population dynamics as a window into how the brain generates movement

Juan Alvaro Gallego (Imperial College London)

Tuesday, July 30, 12:30–13:00, IRC Seminar Room 3

Abstract: Analysing the activity of recordings from hundreds of neurons during behaviour consistently uncovers low-dimensional mathematical structures that capture a large fraction of their variability. These structures or “neural manifolds” are defined by the dominant patterns of covariation across neurons. Recent studies focusing on neural manifolds and the activity within them –the “latent dynamics”– have shed light into questions about cognition, motor control, and learning that had long remained elusive.

In this talk, I will present a recent study that addresses an old question. We know that the brain of each individual from a given species is unique at the level of their cellular composition yet, these different individuals can still produce the same behaviours. What is the basis for these preserved behavioural repertoires? One possibility is that their brains each develop a different “solution” to control their similar bodies; another, is that they implement the same solution, in which case the question becomes: how does this solution look? Using a combination of recordings from several mammalian species and computational models, I will show that different individuals from the same species generate preserved latent dynamics while engaged in the same overt or covert behaviour. Moreover, these latent dynamics are behaviourally relevant: they allow building mathematical models that, trained on one individual, can be used to decode the intended or actual movement of a different individual.

The study of neural manifolds and their associated latent dynamics thus provides insights into the generation of behaviour, may enable comparative studies across groups of individuals from the same or even different species, and hold great potential to accelerate the development of neural interfaces to restore movement after neurological disease or injury.

Ray-tracing the Ulam way

David Chappell (Nottingham Trent University), Martin Richter, Gregor Tanner, Oscar Bandtlow, Wolfram Just, Julia Slipantschuk

Tuesday, July 30, 13:30–14:00, IRC Seminar Room 3

Abstract: Ray-tracing is a well established approach for modelling wave propagation at high frequencies, in which the ray trajectories are defined by a Hamiltonian system of ODEs. An approximation of the wave amplitude is then derived from estimating the density of rays in the neighbourhood of a given evaluation point. An alternative approach is to formulate the ray-tracing model directly in terms of the ray density in phase-space using the Liouville equation. The solutions may then be expressed in integral form using the Frobenius-Perron operator, which is a transfer operator transporting the ray density along the trajectories. The classical approach for discretising such operators dates back to 1960 and the work of Stanislaw Ulam. The convergence of the Ulam method has been established in some cases, typically requiring the trajectory flow map of the dynamical system to be expanding. In this talk we investigate the convergence of the Ulam method for ray tracing in convex polygonal billiards, where the dynamics are parabolic and the flow map contains jump discontinuities.

A conceptual predator-prey model with complicated dynamics

Misha Chai (Max Planck Institut für Physik Komplexer Systeme), Holger Kantz (Max Planck Institut für Physik Komplexer Systeme)

Tuesday, July 30, 14:00–14:30, IRC Seminar Room 3

Abstract: In recent decades, complicated transients in ecosystems have garnered considerable attention. However, it is rare to find a conceptual ecological model that is both theoretical and realistic while simultaneously describing dynamics in the simplest possible way. In this context, drawing on the understanding

of the logistic map, we propose a simple predator-prey model in which the prey responds to predation. Thus, the evolution of the prey can influence predator dynamics, which in turn affects prey evolution. These dynamics can lead to the continuous co-evolution of prey and predators in response to each other's adaptations. Thus displays rich dynamical complexity, such as the persistence and coexistence of population cycles and chaotic behaviors(fluctuation), the emergence of the super-long transient, and regime shifts. It can help us understand the complexity of realistic ecosystems well.

Manifold learning of Poincaré sections for dissipative chaotic flows

Evangelos Siminos (Volvo Group)

Tuesday, July 30, 14:30–15:00, IRC Seminar Room 3

Abstract: It is shown that applying manifold learning techniques to Poincaré sections of high-dimensional, strongly dissipative chaotic dynamical systems can uncover their low-dimensional organization. Manifold learning, also known as nonlinear dimensionality reduction, provides a low-dimensional embedding and intrinsic coordinates for the parametrization of data on the Poincaré section, facilitating the construction of return maps with well defined symbolic dynamics. The method is illustrated by numerical examples for the Rössler attractor and the Kuramoto-Sivashinsky equation. For the latter we present the reduction of the high-dimensional, continuous-time flow to dynamics on one- and two two-dimensional Poincaré sections, for two different values of the hyper-viscosity parameter. We show that even in the two-dimensional embedding case the attractor is still organized by one-dimensional unstable manifolds of short periodic orbits. As a consequence, the dynamics can be approximated by a map on a tree which can in turn be reduced to a trimodal map of the unit interval. In order to test the limits of the one-dimensional map approximation we apply classical kneading theory in order to systematically detect all periodic orbits of the system up to any given topological length.

Computing Chaotic Time-Averages from a Small Number of Periodic Orbits

Sam Quinn (Georgia Institute of Technology), **Joshua Pughe-Sanford** (Georgia Institute of Technology), **Teodor Balabanski** (Georgia Institute of Technology), **Roman Grigoriev** (Georgia Institute of Technology)

Tuesday, July 30, 15:00–15:30, IRC Seminar Room 3

Abstract: The invariant measure describing the long-time average behavior of a dynamical system is a member of a pathologically large dimensional space of measures. Here, we use periodic orbits–unstable time-periodic solutions of the governing equations–to intelligently select a finite-dimensional subspace. We demonstrate that fitting the invariant measure within that subspace produces accurate long-time averages, even when the subspace dimension is quite small.

Minisymposium on Dynamical systems in mathematical biology: Insights into disease modeling and mathematical analysis

Investigating normal and abnormal wound healing: from deterministic nonlocal mathematical models to machine learning

Olusegun Ekundayo Adebayo (Université de Franche Comté), **Raluca Eftimie** (Universite de franche comte), **Dumitru Trucu** (University of Dundee)

Wednesday, July 31, 11:00–11:30, East Hall 1

Abstract: The movement of cells during (normal and abnormal) wound healing results from biomechanical interactions that combine cell responses with growth factors as well as cell-cell and cell-matrix interactions (adhesion and remodelling). In this talk, we consider a non-local partial differential equation model for the interactions between fibroblasts, macrophages, and the extracellular matrix via a growth factor in the context of wound healing. We first investigate numerically the dynamics of this non-local model and the dynamics of the localised versions of this model. Since some of the numerical simulations show solutions of this class of models approaching either spatially homogeneous steady states or spatially heterogeneous states with overgrown cell densities, we investigate analytically the local in-time existence and uniqueness

of solutions for this class of non-local models using the framework of the analytic semigroups of operators. Finally, we focus on clinical images of a particular type of abnormal wound scar (the keloid scar), and propose some machine-learning models that could help us identify keloids (i.e., keloid scar patterns) from other skin disorders (either benign or malignant).

Social Models and Epidemiology

Thomas Götz (University of Koblenz)

Wednesday, July 31, 11:30–12:00, East Hall 1

Abstract: The past COVID—pandemic has shown the need for mathematical models of disease dynamics. Worldwide, researches have developed models and tried to evaluate the effect of potential counter measures like lock-down, testing or vaccinations. However, disease dynamics is not just driven by individual factors but also social aspects. Living conditions, households structures or even beliefs and media consumption play a crucial role in determining the progression of the epidemic and for the impact of counter measures. Individual opinions on the disease and influence the transmission dynamics and are influenced themselves by the prevalence of the disease in the population. In this talk we will discuss the combination of epidemiological models with social aspects based on some exemplary situations.

Investigation of the dynamics of a mathematical model including the virus concentration in the environment

Burcu Gürbüz (Johannes Gutenberg-Universität Mainz), Aytül Gökçe (Ordu University), Alan D. Rendall (Johannes Gutenberg-Universität Mainz)

Wednesday, July 31, 16:00–16:30, East Hall 1

Abstract: In this study, we focus on investigating a mathematical framework tailored to elucidate the dynamics of disease transmission within human populations. This model integrates various factors, including imperfections in vaccination, and accounts for infections originating from viral particles in the environment. We first outline the mathematical model under consideration and highlight the main features of its solutions. It is especially important to include a response mechanism to explain how virus levels influence transmission through this mechanism. This function depends on an integer $n \geq 1$, with its role pivotal in determining the presence of backward bifurcation within the system. We then observe the occurrence of backward bifurcations where, for certain parameter values, an endemic steady state persists despite a basic reproduction ratio R_0 being less than one. Additionally, we demonstrate that multiple endemic steady states can coexist under these circumstances. Lastly, we consider potential updates to the model regarding the impact of climate change.

Critical behavior of the SIR model with mobility and disorder

Shengfeng Deng (Shaanxi Normal University), Géza Ódor (HUN-REN Centre for Energy Research)

Wednesday, July 31, 16:30–17:00, East Hall 1

Abstract: We show that the critical behavior of the 2d SIR model is altered by the mobility of individuals and a novel universality class appears. This is the consequence of breaking the duality symmetry of the model [1,2]. On the other hand quenched disorder is irrelevant for the scaling behavior, thus one can find dynamical percolation-like behavior [3,4]. We also show how the epidemic spreads by varying the graph dimension of a hierarchical modular network, which models containment measures [3].

[1] Shengfeng Deng, Geza Odor, Critical behavior of the diffusive susceptible-infected-recovered model, Phys. Rev. E 107 (2023) 014303

[2] Beatrice Nettuno et al, arXiv:2402.06505.

[3] Géza Ódor, Nonuniversal power-law dynamics of susceptible infected recovered models on hierarchical modular networks, Physical Review E 103 (2021) 062112.

[4] R. Mukhamadiarov, U. C. Täuber, Phys. Rev. E 106, (2022) 034132.

Dynamics of Time Delay Epidemic Model of Noise-Induced Hearing Loss

Karmand K. Ahmad (Soran University), **Bootan Rahman** (University of Kurdistan - Hewler), Grace O. Agaba (Benue State University)

Wednesday, July 31, 17:00–17:30, East Hall 1

Abstract: A mathematical model with time delay is considered to describe the dynamics of hearing loss resulting from noise exposure. The analysis illustrates that solutions consistently maintain positivity, and the model has at most two steady states: Noise-free steady state and Noise-endemic. The existence of stability of the Noise-free steady state and Noise-endemic have shown analytically, demonstrating that the Noise-endemic steady is stable below a critical time delay. Beyond this threshold, it becomes unstable and undergoes a Hopf bifurcation when the time delay matches the critical value. We also did a bifurcation analysis and estimated the delay needed to keep the system stable. Moreover, an assessment of parameter elasticity is performed to measure their influence on hearing loss control. Numerical bifurcation is conducted using DDE-BIFTOOL and traceDDE to explore various dynamical regimes within the model via numerical continuation for varying values of system parameters. Analytical findings are supported by numerical simulations of the model to show different types of dynamical behaviour.

Dynamics of large bacterial metabolic networks

Nicola Vassena (Leipzig University)

Wednesday, July 31, 17:30–18:00, East Hall 1

Abstract: Realistic bacterial metabolic networks comprises hundreds of species and reactions. Understanding the concentration dynamics of their metabolites is a task beyond computer algebra and numerical simulations. In this talk, I will present an approach to investigate the range dynamics of such large networks. The ingredients are a multiscale reduction method and bifurcation theory. In particular, I will discuss how to identify “leading” pattern indicating a certain bifurcation and consequent dynamical behavior.

Minisymposium on Dynamics in energy systems - from analytics to machine learning

Reducing blackout risk by segmenting large power grids with HVDC lines

Damià Gomila (IFISC (CSIC-UIB)), Benjamín Carreras (IFISC (CSIC-UIB) and Universidad Carlos III de Madrid), José Miguel Reynolds-Barredo (Universidad Carlos III de Madrid), María Martínez-Barbeito (IFISC (CSIC-UIB)), **Pere Colet** (IFISC (CSIC-UIB)), Oriol Gomis-Bellmunt (Centre d’Innovació Tecnològica en Convertidors Estàtics i Accionaments (CITCEA-UPC))

Tuesday, July 30, 11:00–11:30, Research V Seminar Room

Abstract: Large electrical transmission networks are susceptible to undergo very large blackouts due to cascading failures, with a very large associated economical cost. Segmenting large power grids using controllable lines, such as high-voltage direct-current (HVDC) lines can help in reducing the risk of blackouts. In this work we present an accurate and efficient method to determine the power that must flow through these lines in order to minimize the load shed. As an example, the method is applied to the case where the European grid is segmented, first, through the Pyrenees separating the Iberian peninsula from the rest of Europe and, second, dividing the network in approximately two halves, Eastern and Western Europe. In both cases the method is very efficient in reducing the size of blackouts involving both sides of the HVDC lines. Globally, this reduction translates in a decrease of the total blackout risk of approximately 46% and 67% respectively. We estimate also the savings due to the risk reduction and the cost of converting the necessary conventional lines to HVDC. The results show that segmentation the European grid using HVDC could be economically viable, specially the case of segmenting the Iberian peninsula, as the cost risk-reduction ratio is lower.

Multistability in electrical networks

Jim Delitroz (Hes-so Valais)

Tuesday, July 30, 11:30–12:00, Research V Seminar Room

Abstract: The dynamics of the electrical grid are governed by the well-known power flow equations, which establish the relationship between power and voltages. Power is specified at each node of the system, and can be either negative (indicating a power consumption) or positive (power generation), whereas nodal voltages are the unknowns of the system. In an alternating current network, such as the European power grid, the voltages are defined by their magnitude and phase angle. However, at high voltage levels, the magnitude is not expected to vary much, thanks to the action of voltage controlling devices. Assuming that these magnitudes are constant is a slight simplification of the power flow equations, that reduces the system to a network of coupled oscillators, akin to the widely-studied Kuramoto model. It is acknowledged that such system of highly nonlinear equations can have multiple stable equilibrium points, an undesirable feature in electrical networks due to potential interference with safe operation. Indeed, operators tend to use numerical techniques such as Newton-Raphson to solve the equations for electrical planification, but this algorithm will only find one solution. Moreover, the Newton-Raphson method is known to be highly sensitive to initialization. Hence, if multiple states are possible, the algorithm may converge to a solution that does not correspond to the actual state of the system, misleading the user and compromising the safe operation of the system.

In this talk, we will investigate the solutions to the power flow equations, by proposing a reformulation of the problem based on the network's underlying graph topology. In particular, we highlight the close connection between multistability and the cycles of the network, which lies in the crucial notion of winding number. This paradigm shift offers an intuitive geometric representation of equilibrium points in the solution space. In particular, we describe how the introduction of Ohmic losses, which are often neglected for the sake of simplicity, will affect the multistability of our dynamical model. Notably, we provide simple examples that only admit one stable solution in the lossless case, and at least two if reasonable losses are taken into account. Finally, we apply some numerical methods to find all solutions to the reformulated equations, with particular emphasis on the relevance of the Krawczyk interval method within this framework. We also provide some insights on how to implement this algorithm in a smart and efficient way, based on our knowledge of the structure of the solution space.

Understanding the synchronization stability of power grids with its community structure

Heetae Kim (Korea Institute of Energy Technology)

Tuesday, July 30, 13:30–14:00, Research V Seminar Room

Abstract: A power grid is one of the world's largest networked systems comprising power producers, power consumers, and transmission lines. Network science has been used to investigate power-grid dynamics, such as cascading failure or synchronization stability. So far, the local (or nodal) network properties have provided numerous clues to understanding the synchronization dynamics of a power grid. Now, we try to expand our understanding of synchronization stability to the community perspective. A community refers to a group of nodes in a network that are densely connected. The dense connection can enhance the interaction between power-grid nodes. However, it is not trivial whether the clustered connection will exacerbate the disturbance, reducing its stability, or promote healing, letting the system relax fast. We show that the node attributes related to constant community membership influence successful synchronization recovery. In particular, the more constant membership a node has, the more stable the node is. In addition, we also find that a group of nodes as a community—a consumer or a producer group—determines the overall distribution of partial synchronization stability. We introduce the Chilean and German power grids as case studies to show how community influences the synchronization stability of power-grid nodes.

Non-trivial synchronization dynamics of simple phase oscillators on power grid models

Timo Bröhl (Uniklinik Bonn), Max Potratzki, Thorsten Rings, Klaus Lehnertz

Tuesday, July 30, 14:00–14:30, Research V Seminar Room

Abstract: We study synchronization dynamics of Kuramoto phase oscillators coupled onto models of European and US-American power grids. Oscillators have heterogeneous natural frequencies. Depending on initial conditions, we observe the Kuramoto order parameter to exhibit either constant or periodic or non-

periodic, possibly chaotic temporal evolutions for a given coupling strength. Surprisingly, topological and spectral characteristics of the networks point to a diminished capability of the networks to support a stable synchronization dynamics. Our observations put into perspective the suitability of the long-time average of the Kuramoto order parameter as a single measure for phase ordering.

Role of asymmetry in the desynchronization of power grids

Yuriy Maistrenko (Research Centre Juelich, Germany), Patrycja Jaros (Lodz University of Technology), Roman Levchenko (Research Center Juelich, Germany), Tomasz Kapitaniak (Lodz University of Technology), Jürgen Kurths (Potsdam Institute for Climate Impact Research)

Tuesday, July 30, 14:30–15:00, Research V Seminar Room

Abstract: Dynamical stability of the synchronous regime remains a challenging problem for secure functioning of power grids. A particular complexity of the power grid dynamics is caused by the fact that the desired synchronous operating regime is generally only locally stable. In the system phase state this synchronous state multiply co-exists with many others, desynchronized states which are also stable, examples of which are given e.g. by the so-called solitary states [1]. The desired grid synchrony can be then secured only against small perturbations, but not against large impacts even applied to a single grid element or to a single connection. With such an action, the system dynamics can switch to another, desynchronized attractor. The essential difficulties of the power grid studies are also caused by intricate, often highly asymmetric architectures of the realistic grids, as for geographical and historical reasons. What is the role of the grid asymmetry for the stability? Which grids, with symmetric or asymmetric topology are more reliable and therefore, more suitable for designing future smart grids?

Based on the symmetric power grid model [1], we demonstrate that the synchronous regime can be typically destroyed by the elementary violations of the network architecture such as cutting a link between two nodes or removing a generator or a consumer. We describe the mechanism for the cascading failure in each of the damaging case considered and show that the desynchronization transition starts in such a case with the frequency deviation in the neighboring nodes causing the cascading splitting of the others more distant elements, leading eventually to the complete splitting of the generator and consumer frequencies. In the transition, the generators and consumers escape, one by one, from the operating synchronous state until the grid is fully disintegrated. The closer to the damage grid point, the earlier node is desynchronized. Our findings reveal that symmetric topology underlines prevailing grid stability, while imposing asymmetry can cause the fatal grid blackout [2].

[1] F. Hellmann, P. Schultz, P. Jaros, R. Levchenko, T. Kapitaniak, J. Kurths and Y. Maistrenko, Network-induced multistability through lossy coupling and exotic solitary states, *Nat. Commun.* 11, 592 (2020).

[2] P. Jaros, R. Levchenko, T. Kapitaniak, J. Kurths, and Y. Maistrenko. Asymmetry induces critical desynchronization of power grids. *Chaos* 33, 011104 (2023).

Improving power-grid systems via topological changes, or how self-organized criticality can help power-grids

Géza Ódor (HUN-REN Centre for Energy Research)

Tuesday, July 30, 15:00–15:30, Research V Seminar Room

Abstract: Cascade failures in power grids occur when the failure of one component or subsystem causes a chain reaction of failures in other components or subsystems, ultimately leading to a widespread blackout or outage. Controlling cascade failures on power grids is important for many reasons like economic impact, national security, public safety and even rippled effects like troubling transportation systems. Monitoring the networks on node level has been suggested by many, either controlling all nodes of a network or by subsets. We identify sensitive graph elements of the weighted European power-grids (from 2016, 2022) by two different methods. We determine bridges between communities and point out "weak" nodes by the lowest local synchronization of the swing equation. In the latter case we add bypasses of the same number as the bridges at weak nodes and we compare the synchronization, cascade failure behavior by the dynamical improvement with the purely topological changes. We also compare the results on bridge

removed networks, similar to islanding, and with the addition of links at randomly selected places. The synchronization improves the best by the bypassing, while the average cascade sizes are the lowest with bridge additions. However, for very large or small global couplings these network changes do not help, they seem to be useful near the synchronization transition region, where self-organization drives the power-grid. Thus, we provide a demonstration for the Braess' Paradox on continental sized power grid simulations and uncover the limitations of this phenomenon. We also determine the cascade size distributions and justify the power-law tails near the transition point on these grids.

[1] Phys. Rev. Res. 6 (2024), 013194

Multilayer control of synchronization and cascading failures in power grids

Simona Olmi (Institute for Complex Systems, National Research Council of Italy (ISC-CNR)), Lucia Valentina Gambuzza (University of Catania), Mattia Frasca (University of Catania)

Wednesday, July 31, 11:00–11:30, Research V Seminar Room

Abstract: In this work, we propose a control scheme for power grids subject to large perturbations that cause the failure of a node of the grid. Under such circumstances, the system may lose synchrony and, in addition, a cascade of line failures can be triggered as an effect of the flow redistribution that activates the protection mechanisms equipped on each line of the grid. To devise a control action for addressing this problem, we adopt a multi-layer network-based description of the power grid that incorporates an overflow condition to model the possibility of cascading failures. The two other layers of the structure are devoted to the control, one implements the distributed proportional control law, and the other the integral control law. To exemplify the application of our model, we study the Italian high-voltage power grid for different parameters and topologies of the control layers.

Modelling Dynamics and Control of Power Grids

Eckehard Schöll (TU Berlin)

Wednesday, July 31, 11:30–12:00, Research V Seminar Room

Abstract: This contribution focusses on modelling the dynamics and stability of power grids. Several methods of control of the dynamics and stability of power grids are introduced. Furthermore, we provide a new perspective on power grids by demonstrating that they can be viewed as a special class of adaptive networks, where the coupling weights are continuously adapted by feedback of the dynamics, and both the local dynamics and the coupling weights evolve in time as co-evolutionary processes. Such adaptive networks are very common in neural networks with synaptic plasticity. In terms of power grids, the power flow into the network nodes from other nodes represent pseudo coupling weights. This modelling approach allows one to transfer methods and results from neural networks, in particular the emergence of multifrequency clusters, which may form in a hierarchical way and destabilize the desirable completely synchronized operating state of the power grid. In this work, the relation between these two types of networks, in particular the model of Kuramoto-Sakaguchi phase oscillators with inertia (swing equation for power grids) and the model of phase oscillators with adaptivity, is used to gain insights into the dynamical properties of multifrequency clusters in power grid networks. This relation holds even for more general classes of power grid models that include voltage dynamics. Building on this relation between phase oscillators with inertia and adaptive networks, a new perspective on solitary states in power grid networks and their influence on network stability is provided and illustrated by the ultrahigh-voltage power grid of Germany.

How strong fluctuations induce tipping in power grids

Julian Fleck (Technische Universität Dresden), Georg Börner (Technische Universität Dresden), Moritz Thümler (Technische Universität Dresden), Malte Schröder (Technische Universität Dresden), Marc Timme (Technische Universität Dresden), **Seungjae Lee** (Technische Universität Dresden)

Wednesday, July 31, 12:00–12:30, Research V Seminar Room

Abstract: The collective nonlinear dynamics of complex systems is fundamental to our daily lives, whether

in biological cells, ecosystems or engineered systems such as electric power grids. In response to external driving forces these systems may undergo tipping, changes of the collective response dynamics as conditions change, often disrupting their intended functionality. Theory so far primarily addressed responses to weak driving signals, yet understanding responses to strong perturbations remains a substantial challenge. Here we demonstrate that the average response exhibits offsets that scale nonlinearly already at infinitesimally small driving amplitudes. Standard perturbation theory of arbitrarily high order is intrinsically incapable of predicting tipping points. To overcome this challenge, we propose an approximate self-consistency criterion that captures the entirety of the nonlinear response dynamics. We illustrate our novel approach for a basic one-dimensional model and highlight its application in understanding the nonlinear voltage shifts of AC power grid networks.

Assessing the Ability of Graph Neural Networks to Predict Power Outages of Cascading Failures In a Power Grid

Tobias Ohlinger (Fraunhofer-Institut für Algorithmen und Wissenschaftliches Rechnen SCAI)

Wednesday, July 31, 13:30–14:00, Research V Seminar Room

Abstract: We conducted experiments using three Graph Neural Network (GNN) models, namely the Graph Isomorphism Network with Edge features (GINE), the Topology Adaptive Graph Convolutional Network (TAG), and the Graph Attention Network (GAT). Our goal was to predict power outages in a potentially damaged power grid caused by single line failures during hurricanes. The power grid data was generated using the AC-CFM model applied to the ACTIVSg2000 synthetic Texan power grid, which consists of 2000 nodes. To simulate initial damages, we considered probabilistic scenarios of line failures based on historical wind data from 7 hurricanes or tropical storms that occurred in Texas over the past 22 years. However, our analysis comparing GNNs with multiple baselines (a model predicting nodal means, a Ridge regression, a Multi-Layer Perceptron (MLP) and a MLP extended with a Node2Vec embedding (Node2Vec+MLP)) revealed that the provided data lacks sufficient structure for GNNs to effectively learn the task. While the Graph Isomorphism Network with Edge features (GINE) showed a significant improvement ($p < 0.05$) over the two best performing baseline (Node2Vec+MLP and MLP), but the improvements fall within the standard deviations. Additionally, the performance improvement over the Node Mean was not significant ($p > 0.05$). The other two GNNs, TAG and GAT, showed no significant improvement over the baselines, except for Ridge Regression, which performed particularly poorly. Our assessment, based on common performance measures, demonstrated strikingly similar performance among Node2Vec+MLP, MLP, TAG, and GINE. Furthermore, when calculating the R2 score between model outputs, we obtained values greater than 0.98 for TAG, GINE, and MLP. This indicates that these GNNs failed to effectively utilize the graph-structured data and learned an underlying topology-dependent structure. This observation is supported by the fact that one of the provided node features exhibited a correlation of 0.59 with node labels, providing sufficient information for MLP to learn some connections between input features and node labels, which we assume were similarly learned by the GNNs.

Predicting Instability in Complex Oscillator Networks: Limitations and Potentials of Network Measures and Machine Learning

Christian Nauck

Wednesday, July 31, 14:00–14:30, Research V Seminar Room

Abstract: A central question of network science is how functional properties of systems arise from their structure. For networked dynamical systems, structure is typically quantified with network measures. A functional property that is of theoretical and practical interest for oscillatory systems is the stability of synchrony to localized perturbations. Recently, Graph Neural Networks (GNNs) have been shown to predict the probabilistic dynamic stability of power grid models successfully; at the same time, network measures have struggled to paint a clear picture. Here we collect 46 relevant network measures and find that no small subset can reliably predict stability. The performance of GNNs can only be matched by combining all network measures and nodewise machine learning. However, unlike GNNs, this approach fails to extrapolate

from network ensembles to several real power grid topologies. This suggests that correlations of network measures and function may be misleading, and that GNNs capture the causal relationship between structure and stability substantially better.

Intelligent Distributed Control of Renewable Energy Systems: From Analytics to Multi-Agent Reinforcement Learning

Qiong Huang (KIT - Karlsruhe Institute of Technology), Kenji Doya (Okinawa Institute of Science and Technology Graduate University), Benjamin Schäfer (KIT - Karlsruhe Institute of Technology)

Wednesday, July 31, 14:30–15:00, Research V Seminar Room

Abstract: The transition towards sustainable energy systems heavily relies on the efficient integration and utilization of renewable energy sources, such as solar power. However, the inherent variability and distributed nature of these sources pose significant challenges for maintaining grid stability and ensuring reliable energy supply. In this talk, I will present my research using machine learning techniques, particularly reinforcement learning (RL), to address these challenges. Initially, I will discuss my previous work on "Multi-Agent Reinforcement Learning for Distributed Solar-Battery Energy Systems," where I explored the use of reinforcement learning algorithms for adaptive control of energy storage in local batteries and energy sharing through energy grids. I will highlight the key design considerations, such as centralized and distributed control, action granularity, information sharing, and the implementation of deep Q-networks (DQN) and prioritized DQN for real-time energy exchange protocols. The simulation results demonstrated the superiority of RL agents over rule-based control and the benefits of information sharing within the community. Building upon these findings, I will also present my vision for the future of intelligent distributed control of renewable energy systems. I will discuss the potential of integrating RL into different energy sources, and how the techniques could be applied to the electricity bid markets in enabling real-time decision-making and efficient information exchange among agents.

Modeling, analysis and coordinated control of district heating systems

Juan E. Machado (Brandenburg University of Technology (BTU) Cottbus-Senftenberg)

Wednesday, July 31, 15:00–15:30, Research V Seminar Room

Abstract: The decarbonization of the energy system is essential for reducing the amount of anthropogenic emissions contributing to global warming. Nonetheless, this requires fundamental transformative changes in the way energy is produced, transported and consumed. In the heating sector, to which an important proportion of the total energy production is allocated, the current state is rather unfavorable, as about two thirds of the overall heat production is based on fossil fuels. Opportunely, modern district heating (DH) systems can substantially increase the amount of renewable energy in the general energy mix by virtue of their efficient and economically appealing heat storage capabilities. However, enabling the robust and efficient incorporation of heat sources based on renewables will require further coordination among production, storage and heat consumption entities, which goes beyond the scope of classic DH systems control paradigms.

In this talk, we will discuss our contributions towards the advancement of coordinated control of DH systems via the development of physics and graph-theoretic based DH systems modeling and the identification of dissipativity-based qualitative model properties that have shown to be instrumental for control synthesis in other energy carriers.

Note: The contents of this talk are the fruit of a long-term partnership with M. Cucuzzella (Univ. of Groningen), J. M. A. Scherpen (Univ. of Groningen) and, more recently, with J. Schiffer (BTU C-S).

Minisymposium on Dynamics of reservoir computers

Programming and Decoding the Dynamical Algorithmic Subspace of Reservoir Computers

Jason Kim (Cornell University)

Monday, July 29, 11:00–11:30, SCC Conference Hall

Abstract: Reservoir computing has emerged as a ubiquitous paradigm for performing complex temporal computations. One particularly impressive feature of reservoir computers (RCs) is inference—the ability to accurately forecast new time-series data—thereby demonstrating an understanding of the underlying equations that generated the data. How do we make exact this notion of “understanding” in the language of dynamical systems? Can we engineer this understanding to design RCs that run custom algorithms, turning RCs into real computers? In this talk, I will first demonstrate a powerful mechanism by which RCs can perform the full nonlinear inference of local bifurcations and global dynamical trajectories from only pre-bifurcated data [1], thereby indicating the ability of RCs to model the generative equations of data. Then, I will demonstrate how to analytically decompose RCs to decode this model from trained RCs, and how to simply design the interactions between RC units to program our own dynamical models and algorithms into the subspace of RCs [2]. I will also discuss some current work on decoding dynamical models from biological neural systems during mental simulation, and how to use RCs to understand the computational subspaces of brain connectomes. The eventual goal is to transform RCs from computational devices into fully-fledged analog computers.

- [1] Kim, J. Z., Lu, Z., Nozari, E., Pappas, G. J., & Bassett, D. S. (2021). Teaching recurrent neural networks to infer global temporal structure from local examples. *Nature Machine Intelligence*, 3(4), 316-323.
- [2] Kim, J. Z., & Bassett, D. S. (2023). A neural machine code and programming framework for the reservoir computer. *Nature Machine Intelligence*, 5(6), 622-630.

Reservoirs as driven dynamical systems

André Röhm (The University of Tokyo), Kazutaka Kanno (Saitama University), Kazuki Takashima (The University of Tokyo), Haruto Ishii (The University of Tokyo), Atsushi Uchida (Saitama University), Makoto Naruse (The University of Tokyo)

Monday, July 29, 11:30–12:00, SCC Conference Hall

Abstract: Reservoir computers are simple yet capable machine learning systems that enable a wide variety of applications. From the perspective of dynamical systems sciences, one of the most intriguing aspects is the ability of reservoirs to autonomously emulate a target dynamical system. Recent research has shown that not only can the reservoir learn to predict the future of a chaotic time series, but also infer the bifurcation structure, exhibit similar Lyapunov exponents and fractal dimensions, as well as infer the existence of unseen attractors [1].

In our current work [2], we obtain new results for predicting driven dynamical systems using reservoir computing. This allows us to model a wider class of systems, but also introduces additional complexity for the task at hand. No longer is it possible to reproduce the target system in any kind of "autonomous mode" as the target system is always driven. A "semi-open" or "semi-driven" mode is needed to allow a RC to model a driven system. Furthermore, as with traditional autonomous attractor reconstruction, issues such as sampling [3], multistability and large phase space distance from the training data present challenges.

Finally, we turn the reservoir computing scheme on itself: Reservoirs can also be seen as driven dynamical systems. We investigate the performance of reservoir computing with related approaches when it comes to predicting the dynamics of driven systems and other reservoirs. This has implications not just for applications using physical reservoirs, but also poses more general questions related to "the kind of computation" that we should imagine the reservoir performing.

- [1] “Model-free inference of unseen attractors: Reconstructing phase space features from a single noisy trajectory using reservoir computing,” André Röhm, Daniel J. Gauthier and Ingo Fischer, *Chaos* 31, 103127 (2021)
- [2] Under preparation
- [3] “Effect of temporal resolution on the reproduction of chaotic dynamics via reservoir computing,” Kohei Tsuchiyama, André Röhm, Takatomo Mihana, Ryoichi Horisaki and Makoto Naruse

Confabulations and consequences of multistability in a reservoir computer

Andrew Flynn (University College Cork), Andreas Amann (University College Cork)

Monday, July 29, 12:00–12:30, SCC Conference Hall

Abstract: Attractor reconstruction is one of the most popular application areas for reservoir computers (RCs), yet our understanding of the processes in which a RC fails or succeeds in this task remains limited. While it has often been shown that when a RC fails at attractor reconstruction it converges towards some other stable state (due to design factors), more recently it has been found that even when a RC is successfully trained to reconstruct an attractor from a given system there can be additional coexisting attractors in the RCs state space that do not exist in the original system; these are known as ‘untrained attractors’ (UAs). In this talk we explore the link between UAs and failed reconstruction dynamics, the reasons why UAs exist, and how such examples of multistability provides new insight to ‘multifunctionality’. Building on this, the main focus of the talk is to examine what are the consequences of multistability and influence of UAs when training RCs to reconstruct the bifurcation structure of dynamical systems. Our results shed further light on the concept of ‘confabulation’ in cognitive science, which describes our ability to generate false memories.

Extending or reducing dimensionalities of reservoir computing to improve time series predictions

Luk Fleddermann (Max Planck Institute for Dynamics and Self-Organization, Göttingen), Gerrit Wellecke (Max Planck Institute for Dynamics and Self-Organization, Göttingen), Kai-Uwe Hollborn (Max Planck Institute for Dynamics and Self-Organization, Göttingen), Ulrich Parlitz (Max Planck Institute for Dynamics and Self-Organization, Göttingen)

Monday, July 29, 13:30–14:00, SCC Conference Hall

Abstract: Chaotic time series can be predicted using read-outs of driven reservoir dynamics. In many practical applications, however, we are confronted with two problems: Firstly, often only low-dimensional time series of incomplete measurements, i.e. partial observations of the dynamic state, are available. In this case, the mean length of valid predictions can be increased by combining delayed values of input and reservoir states. Limitations and advantages of this method are discussed. Secondly, measurements of spatially extended systems are high dimensional and thus reservoir computing becomes (very) expensive. We use a combined method of linear dimensionality reduction and parallel reservoir computing to identify dominant input dimensions and avoid overly high dimensional reservoir states. This approach preserves the performance of time series prediction while significantly reducing computational costs. Further work aims to investigate whether a synergetic effect between dimensionality extension and reduction can be found to improve predictions of spatially extended chaos.

Stability of a reservoir computer during autonomous prediction

Daniel Gauthier (The Ohio State University)

Monday, July 29, 14:00–14:30, SCC Conference Hall

Abstract: Reservoir computing is a machine learning algorithm that is well suited for predicting the behavior of dynamical systems that can be trained using only observations of the dynamical system of interest. A reservoir computer is a recurrent artificial neural network that has the input and internal weights of the network instantiated randomly. Only the output weights are trained, where the output variables are linear superpositions of the internal states of the reservoir. The weight coefficients of the output layer can be found using various methods, where a popular approach is to use regularized linear regression, known as ridge regression. A reservoir computer excels at various tasks, including forecasting the behavior of chaotic dynamical systems. The reservoir can operate in autonomous mode, where the future dynamics are predicted without any external input to the reservoir. This mode-of-operation is sometimes referred to as generative artificial intelligence. For the case of chaotic systems, autonomous long-term forecasting is not possible, but the reservoir still generates behaviors that are consistent with the underlying strange attractor, sometimes referred to as learning the climate of the system.

Operating a reservoir computer in autonomous mode usually requires two distinct phases of operation. The first is a training phase, where a brief dataset of observed time series data is input to the reservoir computer, which promotes the state of the system to a higher dimension and gives the machine greater expressive power. The output weights are adjusted using a linear optimization method, such as ridge regression, where the target goal is to reproduce the input data. In the second phase, the output of the

reservoir computer is feedback to the input, forming an autonomous time-series predictor.

It is known that the various metaparameters of the reservoir computer affect its stability in the autonomous prediction mode even when the training phase gives a highly accurate model. That is, the reservoir can fail to reproduce the climate. In this talk, I will present a stability analysis of a next-generation reservoir computer [1] (an efficient representation of a traditional reservoir computer) and show how the metaparameters affect stability. Of particular interest is the size of the training dataset and the ridge-regression parameter, which helps promote the generalization of the machine-learning model. I will also discuss how feature selection methods, used to reduce the size of the machine learning-model, affect the stability. These results demonstrate how a nonlinear dynamics perspective can give insights into machine learning algorithms.

- [1] D. J. Gauthier, E. Bollt, A. Griffith, W. A. S. Barbosa, 'Next generation reservoir computing,' Nat. Commun. 12, 5564 (2021)

Minimalist Reservoir Computing: Implementation, Dynamics, and Future Directions

Miguel C. Soriano (IFISC (CSIC-UIB))

Monday, July 29, 14:30–15:00, SCC Conference Hall

Abstract: Reservoir computing (RC) offers a computationally efficient framework for addressing time-dependent problems. While conventional RC architectures rely on intricate networks of interconnected nodes, this work explores the potential of a minimalist physical RC implementation. We present a design centered around a single nonlinear node integrated within a delayed feedback loop. This system is realized through an analog electronic implementation, enabling seamless interaction with digital inputs and outputs and the possibility to operate the system in an auto-regressive mode. Our approach provides a uniquely controlled environment to investigate the intricate nonlinear dynamics central to RC systems. By precisely manipulating system parameters, we strive to advance theoretical understanding of the link between reservoir dynamics and computational capability. Furthermore, we illustrate how the recurrent connections of typical neural networks can be time-multiplexed through this feedback architecture, offering a resource-efficient implementation strategy. Finally, we explore the potential of modulated delays to fold deep RC architectures in time. This could unlock enhanced computational power within a remarkably compact physical footprint. Our research explores the boundaries of RC and offers new insights into the nature of recurrent computation.

- [1] L. Appeltant, M. C. Soriano, G. Van der Sande, J. Danckaert, S. Massar, J. Dambre, B. Schrauwen, C. R. Mirasso, and I. Fischer, "Information processing using a single dynamical node as complex system", Nature Communications 2, 468 (2011).
[2] B. Vettelschoss, A. Röhm and M. C. Soriano, "Information Processing Capacity of a Single-Node Reservoir Computer: An Experimental Evaluation", IEEE TNNLS 33, 2714-2725 (2022).

Exploiting chaotic dynamics as a computational resource

Kohei Nakajima

Monday, July 29, 15:00–15:30, SCC Conference Hall

Abstract: Chaos exists ubiquitously in nature, prevalent across domains, from the intricacies of atmospheric dynamics to fluid mechanics. This is also evident in engineered systems, from the simple double-pendulum system to complex neuromorphic devices. Thus, chaos has long been a source of inspiration for researchers who aim to propose a novel method for computation. However, strategies for effectively exploiting chaotic dynamics in information processing remain largely elusive. Complex dynamics of chaos may have rich expressiveness, but they simultaneously avoid reproducible responses against identical inputs, which is characterized by sensitivity to state perturbations. In this talk, based on the reservoir computing framework, we will present several successful scenarios to exploit chaotic dynamics for our computational purposes. We expect that our proposed scenarios will pave the way toward using a wide variety of physical systems that have been left behind in nature for information processing.

Statistics and Dynamics of Attractor Embeddings in Reservoir Computing

Louis Pecora (University of Maryland), Thomas Carroll (Retired)

Tuesday, July 30, 11:00–11:30, SCC Conference Hall

Abstract: A recent branch of AI or Neural Networks that can handle time-varying signals often in real time has emerged as a new direction for signal analysis. These dynamical systems are usually referred to as reservoir computers. A central question in the operation of these systems is why a reservoir computer (RC), driven by only one time series from a driving or source system of many time-dependent components, can be trained to recreate all dynamical time series signals from the drive leads to the idea that the RC must be internally recreating all the drive dynamics or attractor. In addition, there have been some speculations that RCs may be a fundamental type of system that describes how neuronal networks in biology process sensory input. This has led to the possibility that the RC is creating an embedding (in the mathematical sense) of the drive attractor in the RC dynamics. There have been some mathematical advances that move that argument closer to a general theorem. However, for RCs constructed from actual physical systems like interacting lasers or analog circuits or possibly actual neuronal networks, the RC dynamics may not be known well or known at all. And many of the existing embedding theorems have restrictive assumptions on the dynamics. We first show that the best way to analyze RC behavior is to first treat it properly like a dynamical system, which it is. This will lead to some conflict with existing ideas about RCs, but also a clarification of those ideas. Secondly, in the absence of complete theories on RCs and attractor embeddings, we show several ways to analyze the RC behavior to help understand what underlying processes are in place, especially regarding if there are good embeddings of the drive system in the RC. We show that statistics we developed for other uses can help test for homeomorphisms and diffeomorphisms between a drive system and the RC by using the time series from both systems. These statistics are called, respectively, the continuity statistic and the differentiability statistic and they are modeled on the mathematical definition of a continuous and/or differentiable function. We show the interplay of dynamical quantities (e.g. Lyapunov exponents, Kaplan-Yorke dimensions, generalized synchronization, etc.) and embeddings as exposed by the continuity statistic and other statistics based on ideas from nonlinear dynamical systems theory. These viewpoints and results lead to a clarification of various currently vague concepts about RCs, such as fading memory, stability, and types of dynamics that are useful.

Exploiting timescales and bifurcation structures of driven laser systems for efficient reservoir computing

Kathy Lüdge (Technische Universität Ilmenau)

Tuesday, July 30, 11:30–12:00, SCC Conference Hall

Abstract: Driven laser systems with delayed self-coupling are widely explored candidates for photonic reservoir computing (RC). The latter is a machine learning scheme where only one linear regression step at the readout layer is needed while the reservoir itself remains unchanged during training. RC can thus be easily implemented in various hardware systems [1]. It is especially suited for solving complex time-series prediction tasks due to the memory provided by the finite internal timescales of the dynamical system used as the reservoir. The specific RC prediction performance, however, critically depends on the nonlinear system response to the external perturbation by the data, which is strongly linked to the underlying bifurcation structure, and on the specific memory requirements of the chosen task [2].

In this contribution, different ways for exploiting the internal dynamics of the physical RC system, in our case the charge carrier dynamics within the laser, will be described and its impact on the performance will be discussed. On the one hand, a detailed bifurcation analysis of the driven RC system is shown to be a crucial ingredient while, on the other hand, also the specific task requirements and the scheme used for injecting the data have to be taken into account. While the strong task dependence inherent to RC prevents to find operation conditions for a universal reservoir, there are possibilities for external memory augmentation via external delays that provide easily accessible and very effective tuning parameters [3,4].

- [1] M. Cucchi, S. Abreu, G. Ciccone, D. Brunner and H. Kleemann, Hands-on reservoir computing: a tutorial for practical implementation, *Neuromorph. Comput. Eng.* 2, 032002 (2022).

- [2] K. Lüdge and L. Jaurigue, Deriving task specific performance from the information processing capacity of a reservoir computer, T. Hülser, F. Köster, *Nanophotonics* 12, 937 (2023).
- [3] T. Hülser, F. Köster, L. Jaurigue and K. Lüdge, Role of delay-times in delay-based Photonic Reservoir Computing, *Opt. Mater. Express* 12, 1214 (2022).
- [4] L. Jaurigue and K. Lüdge, Reducing reservoir computer hyperparameter dependence by external timescale tailoring, *Neuromorph. Comput. Eng.* 4, 014001 (2024).

Ubiquitous Analogue Reservoir Computing: Modelling, Optimising, and Controlling Dynamics-Driven Hardware AI

Giulia Marcucci (University of Glasgow, Scotland, UK)

Tuesday, July 30, 12:00–12:30, SCC Conference Hall

Abstract: This study explores the modelling, optimisation, and control of Analog Reservoir Computing (ARC) systems to advance energy-efficient hardware AI. This research aims to establish innovative ARC frameworks by utilising complex wave propagation within diverse physical domains – nonlinear, random, and multimodal photonics, hydrodynamics, visual brain-computer interfaces, and quantum optics.

Neuromorphic computing (NMC) is a prominent field dedicated to developing analogue and unconventional computers, with the primary objective of creating machines based on brain-like algorithms, enabling them to possess unique problem-solving capabilities observed in the human brain.

Achieving this goal requires integrating state-of-the-art software architectures and innovative hardware paradigms, which continually interact and advance. Complex systems, which exist in a delicate balance between order and chaos, have proven to be highly suitable for implementing NMC hardware. Such systems demonstrate superior learning capabilities compared to excessively rigid or overly chaotic ones.

In NMC, the line between software and hardware development blurs. From an algorithmic standpoint, the two most significant architectures in NMC are spiking neural networks (SNNs) and reservoir computing (RC). SNNs rely on time-dependent synapses, where neurons transmit information only when they surpass the activation potential threshold. On the other hand, RC represents a specific type of recurrent neural network in which training occurs solely at the readout stage.

ARC represents the in-hardware implementation of modifications of echo state networks (ESNs), i.e. a widespread RC paradigm, where encoding, reservoir, and decoding are actual physical systems. In this presentation, I will briefly overview my recent developments in ARC and discuss future directions for optimising its learning performance. Specifically, I will report on the differences and similarities of ARC systems exploiting:

- Visual BCIs (encoding via frequency multiplexing, reservoir through steady-state visual evoked potentials, decoding via EEG) [1];
- Quantum optics (encoding using linearly polarised Fock states, reservoir through BosonSampling, decoding via photon-number-resolving detector) [2];
- Multimode fibres (encoding via spatial light modulator (SLM), reservoir with gain-controlled multimode fibres, decoding via CCD camera) [3];
- Nonlinear optics (encoding with SLM, reservoir via self-focusing Kerr effect, decoding via CCD camera) [4];
- Free-space optical propagation (encoding via SLM, reservoir with random phase mask, decoding via CCD camera) [5];
- Nonlinear acoustics (encoding via low-amplitude waves, reservoir through solitary wave collision in shallow water, decoding via imaging in the collision region) [6].

My first key contribution to ARC emphasises the significance of complexity in the holistic encoding-reservoir-decoding system [4]. Following this route, I will detail strategies to optimise information encoding

in ARC using evolutionary algorithms [7] and control reservoir nonlinearity by spatially modulating the gain profile in a multimode fibre [3].

- [1] Wang, “Human-Centred Physical Neuromorphics with Visual Brain-Computer Interfaces”, under review (2024).
- [2] Nerenberg, “Photon Number-Resolving Quantum Reservoir Computing”, ArXiv:2402.06339, (2024).
- [3] GM, “Optimal learning in reservoir computing systems based on gain-controlled multimode fibres”, under review (2024).
- [4] GM, Phys. Rev. Lett. 125, 093910 (2020).
- [5] Pierangeli, Photon. Res. 9, 1446 (2021).
- [6] GM, Phys. Fluids 35, 071703 (2023).
- [7] Pierro, Proc. GECCO 2023, 202 (2023).

Minimal reservoir computing

Christoph R ath (DLR Institute for AI Safety and Security)

Tuesday, July 30, 12:30–13:00, SCC Conference Hall

Abstract: Reservoir computers (RC) are machine learning algorithms for predicting complex, nonlinear systems. The traditional setup of RC uses random matrices to define the underlying recurrent neural network and the input layer that transforms the input data to input signals of each node in the reservoir. Thus, a large number of random weights are to be chosen via a set of optimized hyperparameters (e.g. spectral radius of the reservoir). Here, we show that a few simple modifications to the traditional RC-architecture, which eliminate randomness and minimize computational resources, lead to significant and robust improvements in short- and long-term predictive performance compared to similar models, while requiring minimal amount of training data. Specifically, we first introduce block-diagonal reservoirs, which implies that a reservoir can be composed of multiple smaller reservoirs, each with its own dynamics. Furthermore, we take out the randomness of the reservoir by using matrices of ones for the individual blocks. This breaks with the widespread interpretation of the reservoir as a single network [1].

In a further step, we also omit the nonlinear activation function in the nodes of the reservoir. The nonlinearity is only introduced by also taking higher powers of the reservoir response. Thus, this new architecture opens new avenues to explainable and interpretable reservoir computing. For certain parameterizations, we find that the predictions are accurate for more than 10 Lyapunov times and that ordinary least squares regression directly on the embedded data can predict the long-term climate of chaotic systems [2]. We benchmark our results against those obtained with “normal” reservoir computing, next generation reservoir computing (NG RC) and SINDy. We find better and more stable predictions for minimal reservoir computing.

Finally, we discovered that for certain choices of hyperparameters the prediction fails. With only a few parameters, the phase transition between various parameterizations can be analyzed to comprehend the reasons behind the success of a prediction. We find that a crucial parameter is the size of the boxes in the block-diagonal reservoir [3]. Detailed analyses of the reconstructed, underlying equations give further insights into structural prerequisites for successful predictions.

- [1] H. Ma et al., Chaos, 33, 063130 (2023)
- [2] H. Ma et al., Sci. Rep., 13, 12970 (2023)
- [3] H. Ma, PhD thesis, LMU (2024)

Minisymposium on Energy transfers in atmosphere and ocean

Heat transport in rotating Rayleigh-Benard convection

Roland Welter (Universit t Hamburg)

Thursday, August 1, 13:30–14:00, East Hall 8

Abstract: In the study of climate, general circulation models aim to accurately represent the motions of the atmosphere and ocean using equations from fluid dynamics. However, vertical accelerations are often set equal to zero in primitive equation models. This assumption is partially justified from a physical viewpoint, since the vertical accelerations should be small compared to more dominant forces (gravity, etc) which are

in an approximate hydrostatic balance. Furthermore, this yields a significant mathematical benefit since the well-posedness theory for such equations is then much more satisfactory. On the other hand, such an assumption cannot be entirely physically accurate, and hence often in the climate community additional terms are included to account for the discrepancy. This procedure is known as parameterization, and there is not a consensus about a correct or optimal way to parameterize convection.

In this presentation, I will present recent results which develop a mathematically rigorous framework for studying vertical heat transport in turbulent convection. Starting from the paradigmatic Boussinesq-Oberbeck equations, heat transport is investigated via the HKC hierarchy of Galerkin truncated ODE models of increasing dimension. The dynamics of these models are studied, and particular attention is given to stable values of heat transport, as well as the convergence across models where the models accurately represent the PDE. Implications for energetically consistent parameterization of convection will then be discussed.

Power variations methods for parameter estimation in the geophysical context

Radomyra Shevchenko

Thursday, August 1, 14:00–14:30, East Hall 8

Abstract: This talk presents two examples of use of a classical parameter estimation method — power variations — in an applied context. The first setup is the modeling of 2D turbulence, for which the so-called Matérn process has been used in literature due to its distinct power spectrum, capturing the characteristic change of slope between the low- and high frequency regimes. We present estimators for different parameters of this stochastic process and discuss asymptotic properties of these estimators. The second setup aims at estimating the spectral slope of a given Gaussian field from observations on a sphere. Our estimator, in contrast to the intuitive Fourier ansatz, is defined locally and can be computed along a single geodesic. This permits applications in the context of sparse or missing observations, which is often the case in Oceanography.

Determination of the energy spectrum from incomplete data for random fields

Anton Kutsenko

Thursday, August 1, 14:30–15:00, East Hall 8

Abstract: The energy spectrum - energy distribution over scales - is a key diagnostic in the atmosphere and ocean sciences. Even if we have complete information about the diagnosed field, the energy spectrum analysis can be difficult due to mesh irregularity and complex topology. We consider a more complicated case and discuss the recovery of energy spectra from sparse observations. In this case, the spectrum reconstruction procedure is based on a sophisticated statistical analysis of random fields.

Mesoscale Spectral Energy Transfers in Global Storm-resolving Simulations

Yanmichel Alejandro Morfa Avalos (Leibniz-Institut für Atmosphärenphysik e.V. an der Universität Rostock (IAP)), Claudia C. Stephan (Leibniz-Institut für Atmosphärenphysik e.V. an der Universität Rostock (IAP))

Thursday, August 1, 15:00–15:30, East Hall 8

Abstract: The atmospheric horizontal kinetic energy spectrum follows a $\kappa^{-5/3}$ power law at mesoscales (scales smaller than 600 km). The dynamics underlying the observed mesoscale spectrum remain controversial. The prevailing explanations include a downscale cascade mediated by weakly interacting inertia-gravity waves (IGWs), strongly stratified turbulence, or interactions between waves and the balanced flow. This study investigates the mesoscale spectral energy budgets from storm-resolving simulations of two general circulation models. Our analysis reveals different dynamics regarding the contributions from local forcing and spectral transfers within the upper and lower stratosphere. The stratosphere is mainly energized by upward propagating IGWs through the convergence of vertical pressure and momentum fluxes. The stratosphere also exhibits an upscale cascade mediated by nonlinear interactions between rotational modes; however, it is small compared to the energy deposited by IGWs. The primary contribution to the mesoscale energy

spectrum in the troposphere is from nonlinear spectral transfers towards small scales. Furthermore, the decomposition of the spectral transfers into the contributions from Rossby waves and IGWs reveals that their interactions dominate the downscale energy cascade at mesoscales. This result aligns with the hypothesis that explains the downscale cascade based on triad interactions involving vortical and gravity-wave modes. Moreover, we show that weakly interacting IGWs do not contribute to the resolved kinetic energy transfers in the simulations.

Birth of discrete Lorenz attractors in bifurcations of homoclinic and heteroclinic cycles

Ivan Ovsyannikov (Constructor University)

Thursday, August 1, 15:30–16:00, East Hall 8

Abstract: Consider the following three-dimensional Henon map:

$$x \mapsto y, y \mapsto z, z \mapsto M + Ay + Bx - z^2. \quad (1)$$

It is well-known that in some parameter domain map (1) possesses the discrete Lorenz attractor. This fact helps to prove the existence of such attractors in other problems, where map (1) appears e.g. as a first-return map. In particular, these are homoclinic and heteroclinic cycles with quadratic tangencies of manifolds and having special structures, local and global. The latter include the existence of resonant multipliers in the fixed point or non-simple tangencies of invariant manifolds. These additional conditions are required to avoid the existence of lower-dimensional invariant submanifolds and thus to make the dynamics effectively three-dimensional. Otherwise, the three-dimensional chaotic dynamics e.g. Lorenz-like attractors will be not possible.

Another question arises in this context: is it possible to have Lorenz-like attractors in the same systems, but in the backward time direction? The first return map in this case will be the inverse to (1), that is, the map

$$x \mapsto y, y \mapsto z, z \mapsto M + Az + Bx - y^2. \quad (2)$$

It is automatically clear that map (2) has a discrete Lorenz repeller near the same bifurcation point, where map (1) has an attractor. The main result of the present work is the numerical proof that map (2) possesses period-6 points with multipliers $(-1, -1, +1)$ near which indeed there exist period-6 discrete Lorenz attractors.

Minisymposium on Geophysical and Fluid Modelling with PDEs

Long-time behaviour of stochastic quasi-geostrophic models

Giulia Carigi

Wednesday, July 31, 13:30–14:00, Research II Lecture Hall

Abstract: The introduction of random perturbations by noise in partial differential equations has proven extremely useful to understand more about long-time behaviour in complex systems like atmosphere and ocean dynamics or global temperature. Considering additional transport by noise in fluid models has been shown to induce convergence to stationary solutions with enhanced dissipation, under specific conditions. On the other hand the presence of simple additive forcing by noise helps to find a stationary distribution (invariant measure) for the system and understand how this distribution changes with respect to changes in model parameters (response theory). I will discuss these approaches with a two-layer quasi-geostrophic model as example.

Approximating the long-time statistics of SPDEs: general results and applications

Cecilia Mondaini (Drexel University)

Wednesday, July 31, 14:00–14:30, Research II Lecture Hall

Abstract: In analyzing complex systems modeled by stochastic partial differential equations (SPDEs), such as certain turbulent fluid flows, an important question concerns their long-time behavior. In particular, one is typically interested in determining how long it takes for the system to settle into statistical equilibrium,

and in investigating efficient numerical schemes for approximating such long-time statistics. In this talk, I will present two general results in this direction, and illustrate them with applications to the 2D stochastic Navier-Stokes equations. Specifically, our results provide a general set of conditions that guarantee: (i) Wasserstein contraction for a given Markov semigroup and, consequently, exponential mixing rates; and (ii) uniform-in-time weak convergence for a parametrized family of Markov semigroups towards a limiting dynamic. Most importantly, our approach does not require gradient bounds for the underlying Markov semigroup as in previous works, and thus provides a flexible formulation for a broad range of applications. This is based on joint work with Nathan Glatt-Holtz (Tulane U).

On resonances in the inviscid Boussinesq equations

Christian Zillinger (Karlsruhe Institute of Technology (KIT))

Wednesday, July 31, 14:30–15:00, Research II Lecture Hall

Abstract: The 2D Boussinesq equations describe the evolution of a heat-conducting fluid. In this talk, I consider the long-time behavior of these equations near a combination of a shear flow and thermal stratification in the setting without dissipation. Unlike the viscous case, here the coupling by buoyancy here gives rise to linear instability and associated norm inflation on critical time-scales. Moreover, nonlinear resonances, called echoes, turn out to be highly frequency-dependent.

Near resonant approximation in Geophysical Fluid Dynamics

Bin Cheng (University of Surrey)

Wednesday, July 31, 15:00–15:30, Research II Lecture Hall

Abstract: Consider the rotating stratified Boussinesq system on three-dimensional tori with arbitrary aspect ratios, a well-studied model for GFD and foundation to the dynamic core of operational weather forecast. The quasi-geostrophic approximation of this system, introduced by Charney, captures the slow dynamics governing potential vorticity (PV), up to an error at the order of the Rossby/Froude numbers. Fast inertia-gravity waves are filtered out based on the convention that they are "too fast", hence having negligible effect on the slow PV part. GFD literature however has seen counterexamples. In this mathematical study using rigorous PDE analysis, we introduce a novel treatment of near resonance (NR) that no longer fully neglects fast waves, and instead capture the important portion of the nonlinear wave interactions selected using NR criteria. We prove global existence for the proposed NR approximation with a key technique being a sharp counting of the relevant number of nonlinear interactions – such counting is tied to the nonlinear estimates on the advection terms. An additional regularity advantage arises from a careful examination of some slow/fast mixed type interaction coefficients. In a wider context, the significance of our near resonant approach is a delicate balance between the inclusion of more interacting modes and the improvement of regularity properties, compared to the well-studied singular limit approach based on exact resonance.

We have a similar NR approximation applied to the rotating Navier-Stokes equations (without stratification) but with crucial differences which seem to make this an easier case.

Joint work with Zisis N. Sakellaris

Well-posedness and long-term behaviour for geophysical fluid models with a subgrid parametrization

Paul Holst

Wednesday, July 31, 16:00–16:30, Research II Lecture Hall

Abstract: Numerical simulations of large scale geophysical flows typically require unphysically strong dissipation for numerical stability. Towards energetic balance various schemes have been devised to re-inject this energy, in particular by horizontal kinetic energy backscatter. In order to gain insight into the impact of such schemes from a mathematical viewpoint, we view these as a modification of the fluid momentum equations on the continuum level, i.e., as partial differential equations. In this talk, we examine two geophysical fluid models, namely the 2D Euler equations and the 3D QG potential vorticity equations, equipped with a backscatter parameterization. We discuss the global well-posedness of both models and, in particular, their long-term behavior. We prove global well-posedness exploiting that the vector fields are

divergence free and using anisotropic Sobolev spaces. For the long-term behavior of these models, essentially two outcomes emerge dependent on the choice of parameters in the backscatter parameterization: Either the dynamics become trivial, i.e., they asymptotically converge towards the zero attractor, or the so-called 'grow-up' phenomenon occurs, i.e., there exist solutions that grow unboundedly and exponentially over time. We also prove stability of unbounded growth for both models.

On the ill-posedness of the 2D Boussinesq equations in the class of bounded initial data

Lars Eric Hientzsch (Bielefeld Universität)

Wednesday, July 31, 16:30–17:00, Research II Lecture Hall

Abstract: The Boussinesq equations describe the evolution of a stratified fluid under the influence of gravity. We investigate the system in vorticity form and with a stable continuous background stratification increasing with depth (spectrally stable density profile).

We prove that the system is strongly ill-posed in the class of data with bounded initial vorticity and density gradient.

The mechanism that allows us to exhibit the norm-inflation in infinitesimal time is purely nonlinear - in contrast to previous results on mild ill-posedness of the system.

Time permitting, applications to the 3D axisymmetric Euler equations with small bounded initial vorticity are discussed.

Based on joint work with R. Bianchini (IAC Rome) and F. Iandoli (Università della Calabria).

Non-Uniqueness of Generalized Weak Solutions to the Primitive and Prandtl Equations

Daniel W. Boutros (University of Cambridge), Simon Markfelder (Julius-Maximilians-Universität Würzburg), Edriss S. Titi (University of Cambridge)

Wednesday, July 31, 17:00–17:30, Research II Lecture Hall

Abstract: The primitive equations, which describe large-scale oceanic and atmospheric dynamics, are an important model in the field of geophysical fluid dynamics. In order to take their anisotropic structure into account, we introduce new notions of weak solutions for the inviscid primitive equations. We develop a convex integration scheme to prove existence and non-uniqueness of these generalized weak solutions, and also to exhibit examples of generalized weak solutions which do not conserve energy. Here we consider both the three-dimensional and the two-dimensional case. Finally we extend the aforementioned results to the three-dimensional viscous primitive equations, as well as the two-dimensional Prandtl equations. This is joint work with Daniel W. Boutros and Edriss S. Titi.

Optimal regularity for the 2D Euler equations in the Yudovich class

David Meyer (Universität Münster)

Wednesday, July 31, 17:30–18:00, Research II Lecture Hall

Abstract: We analyze the optimal regularity that is exactly propagated by a transport equation driven by a velocity field with a BMO gradient. As an application, we study the 2D Euler equations in case the initial vorticity is bounded. The sharpness of our result for the Euler equations follows from a variation of Bahouri and Chemin's vortex patch example.

Based on joint work with Nicola de Nitti and Christian Seis.

Dissipative measure-valued solutions to the compressible Navier-Stokes equations with potential temperature transport

Andreas Schömer (Johannes Gutenberg University Mainz), Mária Lukáčová-Medvid'ová (Johannes Gutenberg University Mainz)

Wednesday, July 31, 18:00–18:30, Research II Lecture Hall

Abstract: A model for the fluid flow in meteorological applications is given by the compressible Navier-Stokes equations with potential temperature transport (NSPTT). (Global-in-time) weak solutions to this

system are only known to exist in certain nonphysical situations - namely, if the adiabatic index appearing in the pressure state equation is sufficiently large. To circumvent this problem, we introduce the concept of so-called dissipative measure-valued (DMV) solutions. Within the DMV framework we can show the existence of solutions to NSPTT for all physically relevant values of the adiabatic index. The main idea of the proof is to apply the fundamental theorem for Young measures to a sequence of approximate solutions obtained from a suitable numerical scheme. Moreover, we briefly mention further results for NSPTT including (i) a DMV-strong uniqueness principle: if there are a DMV solution and a strong solution emanating from the same initial data, then they coincide as long as the latter exists; (ii) the existence of local-in-time strong solutions.

Passive tracer spectra in turbulent flows

Djoko Wirosoetisno (Durham University)

Thursday, August 1, 11:00–11:30, Research II Lecture Hall

Abstract: We consider passive tracers advected by a random synthetic smooth velocity field. In the small Prandtl/Schmidt number limit, the tracer (power) spectrum was found to obey the scaling predicted by Batchelor, Howells and Townsend (1959), probabilistically and asymptotically in wavenumbers. No assumption is made on the tracer behaviour at larger scales (smaller wavenumbers), believed to obey the Corrsin-Obukhov scaling.

Are turbulent incompressible flows computable ?

Gert Lube (Universität Göttingen)

Thursday, August 1, 11:30–12:00, Research II Lecture Hall

Abstract: Turbulent incompressible flows can be described by the Euler equations in case of an infinite Reynolds number. The solutions are very often subject to anomalous diffusion. An implicit Large-Eddy-simulation (ILES) method has to cope with missing regularity of the solution according to the Onsager conjecture and to provide appropriate numerical diffusion. A H(div)-conforming FEM is applied which guarantees pointwise divergence-free discrete velocities together with pressure-robustness. We provide numerical solutions for turbulent flows of channel type.

Analysis of Hibler's Sea-Ice Model

Karoline Disser

Thursday, August 1, 12:00–12:30, Research II Lecture Hall

Abstract: Hibler's model is used in climate simulations for predicting the dynamics of the arctic sea-ice covers. Mathematically, its typical regularisations are like variants of the compressible Navier-Stokes equations. We discuss first results and difficulties in the analysis.

Minisymposium on Holomorphic Dynamics

Parameter slices in the space of polynomials

Pascale Roesch

Monday, July 29, 11:00–11:30, Research I Lecture Hall

Abstract: We will restrict ourselves to polynomials with one free critical orbit. We will see that, stemming from any relative hyperbolic component, appears the connectedness locus of some model family of polynomials. This is a joint work with Hiroyuki Inou and Jan Kiwi.

Rigidity of Bounded-type Siegel Polynomials

Kostiantyn Drach (Universitat de Barcelona (CIF: Q0818001J))

Monday, July 29, 11:30–12:00, Research I Lecture Hall

Abstract: In this talk, we will discuss bounded-type Siegel polynomials, which are complex polynomials

of arbitrary degree that contain a Siegel disk with a bounded-type rotation number. We will outline the proof of the result stating that non-renormalizable bounded-type Siegel polynomials are quasi-conformally rigid, and under certain natural assumptions, they are even conformally rigid. This means that any topological conjugacy between a pair of such polynomials is automatically quasi-conformal/conformal. This theorem extends the existing results on rigidity for non-renormalizable polynomials of arbitrary degree by incorporating irrational neutral dynamics. This work is based on joint research with Jonguk Yang.

McMullen-like dynamics near degeneracy parameters via singular perturbations

Jordi Canela (Universitat Jaume I)

Monday, July 29, 12:00–12:30, Research I Lecture Hall

Abstract: In some cases, holomorphic families of rational maps may have degeneracy parameters, i.e. parameters for which the degree of the map decreases. In this talk we will explain how singular perturbations can be used to understand the dynamics of families of rational maps near degeneracy parameters. More specifically, we will show how to relate the dynamics of a family of rational maps obtained from Chebyshev-Halley root finding algorithms with the dynamics of the McMullen singularly perturbed maps $z^4 + \lambda/z^2$.

This is a joint work with Antonio Garijo and Pascale Roesch.

Blenders, independence of multipliers and sparsity of PCF maps

Johan Taflin

Monday, July 29, 13:30–14:00, Research I Lecture Hall

Abstract: Postcritically finite rational maps are of particular importance in one-variable complex dynamics. They are related to strong bifurcations phenomena and they form a Zariski dense set in the moduli space of degree d rational maps. In higher dimensions, we prove with Thomas Gauthier and Gabriel Vigny that postcritically finite maps are not Zariski dense in the moduli space of degree d endomorphisms of the projective space P^k as soon as d and k are larger or equal to 2. The proof is a combination of arguments coming from complex analysis, arithmetic geometry and smooth dynamics. Two important ingredients are the use of special hyperbolic sets called blenders and the independence of the multipliers of periodic points. This last point has been recently generalized in a joint work with Igors Gorbovickis.

Collet–Eckmann and hyperbolicity

Mats Bylund (Université Paris-Saclay)

Monday, July 29, 14:00–14:30, Research I Lecture Hall

Abstract: In this talk I will introduce an important class of non-hyperbolic maps – the so-called Collet–Eckmann maps – and discuss how well these maps can be approximated by hyperbolic maps. In particular, within the unicritical family, any Collet–Eckmann map is (in the parameter space) a Lebesgue density point of the complement of the connectedness locus. This talk is based on joint work with Magnus Aspenberg and Weiwei Cui.

Resonances for rational Anosov diffeomorphisms

Julia Slipantschuk

Monday, July 29, 14:30–15:00, Research I Lecture Hall

Abstract: Eigenvalues of transfer operators, known as Pollicott-Ruelle resonances provide insight into the long-term behaviour of the underlying dynamical system, in particular determining its exponential mixing rates. I will present a complete description of Pollicott-Ruelle resonances for a class of rational Anosov diffeomorphisms on the two-torus.

Wild attractors for Fibonacci maps

Denis Gaidashev (Uppsala University)

Tuesday, July 30, 11:00–11:30, East Hall 3

Abstract: Existence of wild attractors - attractors whose basin has a positive Lebesgue measure but is not a residual set - has been one of central themes in one-dimensional dynamics.

It has been demonstrated by H. Bruin et al. that Fibonacci maps with a sufficiently flat critical point admit a wild attractor.

We propose a constructive trichotomy that describes possible scenarios for the Lebesgue measure of the Fibonacci attractor based on a computable criterion. We use this criterion, together with a computer-assisted proof of existence of a Fibonacci renormalization 2-cycle for non-integer critical degrees, to demonstrate that Fibonacci maps do not have a wild attractor when the degree of the critical point is $d = 3.8$ (and, conjecturally, for $2 < d < 3.8$), and do admit it when $d = 5.1$ (and, conjecturally, for $d > 5.1$).

Hyperbolicity of Lorenz renormalization at fixed points of high type

Matteo Tabaro (Constructor University Bremen), Igors Gorbovickis (Constructor University Bremen)

Tuesday, July 30, 11:30–12:00, East Hall 3

Abstract: In the classical setting of unimodal or circle maps, it has been proven that fixed points of renormalization are hyperbolic, and their stable manifolds coincide with their topological classes. This has huge repercussions on the geometry of the parameter space, and it lies behind interesting phenomena such as quantitative universality.

It was shown by Martens and Winckler in 2014 and 2016, that this paradigm does not hold for Lorenz maps, piecewise increasing smooth maps of the interval without critical points other than at the unique discontinuity at which the one-sided derivatives are equal to zero. More specifically, Martens and Winckler showed there exist topologically conjugated maps whose attractors have wildly different geometries and hence, divergent renormalizations. Nevertheless, they conjectured that the Lorenz renormalization is still hyperbolic and showed that if this is the case, then the dimension of the unstable manifold is at least 3.

In this talk, we will explore the renormalization theory of Lorenz maps, together with some recent advancements related to the hyperbolicity of fixed points of high type, and the dimension of their stable and unstable manifolds.

Thurston's iteration for post-singularly infinite entire functions

Konstantin Bogdanov (Saarland University)

Tuesday, July 30, 12:00–12:30, East Hall 3

Abstract: Thurston's Topological Characterization of Rational Functions is an important tool in complex dynamics allowing us to decide when a "topological model" of a post-critically finite rational function is equivalent to a complex analytic rational function. In particular, it is so if and only if Thurston's pull-back map acting on the Teichmüller space of a sphere with finitely many punctures has a fixed point. Analogous problem can be studied for transcendental entire functions of finite type with infinite post-singular orbits. This new setting requires, however, a different approach on the level of techniques. We describe a general family of entire functions (defined by "asymptotic area property") for which the question of existence of a fixed point of Thurston's map can be answered positively in the most natural cases.

Moduli spaces of polynomial maps and multipliers at small cycles

Valentin Huguin (University of Toronto Mississauga)

Tuesday, July 30, 12:30–13:00, East Hall 3

Abstract: In this talk, I will show that the multipliers at small cycles provide a good description of the moduli space of polynomial maps of any given degree. The moduli space P_d of polynomial maps of degree $d \geq 2$ is the space of all polynomials of degree d modulo conjugation by an affine transformation. It is a central object of study in complex dynamics. As an affine variety, one can understand its geometry by studying the regular functions defined on it. Dynamically interesting such functions are given by the symmetric functions of the multipliers at the cycles of any given period. Here, I will show that the corresponding map induced by the multipliers at the cycles with periods 1 and 2 is a finite birational morphism from P_d onto its image. Equivalently, I will show that (1) the multipliers at the cycles with periods 1 and 2 detect degeneration in

P_d and that (2) a generic conjugacy class of polynomial maps of degree d is uniquely determined by its multipliers at its cycles with periods 1 and 2. The point (2) proves a conjecture by Hutz and Tepper and strengthens recent results by Ji and Xie in the particular case of polynomial maps.

Minisymposium on Integrated time series analysis and machine learning to analyze dynamical complex systems

The use of entropy of recurrence micorstates and artificial intelligence to detect abnormal dynamical regimes

Elbert Macau (Universidade Federal de Sao Paulo - UNIFESP), Bruno Boaretto (Universidade Federal de Sao Paulo - UNIFESP), Alexandre Adriani (Universidade Federal de Sao Paulo - UNIFESP), Sergio Lopes (Universidade Federal do Paraná), Thiago Prado (Universidade Federal do Paraná)

Wednesday, July 31, 11:00–11:30, IRC Seminar Room 2

Abstract: Cardiac arrhythmia is a common clinical problem in cardiology defined as the abnormality in heart rhythm. Bradycardia, atrial fibrillation, tachycardia, supraventricular tachycardia, atrial flutter and sinus irregularity are common different classifications for arrhythmia. Here, we present a new approach to distinguishing between these most common heart rhythms. Our approach is based on dynamical system techniques, namely recurrence entropy of microstates, and recurrence vicinity threshold, in association with artificial intelligence. The rhythms and other cardiac conditions of the dataset were labeled by more than one licensed physician. The main contributions of this work are the identification of how different heart rhythms affects the entropy of recurrence microstates and recurrence vicinity threshold parameter, and in doing so, this quantifier may be used as a feature extraction to artificial intelligence classifiers.. Our method involves a significant reduction of the data set to be analyzed by machine learning algorithms and can bring benefits in situations of pre-testing individuals, due to the minimum processing time and hardware required to perform the analysis. The additional information obtained by the two quantifiers may also be put together with the signals, consolidating data from multiple sources, adding more useful information to the dataset.

Learning the dynamics while learning the connections: models of dynamical networks

Michael Small (University of Western Australia), Braden Thorne, Eugene Tan

Wednesday, July 31, 11:30–12:00, IRC Seminar Room 2

Abstract: I will describe two approaches to understand machine learning from the perspective of dynamical systems. First, recurrent neural networks will be shown to perform an embedding of time series data in the sense of Takens' theorem. That is, the internal state of the neural network is diffeomorphic to the underlying (presumed deterministic) dynamical system. Second, I will describe the application of learning techniques to estimate the state of a network dynamical system from observation of the node dynamics. Along the way, the utility of these methods will be demonstrated with application to industrial maintenance, music and detection of ventricular fibrillation.

Predicting fluctuations in a chaotic cancer model through machine learning

Kelly C. Iarosz (University Center UNIFATEB), Elaheh Sayari (UEPG), Sidney T. Silva (UEPG), Antonio Marcos Batista (UEPG), José Danilo Szezech Jr. (UEPG), Ricardo Luiz Viana (UFPR - Federal University of Parana), Paulo Protachevicz (USP), **Eduardo Luís Brugnago** (USP), Ana Paula da Silva Koltun (University of Aberdeen, Aberdeen, UK)

Wednesday, July 31, 13:30–14:00, IRC Seminar Room 2

Abstract: Cancer, the second leading cause of death globally according to the World Health Organization, comprises a diverse group of diseases. Mathematical and computational approaches have been instrumental in delving into the spread of cancer cells and understanding the mechanisms governing their growth. Our focus centers on a cancer model that displays both periodic and chaotic attractors, depicting interactions among host cells, effector immune cells, and cancer cells. Within this model, observable fluctuations arise

in cell populations, with the fluctuation range linked to the emergence of tumor cells. In this study, we leverage machine learning algorithms to predict these fluctuations. Our findings demonstrate that our machine learning classification is adept at identifying fluctuations associated with the growth rate of cancer cells. This approach represents a promising avenue for enhancing our understanding of cancer dynamics and potentially informing strategies for intervention and treatment.

statFEM – dealing with imperfect models in a statistical learning framework

Thomas Stemler (The University of Western Australia), Connor Duffin (Cambridge University), Edward Cripps (The University of Western Australia)

Wednesday, July 31, 14:00–14:30, IRC Seminar Room 2

Abstract: The incorporation of statistical learning into mathematically derived models rooted in physics is increasingly gaining attention in the literature. A recent strategy involves enhancing the fundamental physics of governing equations through data-driven Bayesian statistical techniques. Termed as statFEM, this method recognizes a priori model misspecification by incorporating stochastic forcing into the governing equations. When additional data becomes available, classical Bayesian filtering techniques are employed to update the posterior distribution of the discretized finite element solution. The resulting posterior effectively captures the combined uncertainty related to the common issue of model misspecification and the data. We analyse the performance of the statistical learning method in a variety of cases drawn from fluid dynamics as well as reaction-diffusion equations.

Characterizing the spike timing of a chaotic laser by using ordinal analysis and machine learning

Bruno Boaretto (Universitat Politècnica de Catalunya (UPC)), Cristina Masoller (Polytechnic University of Catalonia (UPC))

Wednesday, July 31, 14:30–15:00, IRC Seminar Room 2

Abstract: Semiconductor lasers with optical feedback are well-known nonlinear dynamical systems. Under appropriate feedback conditions, these lasers emit optical pulses that resemble neural spikes. Influenced by feedback delay and various noise sources, including quantum spontaneous emission noise, the dynamics are highly stochastic. A good understanding of the spike timing statistics is needed to develop photonic systems capable of using the fast-spiking laser output for novel applications, such as information processing or random number generation. Here, we analyze experimental sequences of inter-spike intervals (ISIs) recorded when a sinusoidal signal was applied to the laser current. Different combinations of the DC value and frequency of the signal applied to the laser lead to ISI sequences with distinct statistical properties. This variability prompts an investigation into the relationship between experimental parameters and ISI sequence statistics, aiming to uncover potential encoding methods for optical spikes, since this can open a new way of encoding and decoding information in sequences of optical spikes. By using ordinal analysis and machine learning, we show that the ISI sequences have statistical ordinal properties that are similar to Flicker noise signals, characterized by a parameter α that varies with the signal that was applied to the laser current when the ISIs were recorded. We also show that for this dataset, the $(\alpha, \text{permutation entropy})$ plane is more informative than the $(\text{complexity}, \text{permutation entropy})$ plane because it allows better differentiation of ISI sequences recorded under different experimental conditions, as well as better differentiation of original and surrogate ISI sequences.

Minisymposium on Lost and found: Searching for underlying dynamical relations in data using Machine Learning

A trajectory's guide to the state space – learning missing terms and phytoplankton bloom patterns

Rahel Vortmeyer-Kley (Carl von Ossietzky Universität Oldenburg), Maximilian Berthold (Mount Allison University), Pascal Nieters (University Osnabrueck)

Thursday, August 1, 11:00–11:30, East Hall 1

Abstract: The description of measured dynamical systems - like for example phytoplankton blooms in coastal

waters - by differential equations always includes the challenge of selecting the most important drivers and interactions of state variables to construct a complete but as simple as possible algebraic description of the phenomenon of interest. In practice modelers can end up with terms in the differential equation that do not fully describe the interactions or in worst case with missing terms.

The Scientific Machine Learning approach Universal Differential Equation (UDE) by Rackauckas et al. (2020) tries to overcome this challenge by extracting underlying dynamical relations of state variables from measured data using a neural network as a universal function approximator that provided time series of the state variables and their derivatives. However, the success of this framework in systems that undergo sudden changes in their dynamics crucially depends on the choice of the lossfunction and is not always guaranteed. In case of a successfully trained universal function approximator, Sparse Identification of nonlinear Dynamics (SinDy) by Brunton et al. (2016) can translated the results into algebraic equations.

By applying these approaches, we were able to learn the chlorophyll:nutrient interactions from a 22-year nutrient and chlorophyll data set of the southern Baltic Sea with additional forcing of external temperature, salinity and light attenuation dynamics as drivers. The recovered algebraic relationships between the variables chlorophyll, dissolved and total nutrients and the external drivers temperature, salinity and light attenuation showed the difference in importance of the different state variables and drivers for the six phytoplankton bloom types present in the data set of the southern Baltic Sea.

Foundation models for weather and climate

Martin Schultz (Forschungszentrum Jülich)

Thursday, August 1, 11:30–12:00, East Hall 1

Abstract: Two years ago machine learning revolutionized the domain of weather forecasting with three deep learning models that showed better skill than the best traditional equation-based models. Since then efforts are under way to further improve DL weather models and apply them to other Earth system compartments and to climate time scales as well. Adopting concepts from large language and multi-modal models, we also develop abstract foundation models for Earth system dynamics. With AtmoRep (Lessig et al., 2023), a first prototype of such a foundation model has been built, and in the HClimRep project this is now being enhanced to allow for simulations of seasonal to decadal timescales and include ocean and sea ice dynamics as well as stratospheric tracer transport.

Unraveling kelp forest population dynamics – a machine learning approach

Cheyenne Jarman (Oregon State University), Mark Novak (Oregon State University)

Thursday, August 1, 13:30–14:00, East Hall 1

Abstract: Mathematical models are essential for understanding population dynamics embedded in complex ecosystems. Kelp forests typify such ecosystems, with numerous factors and feedbacks influencing the many species that comprise and reside in kelp forests, many of which hold immense cultural, ecological, and economic value. Global climate change has driven dramatic shifts in foundational kelp species around the world, making it ever more crucial to understand their population dynamics. Many kelp species, including the best-studied species, giant kelp (*Macrocystis pyrifera*), exhibit annual and multi-year oscillations in abundance. Most kelp population models are therefore derived to capture deterministic mechanisms of stable limit cycles, including logistic growth and the time-lagged feedback between intra- and interspecific interactions. Here, rather than derive such models from first principles, we apply a form of machine learning known as symbolic regression to learn the governing equations of a persistently cyclical population of giant kelp directly from simulated and empirical time-series data. Coupling symbolic regression with time-delay embedding, which allows us to encapsulate the multi-dimensionality of kelp dynamics, we evolve equations towards the Pareto front of human-interpretable, single-species equations that best describe kelp population dynamics, specifically the per capita growth rate, over varying levels of equation complexity. Comparing the mathematical forms and simulated dynamics for the empirical time series reveals that parameter values for the per capita growth rate equation are likely being underestimated in the field as a function of sample frequency, and as a result, produce dynamics not representative of the system. Following these results, our

next steps include adjusting the quantity of empirical data provided to symbolic regression and deriving a correction equation to better map the observed per capita growth rate to the expected per capita growth rate.

Neural Networks as Surrogates for Time-Dependent Parameters in Epidemiology

Lukas Stelz

Thursday, August 1, 14:00–14:30, East Hall 1

Abstract: The complex system governing the spread of a disease in a population can be modelled using compartmental epidemiological models. These models usually feature constant parameters and thereby predict a singular outbreak. During the Covid-19 pandemic, it became evident that population-specific parameters continually changed due to policies aimed at "flattening the curve". This led to multiple peaks in the case counts as the population adapted to higher and lower incidences. Additionally, new emergent variants showed increased infectiousness. In some cases, these variants even rendered prior infections insufficient in providing lasting immunity. These features can only be captured as the same disease by time-varying disease-specific parameters like infection or recovery rates.

To account for these model external influences, we propose a data-driven technique that uses neural networks as surrogates for time-varying parameters. Based on the UDE approach a small neural network maps continuous time to a set of dynamical parameters at the corresponding time point. This neural network alongside the constant parameters is trained using one time series of reported data, including cumulative cases, hospital occupancy and deaths depending on the number of compartments in the model.

We demonstrate our modelling approach on the first 14 months of the Covid-19 pandemic in different countries. The reproduction number calculated using the recovered time-varying parameters fits the real time estimate by Robert Koch Institute albeit fewer fluctuations. Time-varying hospitalization and death rates were also recovered. These can be used as indicators for phases where healthcare system operated close to capacity.

Data-Driven Modeling and Model Selection Through Library Adaptation

Gianmarco Ducci (Fritz-Haber-Institut der MPG), **Maryke Kouyate** (Fritz-Haber-Institut der MPG), **Christoph Schreuer** (Fritz-Haber-Institut der MPG), **Karsten Reuter** (Fritz-Haber-Institut der MPG)

Thursday, August 1, 14:30–15:00, East Hall 1

Abstract: Data-driven methodologies facilitate the derivation of concise equations approximating the governing laws of physical phenomena. Nevertheless, they encounter challenges arising from inherent data noise, leading to reduced sparsity in outcomes. Despite substantial advancements in this domain over the past decade, these methodologies commonly rely on the paradigm of employing a predefined set of library functions. Achieving enhanced sparsity necessitates the identification of the optimal basis function set, which represents a challenging task to guess in advance.

In this work, we propose an alternative approach which consists of optimizing the very library of functions while imposing sparsity. The robustness of our results is not only evaluated by the quality of the fit of the discovered model, but also by the statistical distribution of the residuals with respect to the original noise in the data. The model selection is then chosen from a subset of optimal models obtained in a Pareto fashion. We illustrate how this method can be used as a tool to derive microkinetic equations from experimental data, and we propose a systematic approach for network identification.

Computational discovery of human reinforcement learning dynamics from behavioural data

Daniel Weinhardt (Osnabrück University)

Thursday, August 1, 15:00–15:30, East Hall 1

Abstract: In the field of cognitive science, several scientific models conceptualize human cognition as a dynamic process. Among these, models of human reinforcement learning stand out by proposing that humans continually adjust the values assigned to actions based on received rewards. A significant challenge in uncovering these learning dynamics lies in the fact that the variables of the underlying dynamical system,

such as the values for actions, are latent and cannot be directly observed but rather must be inferred from observable human behaviors, like the actions chosen. Here, a novel approach to automate the discovery of these latent learning dynamics from human choice behavior is presented.

Our methodology combines the expressiveness of recurrent neural networks (RNNs) with the interpretability of sparse dynamical systems to decode human choice behavior. In the first step, we fit an RNN to capture the sequence of choices made by individuals in a reinforcement learning task. Specifically, the RNN is trained to predict trial-by-trial human decisions based on the available choices, the received rewards, and the RNN's own hidden state. Further, we are limiting the RNN's hidden state to n variables corresponding to n possible choices. This restriction forces the RNN to encapsulate human decision-making processes within one single evolving variable per choice possibility. After the RNN is fit to human behavior, we extract a time series of these latent variables. In a second step, we use Sparse Identification of Non-linear Dynamical Systems (SINDy) to identify a dynamical system explaining this time series. The resulting dynamical system provides an interpretable model of human learning dynamics.

We evaluate the recoverability of our approach in a two-armed bandit task. In this task, an agent is presented on each trial with two options (arms) to choose from. Each arm is assigned a reward probability, which changes stochastically after each trial according to a drift rate. For validation, we simulate behavior with a reinforcement learning agent that incorporates psychological phenomena observed in human learning, such as reward-based value updates and forgetting over time. We then train an RNN with the generated behavioral data. After the training, we generate a time series of Q-values from the RNN along with its chosen actions and obtained rewards. In the final step, we fit a SINDy model with a candidate library containing all polynomial combinations of the dataset variables up to the third degree.

Our method successfully recovers the correct terms and the Q-value update dynamics. The RNN as well as the recovered dynamical system are able to recover the general update rule. However, in contrast to the RNN, the recovered dynamical system resembles an interpretable learning rule matching the original learning rule used to generate the agent's behavior.

Overall, we demonstrate that a combination of RNNs and SINDy can uncover learning dynamics of simulated agents in a two-step bandit task. This data-driven discovery complements theory-driven approaches for modeling human learning dynamics.

Minisymposium on Modeling, analysis, and control of cardiac dynamics

Dynamics and Role of Rotors in the Onset of Fibrillation

Omer Berenfeld (University of Michigan)

Tuesday, July 30, 11:00–11:30, Research I Lecture Hall

Abstract: Fibrillation is a highly lethal irregular rhythm of the heart with a major clinical-societal burden. To better understand the dynamics and role of rotors in sustenance of the arrhythmia we are focusing on the period of the onset of fibrillation and studying the transition from the regular impulse propagation to the irregular and complex propagation patterns of fibrillation. Optical mapping experiments in animal models demonstrate that the fibrillation onset, with its gradual increase in the complexity of the excitation patterns, is characterized by a concomitant gradual increase in tissue activation rate. Analysis of the spatiotemporal optical data reveals that the acceleration of the activation rate in the tissue occurs during the presence of series of rotors, but further suggests that only a subset of rotors with relatively high drifting speed and distance contribute to that acceleration. Novel analysis of the fibrillation in the energy domain sheds light on the forces induced by the ionic membrane currents and acting on rotors to drift and stabilize.

Impact of Source-Sink Mismatch on a New 3D Cardiac Defibrillation Method

Rhiannan Ruef (Rochester Institute of Technology), Niels Otani (Rochester Institute of Technology)

Tuesday, July 30, 11:30–12:00, Research I Lecture Hall

Abstract: Previously, we have presented a new, 3D low-energy defibrillation method which works by reconfiguring the shape of the filaments around which scroll waves rotate so that they shrink and disappear. Our method sometimes fails due to a process we call “flopping”, where filaments reorient themselves in

an unfavorable way and persist indefinitely. In this talk, we will describe the dynamics of flopping, and its relationship to the source-sink mismatch encountered by the accompanying propagating action potentials.

How Physics-Informed Neural Networks Can Characterise Cardiac Dynamics

Marta Varela (Imperial College London)

Tuesday, July 30, 12:00–12:30, Research I Lecture Hall

Abstract: Physics-informed neural networks (PINNs) combine data learning with knowledge of the differential equations describing the dynamics of a system.

We will demonstrate how PINNs can rapidly and accurately describe cardiac dynamics in 2D and 3D in different dynamic regimes by solving the reaction-diffusion equations that describe cardiac electrophysiology.

We will also show how PINNs can be used for parameter inference, namely to characterise changes in electrophysiological properties caused by the action of anti-arrhythmic drugs using in silico and in vitro data.

We will also briefly discuss future opportunities to use PINNs to characterise biological systems.

Multipulse protocols for the control of spiral wave dynamics

Thomas Lilienkamp (Technische Hochschule Nürnberg Georg Simon Ohm), Daniel Suth (Technische Hochschule Nürnberg Georg Simon Ohm), Ulrich Parlitz (Max-Planck Institut für Dynamik und Selbstorganisation), Stefan Luther (Max-Planck Institut für Dynamik und Selbstorganisation)

Tuesday, July 30, 13:30–14:00, Research I Lecture Hall

Abstract: The efficient control of spiral wave dynamics is particularly interesting in the context of cardiac arrhythmias. Numerical simulations provide an ideal tool to test new control strategies in a safe environment. Multipulse protocols play an important role here. We discuss and compare different approaches, including predetermined pulse sequences as well as feedback solutions.

Insights into the interplay of electrical fields and myocardial heterogeneities in defibrillation

Daniel Suth (Technische Hochschule Nürnberg Georg Simon Ohm), Thomas Lilienkamp (Technische Hochschule Nürnberg Georg Simon Ohm)

Tuesday, July 30, 14:00–14:30, Research I Lecture Hall

Abstract: The difficulty in controlling the complex spatial and temporal dynamics of ventricular fibrillation, a critical cardiac arrhythmia, results from the nonlinear nature of the interaction between the external electrical field and the heterogeneous anatomy of the heart muscle.

Traditional defibrillation techniques utilize external electrical fields to recruit heterogeneities as new wave sources, countering and terminating spiral wave-like patterns in the myocardium. The efficacy of biphasic pulsing over monophasic pulsing has been demonstrated in various numerical, ex vivo and in vivo experiments. Our understanding of the underlying interaction between the external electrical field and single heterogeneities of the myocardium, as well as their contribution to successful defibrillation, however is largely based on hypotheses.

In this study, several hypotheses are investigated at the scale of individual heterogeneities as well as at the tissue level to provide insights into the interaction between external electrical fields and myocardial heterogeneities, which are crucial for understanding fundamental defibrillation mechanisms. Employing 2D simulations of cardiac dynamics, we explore phenomena that are challenging to observe in living heart experiments, shedding light on the complex dynamics of defibrillation processes.

Improving Defibrillation through an Evolutionary Algorithm

Marcel Aron (Max Planck Institute for Dynamics and Self-Organization (MPIDS), Göttingen; German Centre for Cardiovascular Research (DZHK), partner site Göttingen), Ulrich Parlitz (Max Planck Institute for Dynamics and Self-Organization (MPIDS), Göttingen; German Centre for Cardiovascular Research (DZHK),

partner site Göttingen), Stefan Luther (University Medical Center Göttingen (UMG); German Centre for Cardiovascular Research (DZHK), partner site Göttingen; Max Planck Institute for Dynamics and Self-Organization (MPIDS), Göttingen)

Tuesday, July 30, 14:30–15:00, Research I Lecture Hall

Abstract: Cardiac fibrillation results in the lethal, chaotic disruption of the heart’s electricity-coordinated contraction mechanism. Its standard treatment, single-shock defibrillation, can cause lasting damage and may even increase the long-term risk of further fibrillation episodes. This has motivated research into alternative, less intrusive (i.e. low-energy) defibrillation schemes beyond the usual singular, potent shock.

The search for better defibrillation schemes has proven difficult largely due to the sheer amount of possible shock sequences to consider. We sought to reduce this number through the design and application of an optimisation heuristic inspired by evolutionary biology. To guide this heuristic, we consulted simplified defibrillation simulations.

This talk covers our quantification of defibrillation performance and risk, an outline of our optimisation heuristic, and our findings in its application to optimising multi-shock defibrillation. The latter includes the discovery of potentially global optima for specific defibrillation shock-counts.

Transient chaos in heterogeneous excitable media

Melvin Dix (Max Planck Institute for Dynamics and Self-Organization, Göttingen), Paula Luttermann (Max Planck Institute for Dynamics and Self-Organization, Göttingen), Thomas Lilienkamp (Nuremberg Institute of Technology Georg Simon Ohm), Stefan Luther (Max Planck Institute for Dynamics and Self-Organization, Göttingen), **Ulrich Parlitz** (Max Planck Institute for Dynamics and Self-Organization, Göttingen)

Tuesday, July 30, 15:00–15:30, Research I Lecture Hall

Abstract: Chaotic spatio-temporal dynamics in excitable media such as cardiac tissue is often not permanent but limited in time. In this contribution, we discuss the influence of heterogeneities and stochastic perturbations on the mean lifetime of such chaotic transients. Using simulations with the Aliev-Panfilov and the Fenton-Karma model, we show that both forms of perturbations can (significantly) prolong the duration of chaotic transients and may also lead to persistent chaos or stable periodic wave patterns.

Parameter and variable estimation in cardiac dynamics using adjoint optimization

Inga Kottlarz (Max Planck Institute for Dynamics and Self-Organization (MPIDS), Göttingen; German Centre for Cardiovascular Research (DZHK), partner site Lower Saxony), Sebastian Herzog (Georg August Universität Göttingen), Patrick Vogt (Max Planck Institute for Dynamics and Self-Organization), Stefan Luther (Max Planck Institute for Dynamics and Self-Organization (MPIDS), Göttingen; German Centre for Cardiovascular Research (DZHK), partner site Lower Saxony), Ulrich Parlitz (Max Planck Institute for Dynamics and Self-Organization (MPIDS), Göttingen; German Centre for Cardiovascular Research (DZHK), partner site Lower Saxony)

Wednesday, July 31, 11:00–11:30, Research III Lecture Hall

Abstract: Cardiac muscle tissue is an excitable medium that can exhibit a range of dynamics of different complexity, from planar waves to spiral waves to spatiotemporal chaos, the latter being associated with (fatal) cardiac arrhythmia. Both the prediction of such high dimensional chaotic time series, and the reconstruction of their (not yet fully observable) complete dynamical state are ongoing challenges. In recent years, machine learning approaches have gained popularity for solving these problems, which can be advantageous in cases where not much knowledge about the dynamical system in question exists, but are limited by the large amounts of training data that is needed and often not available for biological systems. We present adjointODE, an adjoint optimization framework for estimating model parameters and unobserved variables. We showcase the adjoint method’s effectiveness in optimizing high-dimensional problems with thousands of unknowns, serving as a valuable tool for bridging the gap between empirical data and theoretical models.

Modelling dynamics and feedback control of vortices during defibrillation in cardiac systems

Markus Bär (Physikalisch-Technische Bundesanstalt (PTB))

Wednesday, July 31, 11:30–12:00, Research III Lecture Hall

Abstract: Many relevant experimental systems and applications exhibit irregular dynamics such as spatiotemporal chaos or turbulence. In this talk, I will present fibrillatory cardiac dynamics as an example of such behavior where control of such a state by transforming the dynamics into homogeneous steady state is desirable. Defibrillation of cardiac tissue by periodic pacing is modelled in a two dimensional heterogeneous excitable medium. We used simulations as well as data analysis not only to identify suitable conditions e. g. pacing strength and periods [1, 2], but also to determine the mechanism of defibrillation in this specific modus. We found that the length of the refractory boundary is the best predictor for defibrillation success and that successful defibrillation requires elimination of all vortices as well as prevention of formation of new vortex pairs after an applied shock [3]. Similarities and comparison with periodic pacing of an analytically accessible simple cellular automaton model [4] are discussed. Finally, this newly discovered mechanism is exploited for a feedback control strategy, that allows to identify optimal pacing periods without systematic scanning of the parameter space.

[1] P Buran, M Bär, S Alonso and T Niedermayer. *Chaos* 27, 113110 (2017).

[2] P Buran, T Niedermayer, and M Bär. *New J. Phys.* 24, 083024 (2022).

[3] P Buran, T Niedermayer, M Bär, *bioRxiv preprint*, doi.org/10.1101/2023.03.16.533010 (2023).

[4] N DeTal, F Fenton. *arXiv preprint*, <https://arxiv.org/abs/2109.10861> (2021).

Exploring cardiopulmonary interactions: the importance of improved classification of respiratory pattern

Beata Graff (Medical University of Gdańsk), Grzegorz Graff (Gdańsk University of Technology), Ulrich Parlitz (Max Planck Institute for Dynamics and Self-Organization), Paweł Pilarczyk (Gdańsk University of Technology), Hiromichi Suetani (Max Planck Institute for Dynamics and Self-Organization), Maciej Torhan (Gdańsk University of Technology), Stefan Luther (Max Planck Institute for Dynamics and Self-Organization), Krzysztof Narkiewicz (Medical University of Gdańsk)

Wednesday, July 31, 13:30–14:00, Research III Lecture Hall

Abstract: Cardiorespiratory interaction refers to the complex and dynamic interplay between the cardiovascular and the respiratory systems. This interaction is essential for maintaining homeostasis and optimizing the body's responses to various internal and external stimuli. Disorders that affect either the heart or lungs can disrupt this crucial relation. For example, heart failure can impair the ability of the heart to pump effectively, which can lead to inadequate oxygen delivery and respiratory distress. Conversely, pulmonary diseases can lead to impaired gas exchange and increased cardiac workload.

Studying interactions, especially their impact on heart rate dynamics, is essential to understand the mechanisms of cardiovascular abnormalities and crucial for better prevention, diagnosis and management of diseases that comprise respiratory, cardiovascular and autonomic nervous systems.

However, most of the known cardiorespiratory interactions require further investigation. The results of current research mainly concern healthy individuals and little is known about changes of interactions at the initial stage of cardiovascular disease. There is particularly not enough data on this topic when the respiratory pattern is irregular or changes over time.

The talk will present the variety of respiratory patterns found in healthy subjects and hypertensive patients and how respiratory rate and pattern alterations can impact the interpretation of heart rate and blood pressure variability analyses. In addition, the most important cardiorespiratory interactions will be reviewed. We will show our approach to the analysis of respiration and a proposal for complex qualitative and quantitative assessment of breathing pattern. We will highlight the challenges associated with assessing respiratory variability. Our research aimed at automatic classification of breathing patterns will be discussed in particular.

Characterization of altered electrophysiological properties, spatiotemporal dynamics and arrhythmia mechanisms using high-resolution optical mapping

Nour Raad (Technical University of Munich, Klinikum Rechts der Isar, Munich, Germany), **Philip Bittihn** (Max Planck Institute for Dynamics and Self-Organization)

Wednesday, July 31, 14:00–14:30, Research III Lecture Hall

Abstract: The effects of cardiac diseases on excitation patterns and arrhythmogenicity are often difficult to ascertain, particularly in the absence of structural remodeling. To a large extent, this is due to the conceptual gap between molecular mechanisms on the single-cell level, which can be characterized using traditional electrophysiology measurements, and tissue-level emergent effects which can contribute equally to the generation and maintenance of arrhythmias. To narrow this gap, we employ high-resolution optical imaging with voltage-sensitive dyes to link molecular changes to spatially distributed alterations in action potential morphology and conduction associated with increased susceptibility to arrhythmia initiation. Tailored analysis of wave patterns and rotors allows us to determine stability properties and further characterize the influence of disease-specific alterations in substrate properties on arrhythmia dynamics.

Translational exploration of fully biological restoration of cardiac rhythm via ion channel gene therapy

Tim De Coster (LUMC)

Wednesday, July 31, 14:30–15:00, Research III Lecture Hall

Abstract: Background: Recently it was demonstrated how the heart itself could be enabled to quickly restore its rhythm by realizing a biologically-integrated cardiac defibrillator (BioICD) through modification and subsequent expression of ion channels in cardiomyocytes. By incorporating these frequency-dependent depolarizing ion channels, abnormal cardiac rhythm could be rapidly detected and terminated to restore sinus rhythm in a fully biological and shock-free manner. However, from a translational point of view, it remains unclear how such rhythm restoration can be realized via ion channel gene therapy.

Purpose: To explore and understand the importance of the distribution and number of BioICD-expressing cardiomyocytes in realizing fully biological restoration of cardiac rhythm.

Methods: To this purpose, two different realistic gene therapy configurations, i.e. those corresponding to systemic and local transgene delivery, were tested in a digital twin of human ventricular cardiac monolayers. For the systemic delivery group, BioICD-expressing cells were homogeneously distributed over the tissue with fixed total expression percentage. For the local delivery group, circular areas were given BioICD-expressing cells, randomly patterned in a Gaussian distribution. In both groups spiral waves were initiated and subsequently studied for 10 seconds. For systemic delivery, an additional set of simulations was performed for an adapted BioICD ion channel with a slower opening and faster closing rate.

Results: Upon comparing both gene therapy methods, the systemic approach showed a gradually increasing arrhythmia termination chance with increasing BioICD-expression percentage, while local delivery did not result in termination. Instead, local delivery resulted in islands of ionic heterogeneity, causing attraction and anchoring of the spiral waves in a size and distance-dependent manner. Building on the results of the systemic approach, different ion channel parameters were tried, which resulted in normal rhythm being restored in all cases for more than 50% BioICD expressing cells. Time till termination was inversely related to the percentage, resulting in only 4.3s and 2.5s for 50% and 100%, respectively. Regarding termination, it was observed that conduction blocks appeared throughout the tissue and subsequently connected to force arrhythmic waves to terminate, while this process remained incomplete in the less than 50% groups.

Conclusion: This study reveals that wide-spread distribution of BioICD-expressing cardiomyocytes is required for the realization of fully biological self-restoration of cardiac rhythm, of which the efficiency is ion-channel-dynamics- and dosage-dependent. Local expression, however, results in stabilization of spiral wave activity. Further exploration of this emerging concept of biological cardioversion may not only expand our understanding of cardiac arrhythmias, but also pave the way to breakthrough advances in arrhythmia management.

Development of alternans in cardiac tissues on bio-mimicking extracellular matrices

Julia Erhardt (University of Stuttgart)

Wednesday, July 31, 15:00–15:30, Research III Lecture Hall

Abstract: The extracellular matrix (ECM) is a complex network of structural molecules and proteins that play a vital role in the structure and function of all tissues. In cardiac tissue, maintaining a healthy contractile

rhythm hinges on the ECM's unique protein composition and stiffness. However, an overproduction of ECM components can cause tissue stiffening, making it more susceptible to life-threatening diseases, as for example cardiac arrhythmias. One precursor phenomenon that could shed more light on the subject of cardiac arrhythmias is alternans. Alternans is a phenomenon characterized by beat-to-beat oscillations in the electromechanical conduction of the heart tissue. While previous studies have acknowledged the correlation between ECM stiffness and alternans development, the precise dynamics and onset of alternans in relation to ECM stiffness and tissue composition remain to be elucidated. In this study, we examine alternans of calcium dynamics in bioengineered cardiac tissue sheets using high-speed life-imaging techniques and surface protein-functionalized hydrogels with adjustable rigidity to mimic various physiological ECM conditions. We advance towards gaining deeper insights on the development and transition between different alternans modes in respect to ECM conditions for an early detection of electromechanical instabilities in cardiac tissue.

MODELING MAGNETIC FIELD IN CARDIAC CELL CULTURES

Martina Nicoletti (Università Campus Bio-Medico di Roma), **Alessandro Loppini** (Università Campus Bio-Medico di Roma), **Christian Cherubini** (Università Campus Bio-Medico di Roma), **Alessio Gizzi** (Università Campus Bio-Medico di Roma), **Letizia Chiodo** (Università Campus Bio-Medico di Roma), **Simonetta Filippi** (Università Campus Bio-Medico di Roma)

Wednesday, July 31, 16:00–16:30, Research III Lecture Hall

Abstract: Understanding the origins of cardiac instabilities and the progression of arrhythmias is crucial for clinical applications. Conventional methods in studying cardiac dynamics rely on analyzing voltage and calcium signals to study the transition from regular to irregular rhythms. In this regard, novel quantum sensing technologies are opening the way to investigating the cardiac magnetic field at the cell and tissue scale, possibly leading to deeper insights into arrhythmogenesis [1,2]. These promising magnetometry techniques ask for comprehensive models that can reproduce the complex electromagnetic activity of cardiac tissue, eventually incorporating other multiphysics features. In the present contribution, we introduce a novel theoretical framework combining Maxwell equations with state-of-the-art thermo-electric models of cardiac tissue [3,4], deriving a comprehensive thermo-electro-magnetic model of cardiac activity. We resolve the model on unidimensional cables and quasi-bidimensional domains, with finite differences and finite elements numerical implementations. Specifically, one-dimensional domains are simulated to analyze cardiac magnetic field behavior during restitution protocols, and quasi-bidimensional simulations are conducted to analyze cardiac magnetic response with canonical activation patterns: planar, circular, and spiral waves. The effect of tissue thermal state on the generated magnetic field is further assessed.

- [1] J. Barry et al. "Optical magnetic detection of single-neuron action potentials using quantum defects in diamond". 2016, PNAS, 113 (49) 14133-14138.
- [2] K. Arai et al. "Millimetre-scale magnetocardiography of living rats with thoracotomy". 2022, Communication Physics, 5, (200).
- [3] A. Loppini et al. "Thermal effects on cardiac alternans onset and development: A spatiotemporal correlation analysis". 2021, Phys. Rev. E 103, L040201.
- [4] A. Loppini et al. "Spatiotemporal correlation uncovers characteristic lengths in cardiac tissue". 2021, Phys. Rev. E 100, 020201(R).

Electrophysiological Structural Modeling of Ventricular Tachycardia based on Patient-Specific Electro-Anatomical Mapping

Anna Crispino (Università Campus Bio-Medico di Roma), **Bich Lien Nguyen** (La Sapienza University, Policlinico Umberto I, Italy), **Alessandro Loppini** (Università Campus Bio-Medico di Roma), **Alessio Gizzi** (Università Campus Bio-Medico di Roma)

Wednesday, July 31, 16:30–17:00, Research III Lecture Hall

Abstract: Ventricular tachycardia (VT) is a noteworthy clinical condition observed in patients with structural heart disease, commonly associated with irregular or incomplete scar formation. This pathological state gives rise to reentry circuits that rely on conduction pathways, commonly known as isthmi [1, 2]. Advances in electro-anatomical mapping techniques have facilitated the identification and characterization of such

critical ventricular regions with enhanced spatial resolution [3], integrating subsequent catheter ablation procedures. Detailed anatomical maps of the cardiac chamber offer concurrent information on tissue voltage and local activation times under different pacing regimes. Detecting low-voltage or late-activated zones is in fact essential for locating isthmus sites and highlight reentrant action potential dynamics, thereby establishing the basis for effective VT ablation strategies [4].

In the present contribution, we incorporate clinical data into patient-specific left ventricular geometries reconstructed by means of the CARTO 3 mapping system (Biosense Webster). We propose a voltage-based modeling approach fine-tuning local material diffusivities discriminating healthy and diseased tissue (ischemic regions). The electrophysiological features of the intraventricular excitation are explained by means of a monodomain model [5] with a heterogeneous phenomenological representation of the ionic currents [6].

Extended *in silico* analyses demonstrate the pro-arrhythmic role associated with ischemic regions and border zone transitions, thus enabling a comprehensive examination of post-infarct VT reentrant circuits. It includes characterizing and reproducing isthmi location and size, thus benchmarking advanced electro-anatomic mapping techniques. The developed patient-specific tool provides valuable insights to identify optimal sites for catheter ablation procedures in view of minimizing energy release and patient outcomes.

- [1] E. Ciaccio, et al. "Model of reentrant ventricular tachycardia based on infarct border zone geometry predicts reentrant circuit features as determined by activation mapping," *Heart Rhythm*, vol. 4, n. 8, pp. 1034-1045, 2007.
- [2] V. Gionti, et al. "Role of Scar and Border Zone Geometry on the Genesis and Maintenance of Re-Entrant Ventricular Tachycardia in Patients With Previous Myocardial Infarction," *Frontiers in Physiology*, vol. 13, 2022.
- [3] R. Martin, et al. "Characteristics of Scar-Related Ventricular Tachycardia Circuits Using Ultra-High-Density Mapping," *Circulation. Arrhythmia and Electrophysiology*, vol. 11, n. 10, p. e006569, 2018.
- [4] K. Vlachos, et al. "The value of functional substrate mapping in ventricular tachycardia ablation," *Heart Rhythm O2*, vol. 4, n. 2, pp. 134-146, 2022.
- [5] A. J. Pullan, et al. "Mathematically Modelling the Electrical Activity of the Heart," World Scientific, 2005.
- [6] C. C. Mitchell and D. G. Schaeffer, "A two-current model for the dynamics of cardiac membrane," *Bulletin of Mathematical Biology*, pp. 767-793, 2003.

Physical Processes That Shape Electric Image of Cardiac Muscle

Teodor Buchner (Warsaw University of Technology), Sebastian Wildowicz (Warsaw University of Technology), Tomasz Gradowski (Warsaw University of Technology), **Igor Olczak** (Warsaw University of Technology), Paulina Figura (Childrens's Memorial Health Institute), Judyta Sobiech (Warsaw University of Technology)

Wednesday, July 31, 17:00–17:30, Research III Lecture Hall

Abstract: Electrical activity of cardiac muscle: its generation propagation and measurement is subject to a number of physical principles. Some of them put constraints on the degrees of freedom of this spatially extended dynamical system. The others - on the contrary - add new dimensions that are sometimes quite unexpected. The generation of the cardiac signal, which is probably the one most thoroughly understood has certain peculiarities. Firstly, the epicardium is only connected to the body via capacitive coupling, thus the heart resembles an antenna, which communicates with the outside tissue entirely through displacement currents. Secondly, the response of the tissue can be modelled in one of two languages - following traditional von Hippel approach to the microwaves. Application of molecular language reveals the dependence on properties of the electrolyte, such as ionic strength of the body fluids [1]. Application of Maxwell language allows to determine the propagation velocity. We recently found it to be as low as 1500 m/s, which is in accordance with the refractory index of the body at single Hz frequencies [2]. Many other consequences follow the assumption on a physical structure and properties of the medium. One example is the frequency response, which is limited mostly to the frequency characteristics of the skin-electrode contact. We show, that the property of paramount importance is the impedance spectrum of the tissue. We also show the impact of mesoscopic: i.e. the cellular structure. The physics has to be considered much more than just to label body compartments with single values of conductivity.

A final image which is observed on the surface of the body can be decomposed into separate physical

processes. and we present a method to distinguish two of them: the individual response of an average cardiomyocyte to electric stimulation and the global dynamics of a wavefront of electric activity. We build our theory on an assumption, that the characteristic shape of the ECG comes from spatial derivative, as by the very principle of electric measurements, they are always bipolar. Therefore we are only able to measure a spatially asymmetric component of the cardiac biopotential, which comes from this spatially extended and essentially nonlinear system. Results of an analysis of PTB ECG database will be presented, showing both the normal dynamics and the alterations related to various cardiological aberrations. In consequence we show, why the underlying physics cannot be neglected, when biopotential analysis is considered. We postulate to extend existing models in order to capture more details of the physical nature of biopotentials.

- [1] Buchner T. On the physical nature of biopotentials, their propagation and measurement. *Physica A: Statistical Mechanics and its Applications* 525, 85-95, (2019).
- [2] Buchner, T., Zajdel, M., Pęczalski, K. et al. Finite velocity of ECG signal propagation: preliminary theory, results of a pilot experiment and consequences for medical diagnosis. *Sci Rep* 13, 4716 (2023).

Minisymposium on Multistability and global stability analysis

Introduction to multistability and global stability

George Datsoris (University of Exeter), Karel Luiz Rossi (University of Oldenburg, ICBM)
Monday, July 29, 17:00–17:30, IRC Seminar Room 1

Abstract: This talk will introduce the content of the minisymposium. We will present an overview of central topics and current research in multistability and global stability analysis. We will provide a short introduction to the talks constituting the minisymposium. We will also provide a brief highlight into doing multistability and global stability analysis with the software *DynamicalSystems.jl*.

Resilience Indicators in Dynamical Systems

Hana Krakovská (Medical University of Vienna), Christian Kuehn, Iacopo P. Longo
Monday, July 29, 17:30–18:00, IRC Seminar Room 1

Abstract: Over the recent decades, various frameworks and metrics have emerged to define and explore the concept of “global” (non-local) stability, or “resilience”. Largely motivated by practical applications, there has been an abundance of diverse interpretations and concepts, but often they lacked rigorous definitions. For this reason, a unified mathematical framework with precise definitions is needed. With this objective in mind, we review commonly used metrics from a broad range of fields and formalize them for attractors in continuous dynamical systems. We propose a novel categorization based on their mathematical attributes and extend their applicability to any attractor. This systematic approach aims to enhance the rigor and quantitative comparability of resilience studies in dynamical systems. Additionally, we apply the formalized resilience metrics to a simple ordinary differential equation model from population dynamics with an Allee effect. Our analysis shows that the choice of resilience metrics influences conclusions regarding the attractor’s resilience. Therefore, careful consideration of the nature of perturbations affecting the attractor and the selection of an appropriate resilience metric is important. Furthermore, we explore the predictive capability of these metrics in identifying an approaching bifurcation as the system parameters vary. This investigation underscores the utility of resilience metrics in anticipating critical transitions within dynamical systems.

The space of power grid actors - Characterizing the stability of future power grids

Frank Hellmann (Potsdam-Institut für Klimafolgenforschung (PIK) e.V.)
Monday, July 29, 18:00–18:30, IRC Seminar Room 1

Abstract: Future power grids will be dominated by power electronic devices that provide a synthetic dynamic response to disturbances. I present a technology neutral description of the space of plausible behaviors of such actors. Using this space, we can characterize linear and non-linear stability properties independently of concrete system models. I give results on i) characterizing the single node basin stability of such systems,

ii) using Graph Neural Networks to predict where instabilities may occur, and iii) give sufficient analytic conditions that guarantee the linear stability of the synchronous state in arbitrary topologies.

Solitary states in complex networks: how multistability arises due to synchronization, resonance, or decoupling

Jakob Niehues (Potsdam Institute for Climate Impact Research (PIK), Technische Universität Berlin), Serhiy Yanchuk (Potsdam Institute for Climate Impact Research (PIK), Humboldt-Universität zu Berlin, University College Cork), Rico Berner (Humboldt-Universität zu Berlin, Ambrosys GmbH), Jürgen Kurths (Potsdam Institute for Climate Impact Research (PIK), Humboldt-Universität zu Berlin), Frank Hellmann (Potsdam Institute for Climate Impact Research (PIK)), Mehrnaz Anvari (Fraunhofer Institute for Algorithms and Scientific Computing)

Tuesday, July 30, 11:00–11:30, IRC Seminar Room 1

Abstract: Networks of oscillators exhibit a rich behavior ranging from complete synchronization, over partially synchronized so-called solitary states, cluster synchronization and chimeras, to full disorder. In this talk, we focus on solitary states that occur frequently when a synchronous system of networked oscillators is perturbed locally. Remarkably, several asymptotic states of different frequencies can coexist at the same node. Their understanding is crucial in applications such as power grid stability. Here, we uncover the mechanism underlying this multistability: In addition to a well-understood decoupled state, the resonant back-reaction of the network's modes on the solitary can lead to a large energy transfer between them, and the solitary's frequency adaptation can tune the system to this resonance.

Multistability and malleability in networks of heterogeneous Kuramoto oscillators

Kalel Luiz Rossi (University of Oldenburg), Everton Medeiros (University of Oldenburg), Ulrike Feudel (University of Oldenburg)

Tuesday, July 30, 11:30–12:00, IRC Seminar Room 1

Abstract: Networks of Kuramoto phase oscillators are paradigmatic models for studies on synchronization, and serve as very simple models to study real-world systems like brain circuits. In the case of heterogeneous oscillators coupled under complex topologies, previous works have shown that (i) the systems transition to phase synchronization when their coupling strength is increased, (ii) the systems' sensitivity to parameter changes (malleability) increases considerably during this transition, such that even single-unit changes can radically alter the network's synchronization, (iii) the number of coexisting attractors (multistability) also increases near the transition to phase synchronization. In this work we first investigate the multistability, identifying how the attractors emerge and how they change under different coupling strengths. We then study the relation between multistability and malleability using a global stability framework, measuring the size of the basins of attraction and identifying how they change as the coupling strength changes. We find that for stronger coupling strengths the most synchronized attractor has the biggest basin of attraction, such that it is also the most robust (least malleable attractor). We also investigate how some network properties, such as sparsity, influence the multistability that emerges.

A global stability measure for complex networks and its relation to network topology

Ulrike Feudel (Carl von Ossietzky Universität Oldenburg,), Lukas Halekotte (DLR-Institute for the Protection of Terrestrial Infrastructure), Anna Vanselow (Carl von Ossietzky Universität Oldenburg)

Tuesday, July 30, 13:30–14:00, IRC Seminar Room 1

Abstract: Natural or technical systems possess often several possible stable states of operation. Linear stability theory is the appropriate tool to study the stability properties of such states with respect to small perturbations. However, in nature perturbations are not necessarily small but are finite in size. We discuss a method to investigate the stability with respect to large perturbations such as single shocks. This method aims to determine the distance to the boundary of the basin of attraction by determining the minimal destabilizing perturbation for large dynamical systems such as networks. Besides the size of this perturbation the method allows also to obtain the direction of it. We illustrate this method using two different applications:

(1) plant-pollinator networks in ecology in which the nodes correspond to the plant and pollinator species and the links reflect their mutualistic interactions and (2) power grids in which the nodes represent either generators or consumers of electrical power, while the edges mimic the electricity transportation network.

Moreover, we use such shock perturbations to identify relations between the topology of a network and its global stability properties. For both types of applications, we study the impact of shock perturbations on single nodes to identify the most vulnerable nodes and topological structures in a network.

For the plant-pollinator networks we examine the relation between the endangerment of pollinator species and their position within the mutualistic network. To this end we compare endangerment rankings which are derived from the species' probabilities of going extinct due to random shock perturbations with rankings obtained from different network theoretic centrality metrics. We find that a pollinator's endangerment is strongly linked to its degree of mutualistic specialization and its position within the core-periphery structure of its mutualistic network, with the most endangered species being specialists in the outer periphery. For the example of the power grid of Great Britain we find that the shock perturbations at many nodes only allow for a few outcomes, while the stability landscape of a few nodes show extreme complexity with more than a hundred basins of attraction. Particularly complex domains in the latter case can be related to unstable invariant chaotic sets of saddle type. Most importantly, we show that the characteristic dynamics on these chaotic saddles can be associated with certain topological structures of the network. We find that one particular tree-like substructure allows for the chaotic response to perturbations at nodes in the north of Great Britain.

Data-driven anticipation and prediction of Atlantic Meridional Overturning Circulation collapse using non-autonomous spatio-temporal dynamical modelling

Frank Kwasniok (University of Exeter)

Tuesday, July 30, 14:00–14:30, IRC Seminar Room 1

Abstract: The Atlantic Meridional Overturning Circulation (AMOC) was identified as a potential tipping element in the earth system. In a hierarchy of models of increasing complexity, ranging from simple box models to intermediate-complexity models to state-of-the-art climate models, the AMOC exhibits bistability with an 'on-state' characterised by a functioning overturning circulation and an 'off-state' in which the overturning circulation shuts down. The proximity of the present AMOC to a critical transition threshold is under debate. The impact of an AMOC collapse on the climate in many parts of the world would be enormous.

In this study, a data-driven methodology for identifying, anticipating and predicting critical transitions in high-dimensional model or observational data sets is introduced, based on explicit non-stationary low-order modelling of the tipping dynamics. Unlike the more traditional early-warning signs such as rising autocorrelation (critical slowing down) or rising variance, this allows for dynamical understanding of the underlying tipping mechanism and genuine prediction of the future system state by extrapolation. A set of spatial modes carrying the tipping dynamics are identified and a stochastic model of appropriate complexity is estimated in the subspace spanned by these modes. Analysis of the reconstructed dynamics allows to determine the proximity to a bifurcation point and the type of the impending bifurcation. Different competing tipping mechanisms can be compared and assessed using likelihood inference and information criteria. The technique is inherently probabilistic with the randomness stemming from two sources: the statistical uncertainty in the model estimation and the inevitably stochastic nature of the low-order tipping model. As such, the method allows to quantify the likelihood or risk of a critical transition at some point in the future having observed a certain amount of data up to present.

The methodology is here applied to a data set from a simulation of AMOC collapse with a complex climate model, actually a freshwater hosing experiment with the FAMOUS GCM. The AMOC on-state is found to lose stability via a subcritical Hopf bifurcation; however, the transition to the off-state occurs far ahead of the bifurcation point. The early collapse can be explained by a combination of rate-induced and noise-induced tipping. The finding is relevant in terms of safe operating zones as the risk of AMOC collapse in such a scenario would be higher than anticipated with conventional early-warning signs.

Heteroclinic birth of a preponderance of states in the ring of Janus oscillators

Zachary Nicolaou (University of Washington)

Tuesday, July 30, 14:30–15:00, IRC Seminar Room 1

Abstract: The ring of Janus oscillators, a model consisting of coupled pairs of detuned Kuramoto oscillators, exhibits a surprising extent of multistability. For intermediate coupling constants, previous studies have reported the existence of a multitude of traveling chimera states, which are symmetry-broken periodic states composed of a localized and traveling group of asynchronous oscillators moving through a larger synchronous group. Recently, we recognized the role of parity and time reversal symmetries in this model, which constrain the Floquet multipliers of limit-cycle solutions, giving rise to higher-order bifurcation phenomena. I will detail the results of numerical continuation for these traveling chimera states, documenting their emergence through global bifurcations of heteroclinic cycles and unstable fixed points.

Minisymposium on Non-autonomous dynamics in complex systems: theory and applications to biological, ecological, and climate systems

Data driven methods to identify transitions between metastable states

Klaus Lehnertz

Monday, July 29, 13:30–14:00, Research V Seminar Room

Abstract: In this talk, I shall discuss ways to detect critical transitions in complex systems based on analyses of observations of their non-autonomous dynamics. Topics include segmentation techniques for investigating non-stationary dynamics, univariate and multivariate time-series-analysis techniques as well as surrogate-based concepts to evaluate reliability of detections. Special attention is paid to sources of errors and potential remedies.

Tipping in an ecological system under external forcing

Syamal Kumar Dana (Jadavpur University)

Monday, July 29, 14:00–14:30, Research V Seminar Room

Abstract: We discuss about a kind of tipping from a high population outbreak state to a low population refuge state and vice versa in one-dimensional species model under an external triangular pulse-like forcing. The forcing acts on the carrying capacity with a rising and falling rate of the pulse. Tipping occurs when the carrying capacity crosses the saddle-node bifurcation point of the system. However, such tipping depends upon the critical rate of rise and fall of the pulse, and hence this tipping shows a dual character of bifurcation-induced tipping and rate-induced tipping. When the rate of change of the carrying capacity is faster than the critical value, even though the carrying capacity may cross the saddle-node bifurcation point, the system traces back to its original state after a while although it shows an initial tendency of tipping. In such a case a low amplitude second weaker pulse that never crosses the saddle-point, the system tips to the alternate state. The time interval between the first pulse and the second one is also crucial.

Uncertainty quantification for overshoots of tipping thresholds

Paul Ritchie (University of Exeter), Kerstin Lux-Gottschalk (Eindhoven University of Technology)

Monday, July 29, 14:30–15:00, Research V Seminar Room

Abstract: To tip or not to tip? Many subsystems of the Earth are at risk of undergoing abrupt transitions from their current stable state to a drastically different and often less desired state due to anthropogenic climate change. These so-called tipping events often present severe consequences for ecosystems and human livelihood that are difficult to reverse. One common mechanism for tipping to occur is via forcing a nonlinear system beyond a critical threshold, that signifies the start of self-amplifying feedbacks, inducing tipping. However, previous work has shown that it is possible to briefly overshoot a critical threshold and avoid tipping. Specifically, the peak distance of an overshoot and the time a system can spend beyond a threshold are governed by an inverse square law relationship. In the real world or complex models, critical thresholds

and other system features determining the permitted overshoot are highly uncertain. In this presentation, we look at how such uncertainties affect the probability of tipping from the perspective of uncertainty quantification. We show the importance of constraining uncertainty in the location of the critical threshold and the linear restoring rate to the system's stable state to reduce the uncertainty in the probability of tipping. We will first prototypically analyse effects of an uncertain location of the tipping threshold separately from effects due to an uncertain linear restoring rate. These effects of uncertain system characteristics will be jointly analysed in a simple box model for the Atlantic Meridional Overturning Circulation (AMOC). Thereby, we highlight the need to constrain the high uncertainty attributed to wind-driven fluxes represented by a diffusive timescale within the box model to reduce uncertainty in the tipping probability for overshoot scenarios of the AMOC.

Biological computations with transients

Aneta Koseska (Max Planck institute for Neurobiology of Behavior - caesar)

Monday, July 29, 15:00–15:30, Research V Seminar Room

Abstract: A growing body of empirical evidence suggests that neuronal and biochemical networks are often characterized by long transients which are quasi-stable, with fast switching between them. The duration of the quasi-stable patterns is much longer than one would expect from the characteristic elementary processes of the system, whereas the switching is triggered by external signals or system-autonomously, and occurs on a timescale much shorter than the one of the preceding dynamical pattern. Generalizing the concept of ghost states, we provide a theoretical framework that accounts for emergence of transiently stable phase-space flows generated by ghost scaffolds. We demonstrate that ghost-based phase space objects account for emergence of advanced information processing capabilities, including on-the-fly signal classification, temporal integration and context-dependent responses.

Populations of bacteria exposed to a changing environment

Hildegard Meyer-Ortmanns (Constructor University), Bhumika Thakur (formerly Jacobs University Bremen)

Monday, July 29, 17:00–17:30, Research V Seminar Room

Abstract: We study populations of two phenotypes of bacteria which are exposed to fluctuating environmental conditions as well as to demographic fluctuations due to birth and death processes. We are interested in the impact of this exposure on the mean time to extinction as their final fate. This time should be as large as possible from the perspective of bacteria and as short as possible from the perspective of their host for which their presence has a detrimental effect. As it turns out, the survival probability of the bacteria depends non-monotonically on the frequency of application of adverse environmental conditions, provided, for example, by antibiotic drugs. A minimal survival time of the bacteria is achieved by choosing the frequency out of a small interval of intermediate values, whereas the dose of antibiotics should be chosen as large as possible while still tolerable for the host. In general, the frequency of administration should be smaller than the one for which the system sees only the average birth rate of bacteria and large enough that the bacteria are able to register the change of environmental conditions. Our results are based on Gillespie simulations and, in case of rare extinction events, on the WKB approach applied to classical stochastic systems, here in certain limiting cases.

Transit times for nonautonomous compartmental systems with application to the terrestrial carbon cycle

Martin Rasmussen (Imperial College London)

Monday, July 29, 17:30–18:00, Research V Seminar Room

Abstract: We develop a theory for transit times and mean ages for nonautonomous compartmental systems. We show that the mean ages of mass in a compartmental system satisfy a linear nonautonomous ordinary differential equation that is exponentially stable. We then define a nonautonomous version of transit time

as the mean age of mass leaving the compartmental system at a particular time. We apply these results a nonautonomous compartmental systems modeling the terrestrial carbon cycle.

Joint work with Alan Hastings, George Chappelle, Matthew Smith, Yiqi Luo, Folashade Agosto, Benito Chen-Charpentier, Forrest Hoffman, Jiang Jiang, Katherine Todd-Brown, Ying Wang, Ying-Ping Wang.

The role of edge states for rate-induced tipping of the Atlantic ocean circulation

Reyk Börner (University of Reading), Oliver Mehling (Politecnico di Torino), Jost von Hardenberg (Politecnico di Torino), Valerio Lucarini (University of Leicester)

Monday, July 29, 18:00–18:30, Research V Seminar Room

Abstract: The multistability of Earth's climate implies the risk of critical transitions between competing attracting states, triggered by random fluctuations or changes in external forcing. A major climate subsystem with metastable behavior is the Atlantic Meridional Overturning Circulation (AMOC), a large-scale ocean current believed to have two stable flow regimes: today's vigorous flow and a much weaker flow that would severely alter global climate. While recent studies suggest that the AMOC is approaching a future collapse, the prediction of such a 'tipping event' remains highly uncertain. Particularly, it becomes increasingly clear that early-warning methods derived from the theory of autonomous systems may not be appropriate given the fast rate of ongoing climate change. This requires a better understanding of transition behavior in non-autonomous systems.

Here we explore the global stability of the AMOC through the study of an edge state (or "Melancholia state") — a chaotic saddle embedded in the boundary separating the basins of attraction of the two stable flow regimes. Using an edge-tracking algorithm, we construct the edge state in a fully coupled earth system model of intermediate complexity. We characterize the dynamical and climatological properties of the edge state for different external forcing conditions and analyze its relevance for rate-induced transitions generated under time-varying forcing. An interesting connecting feature are large unstable oscillations of the AMOC strength arising from multiscale interactions in the model. We complement our results with insights from a conceptual ocean-atmosphere model which allows us to discuss rate-induced tipping in the presence of long chaotic transients. Edge states are known to play a central role for transient behavior near bifurcations and as gateways of noise-induced transitions under weak stochastic forcing. More recently, edge states have also been introduced as a concept to understand genuinely non-autonomous transitions. Advancing the theory and practical computation of edge states in non-autonomous settings could offer new ways to anticipate critical transitions in complex systems.

Predictability of tipping the ocean circulation under non-autonomous forcing

Johannes Lohmann

Monday, July 29, 18:30–19:00, Research V Seminar Room

Abstract: Due to the possibility of multi-stability in several parts of the climate system, the evolution of the climate may split into a number of qualitatively different 'storylines' for a given scenario of future climate change. This is because of a phenomenon referred to as 'partial tipping', where non-autonomous forcing can deposit initial conditions belonging to different regions of an attractor onto basins of attraction of different asymptotic states. Due to chaos, there can be a sensitive dependence on initial conditions and forcing trajectory of which storyline is realized by the system. But the probabilities of each storyline may nevertheless be computed by suitable ensemble simulations.

We explore this phenomenon in the context of a tipping point of the Atlantic meridional overturning circulation (AMOC), as simulated by a global ocean model. Different types of time-varying forcing are investigated, including monotonic and non-monotonic increases in meltwater forcing, as well as stochastic temperature forcing, such as from volcanic eruptions and atmospheric extremes. In the latter case, predictability is naturally limited, but we investigate whether ongoing stochastic transitions may be detected by fingerprints of an edge state.

Minisymposium on Nonautonomous dynamical systems

Relating the future to the past - nonautonomous systems that are asymptotically autonomous

Peter Ashwin (University of Exeter)

Thursday, August 1, 11:00–11:30, Research III Lecture Hall

Abstract: Dynamical systems with asymptotic autonomous limits are more analysable than those without: the nonautonomous intermediate dynamics can be related to the limiting dynamics. As such, they are a useful tool to understand applications with for example non-recurrent time dependent forcing and this is particularly true of forced complex systems with chaotic dynamics. In this talk I will discuss some results and applications (especially for climate dynamics) on physical measures and nonautonomous/rate-dependent instabilities and tipping points for such systems.

Chain transitivity properties of affine flows on vector bundles

Fritz Colonius (University of Augsburg)

Thursday, August 1, 11:30–12:00, Research III Lecture Hall

Abstract: Affine skew product flows on vector bundles with chain transitive base flow are lifted to linear flows on an extended state space. Then the decomposition into exponentially separated subbundles provided by Selgrade's theorem is determined. This yields information on the chain recurrent set of the original affine flow. The results are illustrated by applications to affine control systems with bounded control range.

Rate-induced tipping in non-autonomous reaction-diffusion systems: an invariant manifold framework and shifting habitats

Sebastian Wiczorek (University College Cork (UCC))

Thursday, August 1, 13:30–14:00, Research III Lecture Hall

Abstract: Mathematical modelling of tipping points - large and sudden changes in the state of an ecosystem that arise in response to small and slow changes in the external inputs - has mainly focused on ordinary differential equation models. In this talk, we will consider spatially extended ecosystems modelled by reaction-diffusion equations (RDEs), where the reaction term decays in space (asymptotically homogeneous) and varies linearly with time (nonautonomous). Such models are likely to exhibit new and interesting tipping mechanisms, but their analysis is more challenging and requires new mathematical techniques.

As an illustrative example, we analyse a conceptual model of a finite habitat patch in one spatial dimension, that features an Allee effect in population growth and is geographically shrinking or shifting due to human activity and climate change. First, we identify two classes of tipping points to extinction: bifurcation-induced tipping (B-tipping) when the shrinking habitat falls below some critical length, and rate-induced tipping (R-tipping) when the shifting habitat exceeds some critical speed. We then obtain two-parameter R-tipping diagrams to understand how the critical speed depends on the size of the habitat patch and the dispersal rate of the population, uncover parameter regions where the shifting population survives, and relate the critical speed to the invasion speed in an infinite homogeneous habitat.

To facilitate analysis of tipping points in RDEs, such as the moving habitat model, we propose a new mathematical framework. This framework is underpinned by a special compactification of the moving-frame coordinate in conjunction with Lin's method to construct heteroclinic orbits along intersections of stable and unstable invariant manifolds, and allows us to (i) obtain multiple coexisting pulse and front solutions for the RDE by computing heteroclinic orbits connecting equilibria at negative and positive infinity in the compactified moving-frame ordinary differential equation, (ii) detect tipping points as dangerous bifurcations of such heteroclinic orbits, and (iii) obtain tipping diagrams by numerical continuation of such bifurcations

Some remarks on Melnikov chaos for smooth and piecewise smooth systems

Matteo Franca (University of Bologna (UNIBO))

Thursday, August 1, 14:00–14:30, Research III Lecture Hall

Abstract: It is well known that a smooth autonomous system which has a homoclinic trajectory (i.e. a trajectory converging to a critical point as $t \rightarrow \pm\infty$) and subject to a small periodic forcing may exhibit a chaotic pattern. A motivating example in this context is given by a forced inverted pendulum.

Melnikov theory provides a computable sufficient condition for the existence of a transversal intersection between stable and unstable manifolds: in a smooth context this is enough to guarantee the persistence of the homoclinic and the insurgence of chaos.

In this talk we show that in piecewise smooth system with a transversal homoclinic point a generic geometrical obstruction forbids chaotic phenomena which are replaced by new bifurcation scenarios. Further, if this obstruction is removed, chaos may arise again. Piecewise smooth system are motivated by the study of dry friction, state dependent switches, or impacts.

In fact we will also show some results new in a smooth context, concerning multiplicity, position and size of the Cantor set Σ of initial conditions from which chaos emanates. In particular we will see that, even if the perturbation is $O(\varepsilon)$, we may find infinitely many distinct Cantor set Σ located in the same $O(\varepsilon^\nu)$ neighborhood of the critical point, each corresponding to a different pattern, and where $\nu > 1$ is as large as we wish.

A global bifurcation diagram for a one-parameter family of nonautonomous scalar ODE's driven by a minimal flow

Roberta Fabbri (Università di Firenze), Carmen Nunez (UNIVERSIDAD DE VALLADOLID)

Thursday, August 1, 14:30–15:00, Research III Lecture Hall

Abstract: Nonautonomous bifurcation is of growing interest in the scientific community, both for its theoretical interest and for its possible applications to the analysis of mathematical models. In the talk, in the wake of [1] and using results and methods of [2] conditions on the coefficients of the one-parameter family

$$x'(t) = \varepsilon[a(t) + b(t)x(t)] + c(t)x^2(t) - x^3(t)$$

as ε varies are established to describe the global diagram of the motion. Such conditions include the recurrency of the coefficients and the analysis is based on the study of the number and the structure of the minimal invariant subsets of the extended state space for the corresponding skew-product flow induced by the solutions of the family.

[1] R. Fabbri, R. Johnson, F. Mantellini, A nonautonomous saddle-node bifurcation pattern, *Stoch. Dyn.* ,4 (2004), no 3, 335–350.

[2] J. Duenas, C. Nunez., R. Obaya, Bifurcation theory of attractors and minimal sets in d-concave nonautonomous scalar ordinary differential equations, *J. Differential Equations*, 361 (2023) 138–182.

Lagrangian flow networks for transport and mixing studies in fluids

Kathrin Padberg-Gehle (Leuphana Universität Lüneburg)

Friday, August 2, 11:00–11:30, Research III Lecture Hall

Abstract: Transport and mixing processes in fluid flows are crucially influenced by coherent structures and the characterisation of these Lagrangian objects is a topic of intense current research. Recently, computational approaches have been proposed to identify coherent sets directly from an ensemble of trajectories by means of a Lagrangian flow network, where the links are weighted according to spatio-temporal distances between tracer trajectories. In this talk, we propose some extensions to the network-based framework that allow us to study the long-term dynamics of coherent sets, including splits and mergers, and to quantify mixing. We demonstrate the applicability of these methods in a number of example systems, including turbulent convection.

Rate-dependent crossing of bifurcations: The role of unstable states and basin boundaries

Ulrike Feudel (Carl von Ossietzky Universität Oldenburg,), Bálint Kaszás (ETH Zurich), Tamás Tél (Eötvös Loránd University Budapest)

Friday, August 2, 11:30–12:00, Research III Lecture Hall

Abstract: Many systems in nature exhibit the coexistence of different stable states for a given set of environmental parameters and external forcing. Examples for such behavior can be found in different fields of science ranging from mechanical or chemical systems to ecosystem and climate dynamics. As a consequence of the coexistence of a multitude of stable states, the final state of the system depends strongly on the initial condition. The set of initial conditions which all converge to the same stable state is called the basin of attraction. We analyze systems experiencing a parameter drift during which bifurcations are crossed that lead to the emergence of new attractors including their basins of attraction. Using an ensemble of trajectories, we study the role of the relative size of the non-autonomous basins of attraction and the location of their boundaries in rate-dependent tipping. We demonstrate that the decision whether a trajectory tips or tracks the “moving” stable state depends crucially on the changes in the non-autonomous basins of attraction, in particular on their boundaries, that also “move” in state space under a time-dependent variation of a parameter. Our ensemble approach reveals that such bifurcations occurring during the parameter drift might be ‘masked’ because the relative size of the newly formed non-autonomous basins of attractions goes to zero the slower the rate of the parameter drift. As a consequence, the whole ensemble of initial conditions evolving under parameter drift is not signaling the bifurcation. This phenomenon can be observed for smooth basin boundaries as well as for fractal ones. We show that the relationship between the timescale of the parameter drift and the intrinsic dissipative timescale is responsible for this behavior.

Global bifurcation of homoclinic solutions for Carathéodory ordinary differential equations

Iacopo P. Longo (Imperial College London), Christian Pötzsche (University of Klagenfurt), Robert Skiba (Nicolaus Copernicus University)

Friday, August 2, 12:00–12:30, Research III Lecture Hall

Abstract: We establish an alternative classification of the shape of global bifurcating branches of bounded solutions to Carathéodory ordinary differential equations. Our approach is based on the parity associated to a path of index 0 Fredholm operators and the Evans function as a recent tool in nonautonomous bifurcation theory. Similarly to the classical Rabinowitz alternative, we establish that a bifurcating branch of bounded solutions either returns to a given branch, or it fails to be compact. Under further assumptions on the Carathéodory equation (and the known solution branch) one can even establish that the bifurcating branch is unbounded.

Minisymposium on Nonequilibrium phase transitions and critical collective behavior in networks

Self-organized criticality in neural networks

Stefan Bornholdt (University of Bremen)

Wednesday, July 31, 16:00–16:30, Research V Seminar Room

Abstract: I will review mechanisms for self-organized criticality in adaptive networks and discuss their role as toy models for brain criticality. The specific properties of neural networks in the brain motivate an interesting new perspective on the phase transition in the classical spin glass model of random neural networks. This opens possibilities for an alternative SOC mechanism and poses questions about the interplay between topology and self-organization. Finally, I will briefly touch on possible relevance to fields beyond neural networks.

Canard cascading in networks with adaptive mean-field coupling

Serhiy Yanchuk (University College Cork)

Wednesday, July 31, 16:30–17:00, Research V Seminar Room

Abstract: Canard cascading is a fast-slow phenomenon characterised by a recurrent sequence of fast transitions between distinct and slowly evolving quasi-stationary states. We show that it is a genuine adaptive

network effect, scalable with network size, and does not occur without adaptation. We uncover the dynamical mechanisms behind canard cascading using an illustrative example of globally and adaptively coupled semiconductor lasers.

Collective behavior in adaptive networks

Jakub Sawicki (PIK Potsdam Institute for Climate Impact Research)

Wednesday, July 31, 17:00–17:30, Research V Seminar Room

Abstract: Adaptivity is a dynamical feature that is omnipresent in nature, socio-economics, and technology. For example, adaptive couplings appear in various real-world systems, such as the power grid, social, and neural networks, and they form the backbone of closed-loop control strategies and machine learning algorithms. For an interdisciplinary perspective on adaptive systems and its collective behavior [1], it is important to reflect on the concept and terminology of adaptivity in different disciplines. In this way, open challenges can be overcome and inspiration for interdisciplinary research approaches can be gained.

[1] J. Sawicki et al., Perspectives on adaptive dynamical systems, Chaos 33, 071501 (2023)

Preventing critical transitions by microperturbations - lessons learned from epilepsy modelling

Jaroslav Hlinka (Institute of Computer Science, Czech Academy of Sciences), Alberto Pérez-Cervera (Universitat Politècnica de Catalunya (UPC-Barcelonatech))

Thursday, August 1, 11:00–11:30, Research V Seminar Room

Abstract: Many systems show critical transitions, potentially approached by the critical slowing down path providing particular early warning signals. Being able to detect these looming transitions would be even the more beneficial, if reliable methods to prevent the transition by some intervention were available. In this contribution, we discuss some experimental and modelling evidence from studying the dynamics of epileptic seizures, promising potentially transferable insights.

Although epilepsy is the most chronic neurological disorder, the mechanisms underlying the initiation of epileptic seizure remain unknown. Epileptic seizures are related to intense activity of highly synchronized neuronal populations. These phenomena are usually preceded and followed by intervals of reduced activity, known as interictal periods. Importantly, the transient neuronal activity during these interictal periods - known as interictal epileptiform discharges - is considered a key mechanism governing the transition to seizure. However, whether these discharges prevent or facilitate that transition is still a matter of debate. It has been proposed that indeed the same perturbation might have either pro-ictal or anti-ictal effect depending on its timing with respect to the endogenous seizure dynamics [1], and that the mechanisms explaining this dual nature can be demonstrated in relatively basic dynamical models [2]. Note that similar questions arise around the optimization of therapeutical brain stimulation aiming at suppressing seizures.

In this contribution, we introduce theoretical analysis builds upon the increasing acceptance of framing epileptic dynamics as slow-fast systems, studying the phase response exhibited by a generic relaxation oscillator. Given their inherent bistable nature at the fast temporal scale, it is evident that perturbations applied to the non-seizure state with appropriate magnitude and direction can induce an immediate transition to seizure activity. Conversely, albeit less immediately apparent, perturbations of smaller magnitude have been observed to defer the spontaneous initiation of seizures. We demonstrate that this phenomenon is of a general nature. Consequently, depending on perturbation parameters such as magnitude, frequency, and temporal occurrence, a sequence of perturbations can elicit an increase, decrease, or complete suppression of seizure activity. This dependency lends itself to systematic analysis and mechanistic comprehension. We exemplify this methodology by computing isochrons, phase response curves, and the response to perturbations across several epileptic models characterized by diverse slow vector fields. Within the context of epilepsy, the findings provide insight into the mechanisms underlying seizure initiation and termination, thereby suggesting stimulation strategies capable of delay to full suppression of seizures. Moreover, as the theoretical findings are universally applicable to any planar relaxation oscillator, we hope to stimulate discussion of relevance for other systems with critical transitions.

This contribution was supported by the Czech Science Foundation project No. 21-32608S and by the ERDF-Project Brain dynamics, No. CZ.02.01.01/00/22_008/0004643.

- [1] Chang, W. C., Kudlacek, J., Hlinka, J., et al. (2018). Loss of neuronal network resilience precedes seizures and determines the ictogenic nature of interictal synaptic perturbations. *Nature Neuroscience*, 21(12), 1742-1752.
- [2] Pérez-Cervera, A., Hlinka, J. (2021). Perturbations both trigger and delay seizures due to generic properties of slow-fast relaxation oscillators. *PLoS Computational Biology*, 17(3), e1008521.

Competing attractors in multifunctional networks

Andreas Amann (University College Cork (UCC)), Andrew Flynn
Thursday, August 1, 11:30–12:00, Research V Seminar Room

Abstract: The mechanisms by which the brain is able to learn many different behaviours and to switch between them is not yet fully understood. It seems that a high level of multistability in the collective behaviour of neurons plays an important role. In this presentation we aim to better understand these mechanisms by training a reservoir computer to perform a number of different functions. Once the individual tasks are trained, it is possible to switch between them in a controlled manner.

Discrete synaptic events induce global oscillations in balanced neural networks

Alessandro Torcini (CY Cergy-Paris University, France), Denis Goldobin (Perm State University,), Matteo di Volo (Universit  Claude Bernard Lyon 1)
Thursday, August 1, 12:00–12:30, Research V Seminar Room

Abstract: Neural dynamics is triggered by discrete synaptic inputs of finite amplitude. However, the neural response is usually obtained within the diffusion approximation (DA) representing the synaptic inputs as Gaussian noise. We introduce a complete mean-field (CMF) approach for balanced neural networks, taking into account the sparseness of the network and the discreteness of the synaptic pulses, able to reproduce all the possible dynamical states. For simplicity, but without any loss of generality, we consider inhibitory balanced networks subject to an external excitatory drive. Firstly, we illustrate that the DA cannot capture oscillatory behaviours emerging for sufficiently low in-degree in spiking neural networks by considering conductance- and current-based neuronal models. However, this regime is correctly reproduced by a mean-field approach whenever the sparse and discrete synaptic inputs are taken in account. Furthermore, for Quadratic Integrate-and-Fire (QIF) neuronal network via the CMF approach we obtain a complete bifurcation diagram encompassing asynchronous and oscillatory regimes. In particular, for sufficiently low (large) excitatory drive (synaptic amplitudes) the CMF reveals bifurcations from the asynchronous irregular (AI) to the oscillatory irregular (OI) regime as well as a region of coexistence of these two phases not captured by the DA. By irregular we mean that the microscopic evolution is characterized by fluctuations in the instantaneous firing rates associated to coefficient of variations of $O(1)$. Exact event-driven simulations of large QIF networks confirm the sub- and super-critical Hopf bifurcations predicted within the CMF theory. Furthermore, for low in-degrees collective oscillations in biologically relevant frequency bands (from δ to γ band) are observable.

From chimeras to collective chaos in networks of heterogeneous Kuramoto oscillator populations

Pol Floriach (Universitat Polit cnica de Catalunya (UPC)), Jordi Garcia-Ojalvo (Universitat Pompeu Fabra), **Pau Clusella** (Universitat Polit cnica de Catalunya (UPC))
Friday, August 2, 11:00–11:30, Research V Seminar Room

Abstract: Networks of coupled oscillators are known to exhibit a wide range of dynamics, from complete synchronization to exotic chimera and chaotic states. In this work we analyze the dynamics of networks of populations of heterogeneous mean-field coupled Kuramoto-Sakaguchi oscillators and show that the instability that leads to chimera states in a simple two-population model also leads to collective chaos in large networks of interacting populations.

Formally, we describe the system as a complex network where each node represents an oscillator population whose mesoscopic behavior evolves according to the Ott-Antonsen equations for the dynamics of the

complex Kuramoto order parameter. We consider identical parameters for each population and, therefore, the system contains a manifold of homogeneous states corresponding to each population behaving identically to the rest. By means of the Master Stability Function, we study the stability of such homogeneous states to heterogeneous perturbations and obtain an analytical expression relating the eigenvectors of the connectivity matrix with the growth rate of the associated perturbation. We show that the region where non-trivial dynamics emerge occupies a wide region of the parameter space for arbitrary network sizes.

As examples, we first revisit the two-population case and show that the transverse instability corresponds to the pitchfork bifurcation that has been studied as the extension of chimera states in heterogeneous populations of oscillators in previous works. Then we explore the dynamics of a ring topology and Erdős-Rényi networks and show that transverse instabilities of the homogeneous state lead to complex chaotic regimes. In particular, we show that in both regular and random topologies the dimensionality and entropy of the chaotic dynamics scale linearly with the system size, a clear indication of extensive chaos.

In summary, our work provides a unified analytical framework to study the dynamics of networks of Kuramoto populations and shows that the celebrated chimera states transduce to chaotic regimes in large networks of oscillator populations.

Integrability of a globally coupled complex Riccati array: quadratic integrate-and-fire neurons, phase oscillators and all in between

Rok Cestnik (Lund University), Erik Andreas Martens

Friday, August 2, 11:30–12:00, Research V Seminar Room

Abstract: We present an exact dimensionality reduction for dynamics of an arbitrary array of globally coupled complex-valued Riccati equations. It generalizes the Watanabe-Strogatz theory [Phys. Rev. Lett. 70, 2391 (1993)] for sinusoidally coupled phase oscillators and seamlessly includes quadratic integrate-and-fire neurons as the real-valued special case. This simple formulation reshapes our understanding of a broad class of coupled systems - including a particular class of phase-amplitude oscillators - which newly fall under the category of integrable systems. Precise and rigorous analysis of complex Riccati arrays is now within reach, paving a way to a deeper understanding of emergent behavior of collective dynamics in coupled systems.

Adaptive dynamics and phase transitions in self-organization of vascular networks

Erik Andreas Martens (Lund University), Konstantin Klemm (Lund University)

Friday, August 2, 12:00–12:30, Research V Seminar Room

Abstract: The model by Hu and Cai [Hu and Cai, Phys. Rev. Lett., Vol. 111(13) (2013)] describes the self-organization of transport/vascular networks for fluid flow. Diameters of vessel segments evolve so as to minimize a cost functional. The cost is the sum of the power needed for pumping the fluid and of the energy consumption for vessel maintenance. There is thus a trade-off between thin vessels with lower maintenance and thicker vessels with lower dissipation in the flow, i.e., less pumping power required. Terminal nodes in the network are subject to fluctuating load. Depending on the amplitude of these fluctuations, it is found that a subset of vessels in the network transition from a non-conducting to conducting configuration. Subsequent analysis of the model, however, has paid little attention to the foundation of this model in the cost functional E . Here, we consider the adaptation dynamics in relation to the underlying cost functional. In particular, we revisit a minimal network with three nodes composed of one source and two sinks. For cost exponent $0 < \gamma < 1$, a pair of stable and unstable cyclic stationary solutions arise in a saddle-node bifurcation as the amplitude α of fluctuations increases above a critical value α_c [Klemm and Martens, Chaos (2023)]. Interestingly, the stable cyclic solution is not the global minimum of the cost functional E for all $\alpha > \alpha_c$. There is another critical parameter value $\alpha_e > \alpha_c$ such that the stable tree-like solution is the global minimum of E for all $\alpha \in [0, \alpha_e]$. Only for $\alpha > \alpha_e$, the stable cyclic solution minimizes E . Even when not being the global minimum of the cost function, the stable cyclic solution attracts the majority of trajectories. While the above scenarios are all deterministic dynamics. Now we also consider the case of fluctuations due random switching of sinks on a finite time scale. Finally, we study how the effects seen for the system with 1 source and 2 sinks carry over to a large vascular network based on real data.

How do extreme weather events disrupt the normal operational collective behaviour of the power grid?

Mehrnaz Anvari (Fraunhofer Institute for Algorithms and Scientific Computing SCAI)

Friday, August 2, 12:30–13:00, Research V Seminar Room

Abstract: Societies are experiencing rapid and pressing changes in the way they generate and consume energy. As part of the necessary transformation towards carbon dioxide neutral energy networks, power systems are increasingly incorporating renewable energy sources (RES) into the energy mix. However, RES such as wind and solar power are inherently uncertain and intermittent, which can result in rapid transitions from maximum power to no power in just a few seconds. These non-Gaussian characteristics can create vulnerabilities in the power system, leading it to transition from its normal collective behavior to undesired ones. Additionally, when extreme weather events cause infrastructural damages in the power grid, it can result in cascading failures that disrupt the operation of the grid. The identification of critical components within the complex power grid, whose failures contribute to significant cascading failures, is crucial for enhancing the grid's resilience. This topic will be discussed during this presentation.

Rate-dependent tipping in thermoacoustic systems under noise perturbation

Yong Xu (Northwestern Polytechnical University)

Friday, August 2, 13:00–13:30, Research V Seminar Room

Abstract: Thermoacoustic instability in thermoacoustic systems may cause catastrophic consequences for rocket engines, aero-engines, gas turbines and other power combustion devices, so the analysis of thermoacoustic instability and their prediction and control have been one of the key and difficult problems faced in the development of high-performance combustion systems. The transition process of the dynamical behavior of the thermoacoustic system from a low amplitude abrupt to a high amplitude state of thermoacoustic instability actually occurs as a tipping. When the rate of parameter is introduced in a system, the tipping-delay phenomenon may arise, which is beneficial to control undesirable states like thermoacoustic instability. Based on the fundamental theories and methods of nonlinear dynamics and random dynamical systems, this talk addresses the problem of rate-dependent tipping in thermoacoustic systems under noise perturbation, and systematically carries out research in terms of model construction, phenomenon analysis, early warning signals establishment, and control measure design. The stochastic modeling of the thermoacoustic system is optimized, and the mechanism of the influence of the stochastic disturbance and system parameters on the rate-dependent tipping is revealed. The prediction of the tipping point in the thermoacoustic system is realized, and the effective control strategy of the thermoacoustic instability is proposed. It provides a theoretical guarantee for the design of engines and other power combustion devices and the formulation of effective control measures, and is of great significance to aerospace and other related fields.

This is a joint work with Xiaoyu Zhang (Tokyo Institute of Technology, Tokyo 152-8552, Japan) and Juergen Kurths (Potsdam Institute for Climate Impact Research, Potsdam 14412, Germany & Humboldt University Berlin, Berlin 12489, Germany)

Minisymposium on Nonlinear and stochastic dynamics in Earth system science

THE HYPER-PARAMETERISATION APPROACH FOR OCEAN MODELLING

Igor Shevchenko

Tuesday, July 30, 11:00–11:30, Research II Lecture Hall

Abstract: It is typical for low-resolution ocean simulations to miss not only small- but also large-scale patterns of the flow dynamics compared with their high-resolution analogues. It is usually attributed to the inability of coarse-grid models to properly reproduce the effects of the unresolved small-scale dynamics on the resolved large scales. In part, the reason for that is that coarse-grid models fail to at least keep the coarse-grid solution within the region of phase space occupied by the reference solution (the high-resolution solution projected onto the coarse grid).

In order to challenge this problem we propose the Hyper-Parameterisation (HP) approach. The HP approach is an alternative to parameterisations offering a new strategy, which capitalises on the use of

phase spaces. Within HP the main focus is shifted from representing essential local physics of cross-scale interactions towards simulating the resolved, most important flow patterns directly. HP draws upon the phase space as an abstraction layer which includes oceanic eddy effects on all spatiotemporal scales.

In this talk we discuss the HP approach within the context of ocean modelling and report main results obtained for both idealized and comprehensive ocean models.

Understanding Dynamic Energy Backscatter through Pressure and Momentum Perturbations

Ekaterina Bagaeva (Constructor University Bremen)

Tuesday, July 30, 11:30–12:00, Research II Lecture Hall

Abstract: Accurately representing mesoscale eddies in ocean models is crucial for understanding global ocean dynamics. Eddy-permitting resolutions, commonly used in ocean modeling, provide a compromise between computational efficiency and capturing mesoscale dynamics, often employing eddy viscosity parameterizations to address unresolved turbulent processes. However, these parameterizations can introduce numerical over-dissipation, which can be mitigated through methods like ocean kinetic energy backscatter. Alternatively, stochastic perturbations can represent small-scale processes and uncertainties, improving simulation accuracy and providing insights into scale interactions.

In this study, we investigate the impact of dynamic backscatter and stochastic perturbations on the mean flow and eddy components, and their complementarity. Using a linear inverse model (LIM) to generate stochastic perturbations, we assess their effects on ocean dynamics, offering insights into coherent structures and uncertainties. Through diagnostic analyses, we evaluate the effectiveness of different perturbation methods in improving the representation of ocean dynamics, providing valuable insights for future modeling efforts.

Ocean eddy parameterizations: Simulating large scale turbulence in the oceans

Stephan Juricke, Sergey Danilov, Ekaterina Bagaeva, Marcel Oliver

Tuesday, July 30, 12:00–12:30, Research II Lecture Hall

Abstract: So called mesoscale eddies are turbulent motions in the ocean of 10-100km extent. These eddies cascade energy across scales, transport tracers and heat, and interact with the mean currents and the atmosphere. At eddy-permitting resolutions which are close to the Rossby radius of deformation, ocean eddies are barely resolved. However, such grids are still commonly applied for decadal climate simulations. They will also remain state-of-the-art at high latitudes for years to come. Due to their excessive dissipation of kinetic energy, these simulations feature reduced eddy variability, eddy formation and eddy-mean flow interactions. This has implications for the overall representation of the oceanic circulation and climate in general. In this study, ocean eddy parameterizations are introduced that aim at an improved representation of mesoscale ocean turbulence. They reduce the overdissipation at eddy-permitting resolutions by utilizing the inverse energy cascade from smaller to larger scales, consequently rectifying ocean model biases associated to dampened eddy activity.

One option to reduce overdissipation is the optimization of viscous closures. An alternative is the reinjection of parts of the overdissipated energy back into the resolved flow via so called kinetic energy backscatter parameterizations. In our study, we will introduce different viscous and backscatter schemes and discuss both deterministic and stochastic backscatter options. Our results show that incorporation of backscatter schemes can help to substantially improve the kinetic energy and mean flow characteristics in global ocean models. They also lead to substantial reductions of biases in water mass properties such as temperature and salinity. We will provide insights into the crucial aspects of the backscatter parameterizations and how such schemes can be adjusted for different resolutions and dynamical regimes to achieve the best performance.

Nonlinear effects of backscatter parameterisation

Jens Rademacher, Paul Holst, Jichen Yang, Artur Prugger

Tuesday, July 30, 12:30–13:00, Research II Lecture Hall

Abstract: Numerical simulations of large scale geophysical flows typically require unphysically strong

dissipation for numerical stability. Towards energetic balance various schemes have been devised to re-inject this energy, in particular by horizontal kinetic energy backscatter. We study this scheme through its continuum formulation with momentum equations augmented by a backscatter operator, and its the impact on certain flows and waves. We particularly focus on shallow water models and the possible trapping of injected energy. This can lead to exponential and unbounded growth in the idealised setting, or yield bifurcations in the presence of bottom drag.

The combined impact of a potential AMOC collapse and global warming on the Amazon forest

Da Nian (PIK), Sebastian Bathiany (Technical University of Munich), Maya Ben-Yami (Technical University of Munich), Lana Blaschke (Potsdam Institute for Climate Impact Research (PIK)), Marina Hirota (Universidade Estadual de Campinas), Regina Rodrigues (Federal University of Santa Catarina), Niklas Boers (Technical University of Munich)

Tuesday, July 30, 13:30–14:00, Research II Lecture Hall

Abstract: The Amazon forest faces the threat of dieback from climate change, particularly due to the increasing alarm of future droughts and rising temperatures. Moreover, more evidence suggests that the Atlantic Meridional Overturning Circulation (AMOC) is losing system stability and approaching the point of collapse. AMOC changes will significantly affect precipitation and temperature in the Amazon forest area. This study assesses the impacts on South American vegetation under two possible future climate change scenarios: global warming and global warming combined with AMOC collapse. We utilize empirical relationships based on observational data, alongside predictions from the HadGEM3 Earth system model's climate simulation, to estimate the most likely state of vegetation. Our results show that in parts of tropical South America, increases in MAP and decreases in MAT following AMOC collapse tend to stabilize Amazon forests, thereby delaying Amazon dieback compared to the default global warming scenario.

Deep Learning for Seasonal Prediction of Summer Precipitation Levels in Eastern China

Qimin Deng (China University of Geosciences, Wuhan), Peirong Lu

Tuesday, July 30, 14:00–14:30, Research II Lecture Hall

Abstract: Skilled seasonal forecasting will effectively reduce the economic losses caused by droughts and floods. Because of the powerful data mining capability of deep learning networks, it is increasingly applied in studies of seasonal rainfall prediction. However, there remain two prominent issues in the modeling process: the lack of enough training samples and the effect of a small number of extreme values on the model optimization. To tackle these deficiencies, we combine strategies such as principal component analysis, reduction of model hidden layers, and early-stopping with Attention U-Net to construct a rainfall classification forecasting model. These steps reduced the model overfitting and improved the model generalization. The results show that the prediction accuracy of this network with leads of 1–3 months is obviously better than that of the numerical model. Further analysis also supports that the spatial features of precipitation predicted by the network are very close to the observations.

Benchmarking prediction skill in binary El Niño forecasts

Xinjia Hu (University of Oxford)

Tuesday, July 30, 14:30–15:00, Research II Lecture Hall

Abstract: Reliable El Niño Southern Oscillation (ENSO) prediction at seasonal-to-interannual lead times would be critical for different stakeholders to conduct suitable management. In recent years, new methods combining climate network analysis with El Niño prediction claim that they can predict El Niño up to 1 year in advance by overcoming the spring barrier problem (SPB). Usually this kind of method develops an index representing the relationship between different nodes in El Niño related basins, and the index crossing a certain threshold is taken as the warning of an El Niño event in the next few months. How well the prediction performs should be measured in order to estimate any improvements. However, the amount of El Niño recordings in the available data is limited, therefore it is difficult to validate whether these methods are truly predictive or their success is merely a result of chance. We propose a benchmarking

method by surrogate data for quantitative forecast validation for small data sets. We apply this method to a naïve prediction of El Niño events based on the Oscillation Niño Index (ONI) time series, where we build a data-based prediction scheme using the index series itself as input. In order to assess the network-based El Niño prediction method, we reproduce two different climate network-based forecasts and apply our method to compare the prediction skill of all these. Our benchmark shows that using the ONI itself as input to the forecast does not work for moderate lead times, while at least one of the two climate network-based methods has predictive skill well above chance at lead times of about one year.

Causality inference and machine learning modeling for nonlinear time series

Yu Huang (Technical University of Munich)

Tuesday, July 30, 15:00–15:30, Research II Lecture Hall

Abstract: Recent research has demonstrated the benefits of employing machine learning in time series analysis, including enhanced computational efficiency, robustness to noise, and adeptness in managing nonlinear characteristics. Specifically, within the framework of reservoir computing, we examined how the dynamic causal relationships behind time series affect the modeling ability of machine learning on time series data. Conversely, machine learning proves proficient in identifying dynamical causal links, encompassing causal strength, directionality, interaction delay periods, and multivariate causal chains. Emerging evidence underscores the practical utility of machine learning in discerning causal relationships across diverse real-world time series datasets.

Regional coupled and decoupled day-night compound hot extremes over the mid-lower reaches of the Yangtze River: characteristics and mechanisms

Yixuan Guo, Zuntao Fu

Wednesday, July 31, 11:00–11:30, Research II Lecture Hall

Abstract: Hot extremes impose severe effects on human health and the ecosystem, especially when high-temperature extremes sequentially occur in both daytime and nighttime within 1 day, known as Compound Hot Extremes (CHEs). Although a number of studies have focused on independent hot extremes, not enough work is devoted to compound ones, not to mention the coupling strength in covariations between the two variables (daytime and nighttime extreme temperature: T_{max} and T_{min}) over a given region. The instantaneous coupling strength can be derived by Dynamical System (DS) approach from covariations between T_{max} and T_{min} over a given region, and used to classify CHEs into coupled and decoupled types. Results show that more frequent coupled CHEs tend to be more intense with prolonged duration and extensive spatial extent compared with decoupled CHEs. Also, the mechanisms behind these two types of CHEs are largely different. Coupled CHEs are accompanied by a significant intensification and westward extension of the western North Pacific subtropical high (WNPSH), and the extremely high-temperature is mainly caused by receiving more solar radiation under the corresponding anticyclone. It is found that barotropic structure, weak jet stream and developing La Niña are conducive to the enhancement and persistence of WNPSH, in favor of the occurrence of long-lasting CHEs. Decoupled CHEs are associated with strong sea-land breeze (SLB), whose diurnal cycle could weaken the persistent large-scale circulation and suppress covariations between T_{max} and T_{min} . This kind of decoupled hot extremes are attributed to the combined effect of receiving more solar radiation during the day and trapping more long-wave radiation at night, where moisture and cloud cover play an important role.

Causal dependencies and Information entropy budget: Analysis of a reduced order atmospheric model

Stephane VANNITSEM (Royal Meteorological Institute of Belgium (RMIB)), Carlos A. Pires, David Docquier

Wednesday, July 31, 11:30–12:00, Research II Lecture Hall

Abstract: One particularly important approach of causal analysis has been developed in a series of papers by Liang based on the development of a theory of the information entropy evolution and the transfer of information between selected variables in the rather general context of deterministic and stochastic dynamical

systems (see e.g. Liang, 2016, 2021). These studies culminated with the derivation of simple relations that could be directly applied to observational data, under the assumption that the system can be described by a linear stochastic system. This type of approximation is indeed sometimes justified (Sardeshmukh et al, 2015), but it can be a too rough approximation that leads to misinterpretation of interactions. This is particularly true in chaotic dynamical systems for which nonlinearities play a key role in generating the dynamics. In such a context, one must therefore turn back to the original equations of the Shannon entropy evolution, and find ways to evaluate properly the rate of information transfer between observables. An important step in that direction has been recently made in Pires et al (2024), in which the rate of information transfer between variables or groups of variables can be computed by means of conditional expectations in a fully nonlinear framework.

The information entropy budget and the rate of information transfer between variables based on the approach of Pires et al (2024) is studied in the context of a reduced-order atmospheric model in which the key ingredients of the dynamics are present, namely the baroclinic instability, the orographic instability, the dissipation related to the surface friction, and the large-scale meridional imbalance of energy. For the parameters chosen, the solutions of this system display a chaotic dynamics reminiscent of the large-scale atmospheric dynamics in the extratropics. The detailed information entropy budget analysis of this system reveals that: (1) The linear rotation terms play a minor role in the generation of uncertainties as compared to the orography and the dissipation due to friction; (2) by far, the dominant contribution comes from the nonlinear advection terms. Their decomposition in synergetic (co-variability) and single (impact of each single variable on the target one) components further reveals that for some variables the co-variability dominates the information transfer, which can be either positive or negative in some cases. The estimation of the rate of information transfer based on time series is also discussed, and an extension of the Liang's approach to nonlinear observables, is proposed, which looks very promising.

- [1] Liang, X. S., Physical Review E, 94, 052 201, <https://doi.org/10.1103/PhysRevE.94.052201>, 2016.
- [2] Liang, X. S, Entropy, 23, 679, <https://doi.org/10.3390/e23060679>, 2021.
- [3] Pires, C., Docquier, D., and Vannitsem, S., Physica D, 458, 133 988, <https://doi.org/10.1016/j.physd.2023.133988>, 2024.
- [4] Sardeshmukh, P. D. and Penland, C., Chaos, 25, 036 410, <https://doi.org/10.1063/1.4914169>, 2015.
- [5] Vannitsem, S., C. Pires and D. Docquier: Causal dependencies and Shannon entropy budget - Analysis of a reduced order atmospheric model. To be submitted, 2024.

Systematic multi-scale decomposition of ocean variability using machine learning

Christian Franzke (IBS Center for Climate Physics, Pusan National University), Stephan Juricke (GEOMAR - Helmholtz Centre for Ocean Research Kiel), Federica Gugole (Centrum Wiskunde & Informatica (CWI))
 Wednesday, July 31, 12:00–12:30, Research II Lecture Hall

Abstract: Multi-scale systems, such as the climate system, the atmosphere, and the ocean, are hard to understand and predict due to their intrinsic nonlinearities and chaotic behavior. Here, we apply a physics-consistent machine learning method, the multi-resolution dynamic mode decomposition (mrDMD), to oceanographic data. mrDMD allows a systematic decomposition of high-dimensional data sets into time-scale dependent modes of variability. We find that mrDMD is able to systematically decompose sea surface temperature and sea surface height fields into dynamically meaningful patterns on different time scales. In particular, we find that mrDMD is able to identify varying annual cycle modes and is able to extract El Nino–Southern Oscillation events as transient phenomena. mrDMD is also able to extract propagating meanders related to the intensity and position of the Gulf Stream and Kuroshio currents. While mrDMD systematically identifies mean state changes similarly well compared to other methods, such as empirical orthogonal function decomposition, it also provides information about the dynamically propagating eddy component of the flow. Furthermore, these dynamical modes can also become progressively less important as time progresses in a specific time period, making them also state dependent.

Minisymposium on Nonlinear and stochastic dynamics in weather and climate science

Data-driven and physics-constrained subgrid-scale parameterization for atmosphere and ocean models

Frank Kwasniok (University of Exeter)

Monday, July 29, 11:00–11:30, Research II Lecture Hall

Abstract: Data-driven deterministic and stochastic subgrid modelling schemes for atmosphere and ocean models are discussed. A pattern-based approach is taken where pairs of patterns in the space of resolved variables (or functions of these) and in the space of the subgrid forcing are identified and linked in a predictive manner. On top of this deterministic part of the subgrid scheme the subgrid patterns may be forced stochastically. Both the deterministic and the stochastic scheme can be constrained by physically motivated conservation laws, such as momentum conservation or (kinetic) energy conservation but enstrophy dissipation. The schemes are machine-learning-style but unlike black-box approaches such as neural networks the present methodology still allows to understand and interpret the subgrid model.

The subgrid modelling schemes are implemented in a spectral quasi-geostrophic three-level atmospheric model with realistic mean state and variability. The atmospheric model at a horizontal resolution of T30 is regarded as the reference against which coarser-resolution versions at T21 and T15, equipped with the subgrid modelling schemes, are compared. In long-term simulations, the novel subgrid schemes greatly improve on a standard hyperviscosity scheme as evidenced by the mean state, the variability pattern as well as kinetic energy and potential energy spectra. They also show marked skill improvements in an ensemble prediction setting.

Low-frequency variability of the atmosphere and predictability – A reduced order model perspective

Stephane Vannitsem (Royal Meteorological Institute of Belgium (RMIB))

Monday, July 29, 11:30–12:00, Research II Lecture Hall

Abstract: Low-frequency variability in the atmosphere at mid-latitudes is covering a wide range of time scales from seasons to millennia. An example of such variability is the North Atlantic Oscillation which reveals predominant weather patterns in the course of the years over the Atlantic and Western Europe. The source of variability is, however, controversial and several possibilities have been envisaged, including coupled ocean-atmosphere variability; stratospheric influence, possibly related to ENSO in the tropical Pacific; and an even more controversial source, namely the solar activity, as discussed in Smith et al (2014). These sources of low-frequency variability (LFV) provide some hope for extended-range forecasts, but a more complete understanding of the impact of these different sources is still missing. The difficulty in understanding and assigning the proper role to the various sources is also due to the difficulties of global climate models to reproduce this natural LFV and the teleconnection patterns it gives rise to, due to substantial model biases. Understanding the role of the ocean on the development of the LFV and of teleconnections is a central topic of current research.

In the present seminar, we will review our recent works on the analysis of the development of LFV in reduced-order models that we have developed along the years. In particular, we will show how genuinely coupled LFV can emerge in a very simple reduced-order coupled ocean-atmosphere model (Vannitsem, 2015; Vannitsem et al., 2015). We will then discuss the impact of the potential influence of the Tropical Pacific on the extratropics using techniques coming from the theory of Pullback Attractors (Vannitsem et al, 2021). Finally, we will discuss the problem of predictability associated with the presence of initial condition errors in such mid-latitude coupled ocean-atmosphere models, together with the impact of boundary error forcing associated with the influence of the Tropical Pacific (Vannitsem, 2017; Vannitsem, 2023).

- [1] Smith, D. M., A. A. Scaife, R. Eade, and J. R. Knight: Seasonal to decadal prediction of the winter North Atlantic Oscillation: Emerging capability and future prospects, *Q. J. R. Meteorol. Soc.*, DOI:10.1002/qj.2479, 2014..
- [2] Vannitsem, S., J. Demayer, L. De Cruz, M Ghil: Low-frequency variability and heat transport in a low-order nonlinear coupled ocean-atmosphere model. *Physica D*, 309, 71-85, DOI: 10.1016/j.physd.2015.07.006, 2015.
- [3] Vannitsem, S., The role of the ocean mixed layer on the development of the North Atlantic Oscillation: A dynamical system's perspective, *Geophys. Res. Lett.*, 42, DOI:10.1002/2015GL065974, 2015.

- [4] Vannitsem S., Predictability of large-scale atmospheric motions: Lyapunov exponents and error dynamics, *Chaos*, 27, 032101, DOI:10.1063/1.4979042, 2017.
- [5] Vannitsem, S., Demaeyer, J., & Ghil, M. Extratropical low-frequency variability with ENSO forcing: A reduced-order coupled model study. *Journal of Advances in Modeling Earth Systems*, 13, e2021MS002530. DOI: 10.1029/2021MS002530, 2021.
- [6] Vannitsem S., Impact of tropical teleconnections on the long-range predictability of the atmosphere at mid-latitudes: A reduced-order multi-scale model perspective, *J. Phys. Complex.* 4 045006, DOI: 10.1088/2632-072X/ad04e8, 2023.

Simulation of extreme events and abrupt transitions in climate models with rare event algorithms

Francesco Ragone

Monday, July 29, 12:00–12:30, Research II Lecture Hall

Abstract: Extreme events like heat waves, floods or wind storms, as well as rare abrupt transitions associated with tipping elements of the climate system, can have severe impacts on human societies and ecosystems. Studying these events on a robust statistical basis with complex climate models is however computationally challenging, as very long simulations and/or very large ensembles are necessary to sample a sufficient number of events to keep statistical uncertainties small enough. This problem can be tackled using rare event algorithms, numerical tools designed to reduce the computational effort required to sample rare events in numerical models. These methods typically take the form of genetic algorithms, where a set of suppression and cloning rules are applied to the members of an ensemble simulation, in order to oversample trajectories leading to the events of interest. In this talk we first show different applications of these methods to the simulation of heat waves and warm summers in the Northern hemisphere. We show how a rare event algorithm allows to efficiently sample extreme events characterised by persistency of high regional surface temperatures on subseasonal to seasonal scales, and we analyse the emergence of atmospheric teleconnections during the events. We then discuss recent applications to the simulation of extremes of Arctic sea ice reduction, and of abrupt transitions associated to the weakening and collapse of the Atlantic Meridional Overturning Circulation. Finally we discuss how these results open the way to further applications to a wide range of problems.

Early warnings of the transition to a superrotating atmospheric state

Mark Williamson (University of Exeter)

Monday, July 29, 13:30–14:00, Research II Lecture Hall

Abstract: Several general circulation models (GCMs) have showed bifurcations of their atmospheric state under a broad range of warm climates. These include some of the more extreme global warming scenarios. This bifurcation can cause the transition to a superrotating state, a state where its angular momentum exceeds the solid body rotation of the planet. Here we use an idealized GCM to simulate this transition by altering a single non-dimensional control parameter, the thermal Rossby number. For a bifurcation induced transition there is potential for early warnings and we look for these here. Typically used early warning indicators, variance and lag 1 autocorrelation, calculated for the mean zonal equatorial wind speed, increase and peak just before the transition. The full autocorrelation function taken at multiple lags is also oscillatory, with a period of 25 days preceding the transition. This oscillatory behaviour is reminiscent of a local supercritical Hopf bifurcation. Motivated by this extra structure, we use a generalised early warning vector technique based on principle oscillation patterns (POPs) to diagnose the dominant spatial modes of the horizontal windfield fluctuations. We find a zonal wavenumber zero pattern we call the ‘precursor’ mode, that appears shortly before and disappears soon after the transition. We attribute the increase in the early warning indicators to this spatial precursor mode. Although the control parameter used to simulate the transition is unlikely to be relevant to future climate change, the simulations reported here do show early warnings of the transition and serve as a test bed for whether we can detect this transition before it happens.

Limits to predictability of the asymptotic state of the Atlantic Meridional Overturning Circulation in a conceptual climate model

Oliver Mehling (Politecnico di Torino), Reyk Börner (University of Reading), Valerio Lucarini (University of Leicester)

Monday, July 29, 14:00–14:30, Research II Lecture Hall

Abstract: Anticipating critical transitions in the Earth system is of great societal relevance, yet there may be intrinsic limitations to their predictability. For instance, the asymptotic state of a dynamical system possessing multiple chaotic attractors depends sensitively on the initial condition in the proximity of a fractal basin boundary. Here, we approach the problem of final-state sensitivity of the Atlantic Meridional Overturning Circulation (AMOC) using a conceptual climate model, composed of a slow bistable ocean coupled to a fast chaotic atmosphere. First, we explore the occurrence of long chaotic transients in the monostable regime, which can mask a loss of stability near bifurcations. In the bistable regime, we explicitly construct the chaotic saddle using the edge tracking technique. We quantify the final-state sensitivity through the maximum Lyapunov exponent and the lifetime of the saddle and find that the system exhibits a fractal basin boundary with almost full phase space dimension, implying vanishing predictability of the second kind near the basin boundary. Our results demonstrate the usefulness of studying non-attracting chaotic sets in the context of predicting climatic tipping points, and provide guidance for the interpretation of critical transitions in higher-dimensional climate models.

[1] O. Mehling, R. Börner, V. Lucarini, *Physica D* 459, 134043 (2024)

Conditions For Stability in the Climate-Carbon System

Joe Clarke (University of Exeter), Paul Ritchie (University of Exeter), Chris Huntingford (UK CEH), Peter Cox (University of Exeter)

Monday, July 29, 14:30–15:00, Research II Lecture Hall

Abstract: The CMIP6 project revealed that some of the latest generation of climate models show a very strong response to increased concentrations of atmospheric CO₂, as quantified by equilibrium climate sensitivity (ECS). This high sensitivity has implications for the carbon cycle. For example, at higher climate sensitivities, a small CO₂ perturbation leads to larger warming, which in turn causes increased decomposition of organic matter. This increased decomposition tends to increase atmospheric CO₂, leading to a feedback loop. Over the last 10,000 years (the Holocene epoch), atmospheric CO₂ concentrations have remained approximately constant, at least until the anthropogenic perturbation from the industrial revolution. This means that the feedback loop cannot be too strong, placing bounds on equilibrium climate sensitivity. CMIP6 climate models, however, cut this feedback loop out by running with prescribed levels of CO₂. In this talk, I will investigate this feedback loop using models of varying complexity. I will use bifurcation theory to argue that the qualitative behaviour of the climate-carbon system rules out certain values of various earth system sensitivities.

On the early warnings of major climatic events: recent progresses based on climate network analysis and the Fractional Integral Statistical Model

Naiming Yuan (Sun Yat-sen University)

Monday, July 29, 15:00–15:30, Research II Lecture Hall

Abstract: Climate prediction is one of the core issues of concern in the field of atmospheric science. Reliable predictions and early warnings are crucial for disaster prevention and reduction, as well as protecting the safety of the human beings. Although significant progresses have been made in the past decades, it is worth noting that there are often “predictability barriers” in the current climate predictions. The existence of the barriers means that we may have not fully grasped the dynamical mechanisms of the climate system, so there is an urgent need for new theoretical methods to explore these issues from different perspectives. In recent years, the climate network analysis, as an emerging research approach, has received much attention and has been successfully applied in many climate studies. In this talk, I will introduce the recent progresses of the climate network studies, particularly on the early warnings of major climate events, i.e., the phase change of the Pacific Decadal Oscillations (PDO), and the occurrence of the Indian Ocean Dipole (IOD). The concept of “More is different” will be used to explain the appearance of early warning signals. In addition, by

combining the climate network analysis with a generalized stochastic climate model (the Fractional Integral Statistical Model, FISM), I will further introduce a way to identify the climate memory impacts on the early warning signals. This combination may be helpful for improving the interpretability of the climate network results. In the end, I will briefly discuss the current challenges, and give an outlook.

Minisymposium on Nonlinear dynamics in music perception and production

Nonlinear Impulse Pattern Formulation dynamical social and political prediction algorithm for city planning and public participation

Rolf Bader (Universität Hamburg), **Simon Linke** (Ligeti Center), Stefanie Gernert (City of Hamburg)
Thursday, August 1, 11:00–11:30, IRC Seminar Room 3

Abstract: Modern city planning as well as social or political decisions depend on prediction mainly based on statistical data or expert opinions. Still, the complexity of modern societies is steadily increasing while resources are getting scarce. Also, public participation becomes more and more important, as modern city planning is often based on upcycling of material and increased densifying of cities rather than planning of new urban areas. A nonlinear-dynamical algorithm of an Impulse Pattern Formulation (IPF) is proposed for predicting relevant parameters like health, artistic freedom, or financial developments of different stakeholders over the course of a planning process. The IPF has already shown high predictive precision with low computational cost in musical instrument simulations, brain dynamics, and human - human interactions. The social and political IPF consists of three basic equations of system state developments, self-adaptation, and interactions with a set of equations suitable for respective planning situations. Typical scenarios of stakeholder interactions and developments can be modelled by adjusting a set of system parameters. Using machine learning, a suitable set of parameters is estimated from a desired development of the planning process.

Listening to music gives rise to partial brain synchronization

Jakub Sawicki (PIK Potsdam Institute for Climate Impact Research), Ekehard Schöll
Thursday, August 1, 11:30–12:00, IRC Seminar Room 3

Abstract: It is well known that synchronization patterns and coherence play a major role in the functioning of brain networks, both in pathological and in healthy states. In particular, in the perception of sound, one can observe an increase in coherence between the global dynamics in the network and the auditory input [1]. In this work, we show that synchronization scenarios are determined by a fine interplay between network topology, the location of the input, and frequencies of these cortical input signals [2]. To this end, we analyze the influence of an external stimulation in a network of FitzHugh-Nagumo oscillators with empirically measured structural connectivity, and discuss different areas of cortical stimulation, including the auditory cortex.

[1] J. Sawicki, L. Hartmann, R. Bader, and E. Schöll: Modelling the perception of music in brain network dynamics, *Front. Netw. Physiol.* 2, 910920 (2022)

[2] J. Sawicki and E. Schöll: Interplay of synchronization and cortical input in models of brain networks, *Europhys. Lett.* (2024), invited Perspective.

Mapping approach to musical instruments: generalized impulse pattern formation

Markus Abel (Ambrosys GmbH), Rolf Bader (University of Hamburg), Robert Mores, Simon Linke
Thursday, August 1, 13:30–14:00, IRC Seminar Room 3

Abstract: In recent work, the so/ called Impulse Pattern Formulation (IPF) was introduced as a model for the dynamics of musical instruments. In this publication, we aim at a generalization of the previous formulation and point out how the model can be used numerically to represent the complicated processes of excitation mechanisms. Such excitation, often transient and chaotic is hard to capture, since it is often transient chaotic, damped, and of short duration.

Dynamically regime changes of the Impulse Pattern Formulation when modeling musical instruments and human interaction

Simon Linke (Hamburg University for Applied Sciences)

Thursday, August 1, 14:00–14:30, IRC Seminar Room 3

Abstract: The Impulse Pattern Formulation (IPF) was introduced as a general modeling approach for musical instruments and has already been extended to further topics, such as modeling the interaction between musicians. The nonlinear recursive equation is based on the idea that the dynamical behavior of mutually coupled systems can be modeled based on interacting impulse trains. Thus, the IPF can, e.g., describe nonlinear phenomena of musical instruments that ordinary analytical or numerical methods can hardly explain, relying on transitions between regular periodicity at a nominal pitch, bifurcation scenarios, and noise. However, while the boundaries of those regime changes have already been modeled and understood for quasi-stationary transitions, their behavior may drastically change when looking at realistic, dynamical conditions. For example, when modeling the tone production of a saxophone, there is a big difference between changing the pitch due to a different fingering resulting in sudden changes of the modeling parameters or increasing the blowing pressure, which changes modeling parameters continuously. In this talk, several examples of dynamically changing modeling parameters are shown, originating from musical instruments to human interaction, allowing a first systematical description of those phenomena and new insights into the overall system stability of the IPF.

Vocal tract and recorder sound

Naomi Nordblom (University of Hamburg; Institute of Systematic Musicology)

Thursday, August 1, 14:30–15:00, IRC Seminar Room 3

Abstract: Musical wind instruments driven by an air-jet hitting a labium, like organs or flutes, produce sound through air turbulence at the labium coupled to a tube and a players mouth cavity. Complex nonlinear dynamic behaviour like hysteresis, sudden phase changes, bifurcations, or noise are part of players articulation. Additionally, the coupling to the mouth cavity can be part of players individual style and sound. The paper presents state-of-the-art in fluid dynamic modeling of such wind instruments solving the Navier-Stokes equation in the presence of turbulence, flow-pressure interactions, and coupling to players mouth cavity. The simulation is carried out with openfoam, an open source software that uses the finite volume method. Furthermore, the interaction between the jet and the players' mouth cavity is discussed. The simulations allow a first estimation of the different sound quality of different players through different mouth cavities.

Neural synchronization dynamics correlate with musical form driving parameters

Lenz Hartmann (Hamburg Universität)

Thursday, August 1, 15:00–15:30, IRC Seminar Room 3

Abstract: The large-scale form of music, i.e. the structure of musical units extending over the whole piece of music, is investigated by EEG measurements of 25 subjects who listened to three full-length pieces of music from three different genres (tech-house, hip-hop, jungle). The grand-averages [event-related potentials, ERPs] of the individual recordings were calculated and the correlations of successive time windows between the ERPs of all electrodes and thus of different cortical regions or neural networks, were used as a measure of the synchronization between these regions. It was shown that the dynamics of synchronization between globally distributed neural networks in different frequency bands correlate strongly with the course of musical form parameters such as amplitude, fractal correlation dimensions and the spectral centroid. Linear regression showed that these parameters could explain up to 65% of the variance in the dynamics of neural network synchronization.

A physics-based Self-Organizing Map (PISOM) to study viscoelastic physical modeling parameters of musical instruments.

Cristhiam Martínez (University of Hamburg), Rolf Bader (University of Hamburg)

Thursday, August 1, 15:30–16:00, IRC Seminar Room 3

Abstract: The increasing availability of computational resources and data acquisition has led to the widespread adoption of techniques such as deep learning and reservoir computing for predicting the behavior of dynamic systems. However, interpretability limitations, data requirements, and computational costs associated with these methods pose significant challenges. Addressing these concerns, particularly in fields characterized by complex nonlinear dynamic behavior, motivates the exploration of alternative approaches, such as unsupervised learning. In this presentation, we introduce a Physics-Informed Self-Organizing Map (PISOM) tailored to the study of the dynamics of complex non-linear acoustic phenomena. This approach involves the use of a physical model to create a data set by systematically modifying the variables that govern the underlying physics, and a data pipeline whose parameters can be adapted to analyze the behavior of the system under such modifications. To illustrate the effectiveness of the approach, we present a case study on the analysis of viscoelastic damping in musical instruments, whose complexity lies in the influence of its time dependent response upon the resulting frequency spectrum due to the memory effect of viscoelasticity. By leveraging the PISOM, valuable insights into the dynamics of the damping process are gained, enhancing our understanding of the intricate interplay between material properties and acoustic performance.

Minisymposium on Patterns of synchrony in complex networks

Dynamics of oscillator populations with disorder in the phase shifts in coupling

Arkady Pikovsky (University of Potsdam)

Wednesday, July 31, 16:00–16:30, IRC Seminar Room 1

Abstract: In the classical Kuramoto-Sakaguchi-Daido setup, one considers an ensemble of noisy oscillators with different natural frequencies with a global coupling containing a phase shift. Here, we generalize this setup to the case where the phase shifts are different for all pairs of oscillators and are taken as random numbers from a specific distribution. First, it is argued that for a non-trivial distribution of the phase shifts, in the thermodynamic limit under the assumption of independence of the phases and the phase shifts, one can reduce the system to one without phase shifts but with a new effective coupling function. This new function is the convolution of the original function and the distribution of the phase shifts. In the second part, a situation of maximal frustration is considered, where the distribution of the phase shifts is uniform. Thus, the effective coupling function vanishes and there is no synchronization transition in the usual sense. Nevertheless, one can follow partial synchrony in the population by virtue of a novel correlation order parameter and by quantifying the entrainment of the frequencies. We demonstrate the scaling of the correlation order parameter with the system size and reveal a nontrivial behavior of the frequencies with maximal concentration at a certain value of the coupling strength.

Synchronization patterns induced by short range attraction and long range repulsion

Matthias Wolfrum (WIAS Berlin)

Wednesday, July 31, 16:30–17:00, IRC Seminar Room 1

Abstract: We study a system of active rotators interacting by short range attractive and long range repulsive coupling. We demonstrate that in such systems one can observe a huge variety of emerging self-organized patterns. This includes coherent Turing patterns and coherence incoherence patterns such as bumps and chimera states. Depending on the parameter regime there are also drift instabilities that generate various types of traveling patterns. In particular, we present new types of synchronization patterns where locked regions with different effective frequencies appear. These regions are separated by incoherent transition layers, which can induce a random drift of the pattern.

Complexified Synchrony – a hidden collective dynamics of complexified oscillators

Seungjae Lee (Technische Universität Dresden), **Moritz Thümler** (Technische Universität Dresden), **Malte Schröder** (Technische Universität Dresden), **Marc Timme** (Technische Universität Dresden)

Wednesday, July 31, 17:00–17:30, IRC Seminar Room 1

Abstract: The Kuramoto model and its variations have been used extensively to understand collective phenomena like synchrony among oscillators. Recent interest has been on generalizations of the Kuramoto model in various directions. Here, we discuss an extension of the Kuramoto model with dynamical variables and system parameters analytically continued to the complex plane. We first introduce this model comparing to other generalizations and then discuss observable collective behaviours of the complexified Kuramoto oscillators that without complexification would remain hidden. For example, a complex locked state reveals a hidden type of synchrony. We discuss possible applications of the complexified Kuramoto model to power-grid models, chimera states, and other nonlinear dynamical systems.

- [1] M. Thümler et al., Synchrony for Weak Coupling in the Complexified Kuramoto Model, Phys. Rev. Lett. 130, 187201 (2023).
- [2] S. Lee et al., Complexified Synchrony, arxiv.org arXiv:2403.02006 (2024), under review at Chaos.

Exploring the Emergence of 3-Cluster Solutions in Stuart-Landau Oscillators

Nicolas Thomé (Technische Universität München)

Wednesday, July 31, 17:30–18:00, IRC Seminar Room 1

Abstract: Understanding the formation of clusters in coupled oscillator systems is highly interesting as it provides insights into the spontaneous organization of complex dynamical systems. Studying the emergence of cluster solutions is typically difficult due to the high coexistence of clustered states. We, therefore, use a reduced-dimension manifold to study the creation/annihilation of these solutions. As a toy model, we use a system of Stuart-Landau oscillators under linear global coupling where we assume the thermodynamic limit.

We find a hierarchy of cluster singularities that organize the different cluster solutions. Whereas a single cluster singularity exists for the formation of 2-cluster solutions (as Kemeth et al. showed [1]), a series of secondary cluster singularities are responsible for the formation of 3-cluster solutions. We base our arguments on bifurcation diagrams and various numerical experiments.

- [1] Felix P Kemeth et al., Chaos 29, 023107 (2019)

Complex dynamics of superconducting neurons

Ioanna Chitzanidi (FORTH, Institute of Electronic Structure and Laser)

Wednesday, July 31, 18:00–18:30, IRC Seminar Room 1

Abstract: Neuromorphic computing exploits the dynamical analogy between many physical systems and neuron biophysics. Superconductor systems, in particular, are excellent candidates for neuromorphic devices due to their capacity to operate at great speeds and with low energy dissipation compared to their silicon counterparts. In this talk, we present two types of Josephson-junction-based neuromorphic circuits, one resistively and one inductively coupled. We review the differences in their dynamical and synchronization properties and identify how our findings relate to neurocomputation. Finally, we present preliminary results on a Josephson-junction with a delayed feedback loop as an implementation of a neural autapse. Autapses, i.e. synaptic couplings of neuron axons to their own dendrites, are known to be common in the brain and play an important role in neural activity.

Dynamics of a network of QIF neurons

Carlo Laing, Oleh E. Omel'chenko (University of Potsdam)

Thursday, August 1, 13:30–14:00, IRC Seminar Room 1

Abstract: We present a mean field description of a network of quadratic integrate-and-fire (QIF) neurons with both synaptic and gap junction coupling. The network shows a variety of states including standing waves, travelling waves and lurching waves. We show a computationally efficient way to study these solutions, using the fact that the dynamics are governed by a Riccati equation. We do this by deriving a self-consistency equation for the current applied to neurons, rather than the dynamic variables of the network. Our results give insight into the effects of gap junctional coupling on the dynamics of such networks.

Phase reduction explains chimera shape: when multi-body interaction matters

Erik T. K. Mau (University of Potsdam), Oleh E. Omel'chenko (University of Potsdam), Michael Rosenblum (University of Potsdam)

Thursday, August 1, 14:00–14:30, IRC Seminar Room 1

Abstract: We present an extension of the Kuramoto-Sakaguchi model for networks, deriving the second-order phase approximation for a paradigmatic model of oscillatory networks - an ensemble of non-identical Stuart-Landau oscillators coupled pairwise via an arbitrary adjacency matrix. We explicitly demonstrate how this matrix translates into the coupling structure in the phase equations. To illustrate the power of our approach and the crucial importance of high-order phase reduction, we tackle a trendy setup of non-locally coupled oscillators exhibiting a chimera state. We reveal that our second-order phase model reproduces the dependence of the chimera shape on the coupling strength that is not captured by the typically used first-order Kuramoto-like model. Our derivation contributes to the rapidly developing field of hypernetworks, establishing a relation between the adjacency matrix and multi-body interaction terms in the high-order phase model.

Insights into oscillator network dynamics using a phase-isostable framework

Rachel Nicks (University of Nottingham, United Kingdom), Robert Allen (University of Nottingham, United Kingdom), Stephen Coombes (University of Nottingham, United Kingdom)

Thursday, August 1, 14:30–15:00, IRC Seminar Room 1

Abstract: The utility of the classical technique of phase reduction for describing the dynamics of networks of coupled oscillators is limited by the assumption that the dynamics for each node remain on the stable limit cycle of the uncoupled system. Here we introduce reduced equations for networks of arbitrary finite size where the dynamics of each node is described in terms of its phase and the slowest decaying isostable coordinate, allowing for representation of trajectories away from (but near) the limit cycle. Specifically, we consider conditions for the existence and stability of phase-locked states generalising existing results for phase-reduced equations. We show that phase-isostable network equations provide the most accurate qualitative description of dynamics of the mean-field Ginzburg-Landau equation when compared with alternative frameworks such as higher-order phase reduction. We further demonstrate the power of the general framework by considering networks of Morris-Lecar neurons. We observe phenomena including the emergence of quasiperiodic behaviour that cannot be captured using first-order phase reduction. The results are shown to be in good qualitative agreement with the dynamics of the original network through numerical simulations and bifurcation analysis. Delays in both the node dynamics and network interactions can strongly influence patterns of phase-locked states and their bifurcations. We will briefly discuss ongoing work incorporating delayed interactions as well as phase and amplitude response of delay induced node oscillations within a phase-amplitude network setting.

Stability of partially synchronized patterns in coupled oscillator networks

Oleh E. Omel'chenko (University of Potsdam)

Thursday, August 1, 15:00–15:30, IRC Seminar Room 1

Abstract: Networks of coupled oscillators are popular mathematical models used in various fields of physics, chemistry, and biology. If the individual oscillators are non-identical or their coupling topology is complex enough, the typical states of such networks are partially synchronized patterns, the best known examples of which are probably chimera states. A common method of analyzing the appearance and stability of these patterns is the continuum limit approach, which in many cases is based on integro-differential equations obtained by the Ott-Antonsen manifold reduction. In this talk, we discuss which mathematical settings are appropriate for characterizing the stability of different types of partially synchronized patterns, including stationary, breathing and traveling chimera states. In addition, we consider the relationship between the predictions of the continuum limit approach and the actual behavior of coupled oscillator networks.

Minisymposium on Quantum simulation and computation at the edge of chaos

Thermalization Universality Classes for Weakly Nonintegrable Many-Body Dynamics

Sergej Flach (Institute for Basic Science (IBS))

Tuesday, July 30, 11:00–11:30, East Hall 8

Abstract: We observe different universality classes in the slowing down of thermalization of many-body dynamical systems upon approaching integrable limits. Two fundamentally distinct long-range (LR) and short-range classes (SR) are distinguished by the nonintegrable perturbation network spanned amongst the (set of countable) actions of the corresponding integrable limit. Weak two-body interactions (nonlinearities) induce LR networks in translationally invariant lattices. Weak lattice coupling (hopping) instead induce SR networks [1,2,3].

For classical systems we study the scaling properties of the full Lyapunov spectrum [4,5,6]. The LR class results in a single parameter scaling of the Lyapunov spectrum, with the inverse largest Lyapunov exponent being the only diverging time control parameter and the rescaled spectrum approaching an analytical function [4,6]. The SR class results in a dramatic slowing down of thermalization and a rescaled Lyapunov spectrum approaching a non-analytic function. An additional diverging length scale controls the exponential suppression of all Lyapunov exponents relative to the largest one [4,6]. Disorder induces transitions from LR to SR classes [7].

For quantum spin chains we compute ergodization time scales within the framework of the Eigenstate Thermalization Hypothesis and the Lyapunov time from operator growth methods using Krylov Complexity [8]. The comparison of both time scales confirms the existence of the above universality classes for quantum many body dynamics as well.

- [1] Mithun Thudiyangal, Carlo Danieli, Yagmur Kati and Sergej Flach, Dynamical Glass Phase and Ergodization Times in Classical Josephson Junction Chains, *Phys. Rev. Lett.* 122 054102 (2019)
- [2] Carlo Danieli, Thudiyangal Mithun, Yagmur Kati, David K. Campbell and Sergej Flach, Dynamical glass in weakly non-integrable Klein-Gordon chains, *Phys. Rev. E* 100 032217 (2019)
- [3] T. Mithun, C. Danieli, M. V. Fistul, B. L. Altshuler and S. Flach, Fragile Many Body Ergodicity From Action Diffusion, *Phys. Rev. E* 104, 014218 (2021)
- [4] Merab Malishava and Sergej Flach, Lyapunov spectrum scaling for classical many-body dynamics close to integrability, *Phys. Rev. Lett.* 128, 134102 (2022)
- [5] Merab Malishava and Sergej Flach, Thermalization dynamics of macroscopic weakly nonintegrable maps, *Chaos* 32, 063113 (2022)
- [6] Gabriel Lando and Sergej Flach, Thermalization Slowing-Down in Multidimensional Josephson Junction Networks, *Phys. Rev. E* 108, L062301 (2023).
- [7] Weihua Zhang, Gabriel M. Lando, Barbara Dietz and Sergej Flach, Thermalization Universality-Class Transition Induced by Anderson Localization, *Phys. Rev. Res.* Letter in print; arXiv:2308.08921
- [8] Budhaditya Bhattacharjee, Alexei Andreanov and Sergej Flach, in preparation.

Quantum computers challenged by many-body chaos

Alexander Altland

Tuesday, July 30, 11:30–12:00, East Hall 8

Abstract: From the perspective of many-body physics, the transmon qubit architectures currently developed for quantum computing are systems of coupled nonlinear quantum resonators. A significant amount of intentional frequency detuning (disorder) is required to protect individual qubit states against the destabilizing effects of nonlinear resonator coupling. In this talk, we will discuss the stability of this variant of a many-body localized phase for system parameters relevant to current quantum processors. An essential element in our diagnostic toolbox are classical simulations, which can be run, e.g., for IBM designs comprising hundreds of qubits. The overall conclusion of this study is that it will take considerable engineering efforts to protect transmon quantum computers from the destructive influence of chaotic fluctuations.

Implementing quantum neural state methods to study "chaotic melting" in Bose-Hubbard systems

Mathias Steinhuber (University of Regensburg), Jonas Rigo, Juan-Diego Urbina (Uni Regensburg), Markus

Schmitt, Klaus Richter (University of Regensburg)
Tuesday, July 30, 12:00–12:30, East Hall 8

Abstract: We aim to understand the concept of ‘chaotic melting’ of a many-body quantum system, where the equilibrium between many-body localization (MBL) and scrambling required by quantum simulators breaks down. This so-called ‘chaotic melting’ effect has been numerically observed in models describing quantum computers [1,2] for small system sizes, and it could make them infeasible for practical applications pretty soon as the system size increases. Our angle is to study the Bose-Hubbard model, a chaotic system, and to look for numerical evidence of chaotic melting in large systems using neural quantum states (NQS) to push the limit of numerical accessibility. To achieve this, we needed to adjust NQS-techniques for bosonic systems by implementing the so-called Transformer architecture [3] (e.g. found in large language models like ChatGPT) to represent low energy states efficiently. Furthermore, as work in progress, we present first results on the way introducing disorder can stabilize MBL against scrambling for large system sizes.

- [1] S.-D. Børner, C. Berke, D. P. DiVincenzo, S. Trebst, and A. Altland. Classical Chaos in Quantum Computers. 2023. arXiv: 2304.14435
- [2] J. Chávez-Carlos et al. Driving superconducting qubits into chaos. 2024. arXiv: 2310.17698
- [3] A. Vaswani et al. Attention Is All You Need. arxiv: 1706.03762

Quantum chaos and quantum phase transitions in superconducting qubits

Jorge Chávez-Carlos (Universidad Nacional Autónoma de México), Lea Santos (University of Connecticut)
Tuesday, July 30, 13:30–14:00, East Hall 8

Abstract: Transmon qubits are the predominant element in circuit-based quantum information processing, such as existing quantum computers. But more than qubits, they are multilevel driven nonlinear (Kerr) oscillators that can be used to investigate fundamental physics questions. We show that in the regime where Kerr-cat qubits are generated, these Kerr parametric oscillators can be used as simulators of excited state quantum phase transitions. On the other hand, by increasing nonlinearities with the goal enabling faster gate times, one may end up inducing chaos, which melts the qubit away. We determine the region of validity of the Kerr-cat qubit and discuss how its disintegration could be experimentally detected. The danger zone for parametric quantum computation is also a potential playground for investigating quantum chaos with driven superconducting circuits.

Multi-unitary circuit dynamics

Pieter Claeys (Max Planck Institute for the Physics of Complex Systems)
Tuesday, July 30, 14:00–14:30, East Hall 8

Abstract: Dual-unitary circuits present minimal models of chaotic quantum many-body dynamics in which an underlying space-time symmetry restricts the dynamics to be unitary along both the time and the space directions. Remarkably, this space-time symmetry results in exactly solvable yet chaotic dynamics, tailor-made for implementation in current digital quantum computing and simulation platforms. In this talk I will review recent moves to generalize dual-unitarity to dynamics that exhibits multiple unitary directions, in this way relaxing some of the constraints of dual-unitarity, and discuss the implications on entanglement and operator dynamics.

Minisymposium on Random Dynamical Systems

Lower bounds to Lyapunov exponents in passive scalar advection

Tommaso Rosati (Warwick University)
Wednesday, July 31, 16:00–16:30, IRC Seminar Room 2

Abstract: We consider a passive scalar advected by a stochastic velocity field. Under some non-degeneracy assumptions on the noise, we prove a lower bound on the energy dissipation that is quantitative in the diffusivity of the scalar. The proof is based on dynamics of energy level sets, a refined short-time and

high-frequency expansion and the introduction of suitable concentration norms. Joint work with M. Hairer, S. Punshon-Smith and J. Yi.

Regularization properties of passive scalar driven by Kraichnan Log-Lipschitz velocity fields

Marco Bagnara (Scuola Normale Superiore), Mario Maurelli (University of Pisa)

Wednesday, July 31, 16:30–17:00, IRC Seminar Room 2

Abstract: We consider a passive scalar driven by a random Kraichnan velocity field. The covariance of the Kraichnan noise is perturbed by a logarithmic correction, corresponding to a stochastic Log-Lipschitz velocity field advecting the passive scalar. We show that such transport noise is able to provide regularizing properties for the passive scalar in terms of negative fractional Sobolev norms.

On the Infinite Dimension Limit of Invariant Measures and Solutions of Zeitlin's 2D Euler Equations

Milo Viviani (Scuola Normale Superiore, Pisa), Umberto Pappalettera (Universität Bielefeld), Franco Flandoli (Scuola Normale Superiore, Pisa)

Wednesday, July 31, 17:00–17:30, IRC Seminar Room 2

Abstract: In this talk we consider a finite dimensional approximation for the 2D Euler equations on the sphere S^2 , proposed by V. Zeitlin, and show their convergence towards a solution to Euler equations with marginals distributed as the enstrophy measure. The method relies on nontrivial computations on the structure constants of S^2 . In the second part of the talk, we discuss the problem of extending our results to Gibbsian measures associated with higher Casimirs. In particular, we focus on the link between the invariant measures in the Zeitlin model and in the Euler equations.

Evolution of negative Sobolev norms in the Kraichnan model

Lucio Galeati (EPFL)

Wednesday, July 31, 17:30–18:00, IRC Seminar Room 2

Abstract: The Kraichnan model is a celebrated SPDE from synthetic turbulence, consisting in a passive scalar advected by a Gaussian velocity field, which is taken white in time and coloured, divergence-free in space. More precisely, it is a family of equations, indexed by a parameter $\alpha > 0$ measuring the spatial regularity of the noise. For $\alpha > 1$, Lagrangian trajectories are unique and one can show the existence of Lyapunov exponents; but for $\alpha \in (0, 1)$, one can show non-uniqueness of trajectories, intrinsic stochasticity and anomalous dissipation of energy still coexisting with uniqueness of solutions to the transport equation. In this talk I will present some recent advances on the topic, based on the analysis of the evolution of negative Sobolev norms H^{-s} of the tracer. It is shown that, for a suitable range of values s , a consistent regularity gain of order $1 - \alpha$ can be observed, which further allows to uniquely extend the dynamics to H^{-s} -valued initial data, which become instantaneously L^2 -regular. Based on a joint work with Francesco Grotto and Mario Maurelli.

Time-shifted Synchronisation in Chemical Reaction Networks

Guillermo Olicón Méndez (Universität Potsdam), Maximilian Engel (University of Amsterdam), Robin Chemnitz (Freie Universität Berlin), Steffanie Winkelmann (Zuse Institute Berlin), Nathalie Wehlitz (Zuse Institute Berlin)

Wednesday, July 31, 18:00–18:30, IRC Seminar Room 2

Abstract: In this talk we consider dynamical models of chemical reaction networks, in which we count the number of individuals of each chemical species. The number of such individuals change when one of the reactions involved occurs randomly (according to a specific law, depending on the physicochemical problem in question).

A realisation of these type of models is given by Gillespie's algorithm. In different examples we observe that different trajectories, under the influence of the same noisy perturbations but changing the initial conditions, seem to follow each other with a delay.

The study of this time-delayed synchronisation phenomenon in our set-up leads to the study of random attractors in discrete time random dynamical systems (RDS) on a countable state space. We present some features of these random attractors, and we emphasize the close relationship between the dynamics and the associated Markov chain. As an example, we compare different birth-death chains whose attractors have qualitative differences only at the level of the so-called two-point motion.

Minisymposium on Reaction networks

Biomass transfer and autocatalytic waiting time on growing reaction networks

Wei-Hsiang Lin (Academia Sinica)

Tuesday, July 30, 13:30–14:00, SCC Conference Hall

Abstract: In this work, I analyzed the biomass transfer process on scalable reaction networks [1]. Typically, reaction network dynamics can be described by ordinary differential equations, while there are corresponded stochastic processes for biomass transition between nodes of the networks. The idea here is to characterize the autocatalysis pathways and the waiting times distribution for biomass to complete these pathways. By introducing the concept of autocatalytic waiting time (AWT) kernel, the long-term growth kinetics of an n-dimensional scalable reaction network can be studied via a 1-dimensional delay-differential equation. The AWT kernel is a useful tool for comparing growing networks with different complexities or under different coarse-graining schemes. It helps us to develop theories that are not limited to specific network dimension or topology.

[1] <https://pubmed.ncbi.nlm.nih.gov/33093194/>

From complex-balanced mass-action systems to binomial differential inclusions

Stefan Müller (University of Vienna (Austria))

Tuesday, July 30, 14:00–14:30, SCC Conference Hall

Abstract: We extend a classical result by Horn and Jackson on the asymptotic stability of complex-balanced equilibria of mass-action systems. As it turns out, all dynamical systems are asymptotically stable that can be embedded in "binomial differential inclusions". For the proof, we use a new decomposition of the Laplacian matrix which allows to write weakly reversible mass-action systems as sums of binomial terms (just like reversible systems). In turn, this allows to consider regions in the positive orthant with given monomial evaluation orders (and corresponding polyhedral cones in logarithmic coordinates).

Implicit universal structural properties of CRNs in chemistry: implications and open questions for theory and experiment

Alexander Blokhuis (IMDEA Nanociencia)

Tuesday, July 30, 14:30–15:00, SCC Conference Hall

Abstract: Experimentally, a reaction network is more than a model: it is the interpretation of experiments, a falsifiable scientific claim in terms of a language in which (much of) chemistry is to be understood. Theoretically, CRN theory is used more broadly to understand various classes of dynamical systems and link them to structural network properties. In the context of chemistry, there exist a variety of properties that chemists use on a daily basis, but which are presently not implemented in CRNs, and whose consequences are not well understood.

Here, we focus on the property that chemists do not consider CRN descriptions for a system to be unique, but that it is always assumed that CRNs are part of hierarchy of mutually consistent descriptions which each match experimental detail at different levels of resolution, with fine-grained descriptions involving simpler reactions with fewer species. Extrapolating, we may study 'stretched' CRNs in chemistry as built up using enough of only the simplest reversible reactions: 1-to-1 and 2-to-1 reactions (3 distinct species) with mass-action kinetics.

Doing so, commonly employed mathematical objects (e.g. dynamical equations, species-reaction graphs, stoichiometric matrix) used to study reaction networks can suddenly fully characterize the CRN and be

mapped one-to-one, facilitating problem-solving for problems in CRNs. The regular structure and fewness of fundamental objects (unimolecular, bimolecular reactions) limits the number of cases one needs to check for proofs, and also enables the formulation of many new mathematical objects fully characterizing CRNs, such as nonlinear Jacobians for species and for currents, each tailored to study particular types of properties. By combining these objects, remarkably simple proofs of new general properties, such as 'handshake stability' can be found.

Structural features and properties of stretched CRNs can be characterized in terms of topological indices, which are related by a large number of independent Euler-like laws. I will discuss how these indices can be used to theoretically characterize allowed CRN behavior and under what circumstances they are robust to a hierarchy of agreeing descriptions for the same system.

Experimentally, finding a reaction network description consistent with all observation is a hard, open problem: current methodologies cannot assess the wealth of conceivable reaction networks and instead have to opt for postulating and evaluating a handful of them. A subset of topological indices that is experimentally accessible can be harnessed to address this problem: each additional index further characterizes the CRN and exponentially filters hypotheses. We illustrate through a variety of real-world examples how this has recently allowed to find scientific explanations for heretofore open problems in chemistry.

Oscillatory reaction networks: a primer on global Hopf bifurcation

Bernold Fiedler (Free University Berlin)

Wednesday, July 31, 11:00–11:30, IRC Seminar Room 1

Abstract: Detecting periodic oscillations in chemical and metabolic networks remains difficult. Is local Hopf bifurcation, i.e. linearization with purely imaginary eigenvalues etc., really a good idea? Large systems are practically inaccessible to a symbolic Routh-Hurwitz approach. We suggest how, and at what price, methods of global, rather than local, Hopf bifurcation may be able to overcome this difficulty.

Mass action systems with many limit cycles

Josef Hofbauer (Universität Wien)

Wednesday, July 31, 11:30–12:00, IRC Seminar Room 1

Abstract: I will present examples of bimolecular n species mass action systems with $\sim 3^{n/3}$ stable limit cycles.

Hopf bifurcation in mass action kinetics networks

Maya Mincheva (Northern Illinois University), Carsten Conradi (HTW-Berlin)

Wednesday, July 31, 13:30–14:00, IRC Seminar Room 1

Abstract: We present a method for identifying Hopf bifurcation points in parametric ordinary differential equations (ODE) systems modeling reaction networks with n species. The method is based on a Hopf bifurcation theorem, algebraic geometry, majorization theory and convex analysis. Selecting parameter values such that the next to last Hurwitz determinant $\det H$ is zero is a condition of the Hopf bifurcation theorem. The existence of a vertex of the Newton polytope of $\det H$ associated with a negative monomial usually guarantees finding parameter values such that $\det H$ is zero. We show that a vertex of the Newton polytope of $\det H$ exists among the monomial exponents in the product of determinants of diagonal submatrices of H .

Absence of Hopf bifurcation in special reductions of dual phosphorylation networks.

Nidhi Kaihnsa (University of Copenhagen), Elisenda Feliu (University of Copenhagen)

Wednesday, July 31, 14:00–14:30, IRC Seminar Room 1

Abstract: Dual-site phosphorylation networks have rich dynamics. When phosphorylation occurs as mixed mechanism (both processive and distributive) network can exhibit oscillations. However, it is as yet unknown whether fully sequential and distributive process admits such behaviour. In this article we provide evidence

in favour of absence of oscillations by considering certain reductions of this process. Our results rely on analyses of semi-algebraic conditions obtained from Hurwitz matrices of the characteristic polynomial of the Jacobian matrix.

On the qualitative dynamics of reaction networks that admit a monomial parameterization

Carsten Conradi (Hochschule für Technik & Wirtschaft Berlin)

Wednesday, July 31, 14:30–15:00, IRC Seminar Room 1

Abstract: Polynomial Ordinary Differential Equations are an important tool in many areas of quantitative biology. Due to high measurement uncertainty, few experimental repetitions and a limited number of measurable components, parameters are subject to high uncertainty and can vary in large intervals. One therefore effectively has to study families of parametrized polynomial ODEs. Multistationarity and Hopf bifurcations have been recognized as important features of these ODEs. As parameter values are confined to large intervals one is generally interested in parameter conditions that guarantee multistationarity or a Hopf bifurcation and further constrain the parameter values. The focus of this talk are mass action ODEs that admit a monomial parameterization of positive steady states. For such systems it is straightforward to derive rate constants where multistationarity or Hopf bifurcations exist. To this class belong, for example, multisite phosphorylation systems, key players in intracellular signaling and regulation.

Hopf bifurcation in substructures of MAPK

Casian Pantea (West Virginia University), Lorand Parajdi

Wednesday, July 31, 15:00–15:30, IRC Seminar Room 1

Abstract: The MAPK cascades are principal kinase transduction pathways in eukaryotic cells. An important open question is whether the building block of these cascades, i.e. the double phosphorylation network (or MAPK for shortness) admits oscillations. Inspired by recent results on the inheritance of oscillations from substructures in reaction networks, we analyze the existence of Hopf bifurcations in subnetworks of MAPK. We identify a new oscillatory motif of MAPK, which underlies versions of MAPK that have recently been shown to admit oscillations. This is joint work with Lorand Parajdi.

Minisymposium on Realization of connection structures in phase space

Finite switching near heteroclinic networks

Sofia Castro (Faculdade de Economia da Universidade do Porto), **Liliana Sofia Garrido da Silva** (Universidade do Porto)

Monday, July 29, 13:30–14:00, East Hall 4

Abstract: We address the level of complexity that can be observed in the dynamics near a robust heteroclinic network. We show that infinite switching, which is a path towards chaos, does not exist near a heteroclinic network such that the eigenvalues of the Jacobian matrix at each node are all real. Furthermore, for a path starting at a node that belongs to more than one heteroclinic cycle, we find a bound for the number of such nodes that can exist in any such path. This constricted dynamics is in stark contrast with examples in the literature of heteroclinic networks such that the eigenvalues of the Jacobian matrix at one node are complex.

Arbitrary Sensitive Transitions in Recurrent Neural Networks

Muhammed Fadera

Monday, July 29, 14:00–14:30, East Hall 4

Abstract: An Excitable Network Attractor (ENA) is a forward-invariant set in phase space composed of two or more attractors and parts of their basins that allow transitions between them under input or noise perturbations. ENAs have recently been used to explain the input-driven dynamics of RNNs trained on sequence-to-sequence classification problems. For example, errors in performance of such trained RNNs are related to transitions to attractors outside the associated ENA. Typically an ENA is extracted from a trained

RNN by finding fixed points in the autonomous dynamics of the RNN and using input-driven trajectories to infer transitions between these fixed points. While successful, this approach is computationally expensive. An alternative approach is to train a model which can realise arbitrary ENAs, and tune the number and sensitivity of attractors to inputs to match the dynamics of the trained RNN. Previous work has demonstrated that ENAs of arbitrary sensitivity can be realised in a RNN by suitable choice of connection weights and nonlinear activation function. The issue with this approach is that RNNs are trained by adapting the weight matrix while the activation function is held fixed. In this talk, I will show that ENAs of arbitrary sensitivity and structure can be realised even using a suitable fixed nonlinear activation function, i.e. by suitable choice of weights only. Furthermore, the weight matrix can be chosen so that the probability of transitions that do not follow the ENA is small.

Robust heteroclinic cycles in pluridimensions

Sofia Castro (Universidade do Porto), **Alastair Rucklidge** (University of Leeds)

Monday, July 29, 14:30–15:00, East Hall 4

Abstract: Robust heteroclinic cycles are sequences of equilibria along with trajectories that connect them in a cyclic manner. We investigate a class of robust heteroclinic cycles that does not satisfy the usual condition that all connections between equilibria lie in subspaces of equal dimension. We refer to these as robust heteroclinic cycles in pluridimensions. With a few reasonable assumptions, we show that such cycles require a state space of at least four dimensions and, in any dimension, a minimum of four equilibria in the cycle. We also show that there are four distinct examples of robust heteroclinic cycles in pluridimensions between four equilibria in four dimensions, and study their stability. This involves generalizing the usual Poincaré return map approach by allowing non-square transition matrices. We provide numerical illustrations of each of the four examples. Although our examples are in four dimensions, we present them in a manner that can be readily adapted to other problems in higher dimensions. Potential applications include modelling the dynamics of evolving populations when there are transitions between equilibria corresponding to mixed populations with different numbers of species.

Higher-order interactions, heteroclinic dynamics, and aperiodicity

Christian Bick (Vrije Universiteit Amsterdam), Sören von der Gracht, Alexander Lohse

Monday, July 29, 15:00–15:30, East Hall 4

Abstract: We consider two aspects of heteroclinic dynamics. First, we consider heteroclinic dynamics in network dynamical systems to understand how nonpairwise "higher-order" interactions allow for (or prevent) the emergence of heteroclinic cycles. Second, we turn to more general heteroclinic structures in phase space: Specifically, we ask whether (and if so, how many) trajectories can approach a heteroclinic network passing by an aperiodic sequence of equilibria (in contrast say to a cycle that gives a periodic sequence). This is joint work with Sören von der Gracht and Alexander Lohse.

Minisymposium on Stability and Bifurcations of Hamiltonian Systems. Theory and Applications

KAM theory in application of multi-scale systems

Lu Xu (Jilin University)

Wednesday, July 31, 13:30–14:00, East Hall 4

Abstract: In this talk, I will initially present the KAM theorems for multi-scale Hamiltonian systems. These multi-scale Hamiltonians arise from celestial mechanics, and the stability of such systems holds significant importance in study of the properties of the dynamics. I will show the persistence results of both full-dimensional and lower-dimensional tori, which bear similarities to the findings observed in classical nearly integrable Hamiltonians.

Subsequently, I will show the applications of these results. For instance, in the context of quasi-periodically forced differential equations, response solutions refer to quasi-periodic outcomes where the

frequency vector aligns with that of the forcing function. These solutions are recognized as the most robust ones so that the existence of such solutions are comprehensively applied in studying the harmonic and synchronizing behaviors exhibited by quasi-periodically forced oscillators, also in scenarios involving substantial damping or non-degenerate cases with minimal or absent damping.

I will show recent results in demonstrating the existence of responsive solutions within degenerate, quasi-periodically forced nonlinear oscillators featuring minimal or no damping. These findings are substantiated through the application of the Poincare mechanism and the KAM theorems of multi-scale systems.

Bifurcations of Hill regions and relative equilibria in the charged three-body problem

Holger Waalkens (University of Groningen, The Netherlands)

Wednesday, July 31, 14:00–14:30, East Hall 4

Abstract: The charged three-body problem is a generalization of the Newtonian three-body problem where the coefficients in the potential are not necessarily given by products of masses but arbitrary real numbers that in addition to gravitational interactions can also describe e.g. Coulomb interactions. For such systems, we discuss the admissible configurations and orientations for fixed values of the constants of motion. These Hill regions bifurcate at relative equilibria which we discuss in this talk. The presented work is based on a collaboration with Igor Hoveijn.

Periodic magnetic geodesics on compact surfaces

Luca Asselle (Rub (Ruhr-Universität Bochum)), **Gabriele Benedetti** (Vrije Univ. Amsterdam)

Wednesday, July 31, 14:30–15:00, East Hall 4

Abstract: In this joint work with Luca Asselle, we study the dynamics of a charged particle moving on a compact surface under the effect of a strong magnetic field. On the one hand, we show that, generically, there exist infinitely many periodic orbits. On the other hand, we observe that only in the case of constant field and constant curvature, all orbits are periodic. This result contrasts with the fact that there are exotic weak magnetic fields on a flat torus for which all orbits are periodic. The main tool for our study is an enhanced version of a normal form for the dynamics which goes back to the work of Vladimir Arnold.

Nonlinear stability of equilibrium points and its bifurcations in the planar equilateral restricted mass-unequal four-body problem

Martha Alvarez-Ramírez (Metropolitan Autonomous University - Iztapalapa), Alejandro Zepeda-Ramirez (Metropolitan Autonomous University - Iztapalapa)

Wednesday, July 31, 15:00–15:30, East Hall 4

Abstract: In this talk we consider the planar restricted equilateral four-body problem in the case that one particle of negligible mass is moving under the Newtonian gravitational attraction of three positive masses m_1 , m_2 and m_3 (called primaries). These always lie at the vertices of an equilateral triangle (Lagrangian configuration) and move with constant angular velocity in circular orbits around their center of masses.

We study the case where all primaries have unequal masses, investigate the bifurcation of equilibrium points, and determine the nonlinear stability of elliptical equilibria (in the Lyapunov sense). Furthermore, a systematic numerical investigation is performed to obtain the resonance curves in the mass space. We use these curves to answer the question about the existing boundary between the domains of linear stability and instability. The characterization of the total number of stable points found inside the stability domain is discussed.

Joint work with J. Alejandro Zepeda Ramirez.

Stability analysis of a one degree of freedom periodic Hamiltonian system in a case of strong degeneracy

Victor Lanchares (Universidad de La Rioja), Boris Bardin (Moscow Aviation Institute)

Wednesday, July 31, 16:00–16:30, East Hall 4

Abstract: We consider the stability of the equilibrium position of a periodic Hamiltonian system with

one degree of freedom. It is supposed that the series expansion of the Hamiltonian function, in a small neighborhood of the equilibrium position, does not include terms of second and third degree. Moreover, we focus on a degenerate case, when fourth-degree terms in the Hamiltonian function are not enough to obtain rigorous conclusions on stability or instability. A complete study of the equilibrium stability in the above degenerate case is performed, giving sufficient conditions for instability and stability in the sense of Lyapunov. The above conditions are expressed in the form of inequalities with respect to the coefficients of the Hamiltonian function, normalized up to any finite order.

Bifurcations of Families of Ejection/Collision Orbits in a Molecular Model

Óscar Rodríguez (Universitat Politècnica de Catalunya)

Wednesday, July 31, 16:30–17:00, East Hall 4

Abstract: In this presentation, we will delve into the dynamics of ejection/collision orbits within the framework of the hydrogen atom interacting with a circularly polarized microwave field. This problem can be effectively modeled as a perturbed Kepler problem, which, upon suitable transformations of coordinates and time, manifests as a Hamiltonian system with two degrees of freedom, dependent on a single parameter ($K > 0$).

Our focus will be on the analysis of n -ejection-collision orbits (n -ECOs), depicting the trajectory of the electron as it undergoes ejection from the nucleus and subsequent collision at the n relative minima in distance with respect to the nucleus.

We will analyze the behavior and evolution of the families of n -EC orbits, with a particular emphasis on the properties and characteristics of the bifurcation families that arise as the energy levels of the problem vary. We apply the Levi-Civita regularization which presents two clear advantages compared with the McGehee's one: the ECOs require a finite interval of time, and the initial conditions are exact at the collision point. By employing Levi-Civita variables, we can analytically establish the existence of exactly two families of n -ECOs for any value of n , under the condition that the energy level is sufficiently constrained.

In addition, we will conduct a thorough examination of the evolution of n -ECO families, supplemented by a comprehensive numerical analysis of bifurcations. This analysis entails precise computations to delineate the successive emergence of bifurcation families. Furthermore, we will explore the periodic and quasi-periodic motions exhibited by n -ECOs within these bifurcation families.

This is a joint work with Esther Barrabés and Mercè Ollé.

Integrability on singularly reduced spaces

Nikolay Martynchuk (University of Groningen)

Wednesday, July 31, 17:00–17:30, East Hall 4

Abstract: As is well known, integrable systems come with a set of commuting integrals of motion. In 4 dimensions, an integrable system is specified by a pair of functions. Fixing one of these functions, gives an invariant manifold and in the presence of a circular group action, one can reduce the dimension further to obtain a 1 degree of freedom system.

In this talk, we shall discuss such a reduction in the case when the circle action has a singularity (an isotropy subgroup \mathbb{Z}_k or \mathbb{S}^1). We will present specific examples and attempt their symplectic classification.

Bifurcations of Riemann Ellipsoids

Patricia Yanguas (Universidad Pública de Navarra), Fahimeh Mokhtari (VU Amsterdam), Jesús Palacián (Universidad Pública de Navarra)

Wednesday, July 31, 17:30–18:00, East Hall 4

Abstract: We give an account of the various changes in the stability character in the five types of Riemann ellipsoids by establishing the occurrence of different quasi-periodic Hamiltonian bifurcations. Suitable symplectic changes of coordinates, that is, linear and non-linear normal form transformations are performed, leading to the characterisation of the bifurcations responsible of the stability changes. Specifically we find three types of bifurcations, namely, Hamiltonian pitchfork, saddle-centre and Hamiltonian-Hopf in the

four-degree-of-freedom Hamiltonian system resulting after reducing out the symmetries of the problem. The approach is mainly analytical up to a point where non-degeneracy conditions have to be checked numerically. We also deal with the regimes in the parametric plane where Liapunov stability of the ellipsoids is accomplished. This strong stability behaviour occurs only in two of the five types of ellipsoids, at least deductible only from a linear analysis.

Minisymposium on Stability, long term behaviour, and data assimilation in infinite dimensional stochastic systems for weather, climate, and ocean

Well posedness and long time dynamics for a quasi-geostrophic ocean-atmosphere model

Federico Fornasaro (Sapienza Università di Roma)

Monday, July 29, 13:30–14:00, IRC Seminar Room 3

Abstract: The interaction between the atmosphere and the ocean is a crucial mechanism driving weather and climate. We consider a system of equations proposed in literature to treat both the mechanical and thermal interaction, which are used to derive the MAOOAM model presented by Vannitsem et al. in [1]. This system is composed of two layer quasi-geostrophic equations for the atmosphere which interact mechanically and thermally with a shallow water ocean layer. The model has been extensively studied numerically, less so analytically. In this talk I will present some results on existence and uniqueness for different classes of solutions for this model, together with a discussion about the global attractor and its properties: an interesting asymmetry in the regularity of the unknowns emerges.

[1] Lesley De Cruz, Jonathan Demaeyer, Stephane Vannitsem, The Modular Arbitrary-Order Ocean-Atmosphere Model: MAOOAM v1.0, Geoscientific Model Development, 2016.

McKean-Vlasov SDEs in nonlinear filtering

Wilhelm Stannat (TU Berlin)

Monday, July 29, 14:00–14:30, IRC Seminar Room 3

Abstract: Estimating Markovian signals X from noisy observations is an important problem in the natural and engineering sciences. Within the Bayesian approach the underlying mathematical problem essentially consists in the (stochastic) analysis of the conditional law of X with a view towards its efficient numerical approximation.

In this talk I will discuss mean-field type descriptions of the conditional law of X , when X is the solution of a stochastic differential equation, and present recent results on corresponding ensemble-based numerical approximations in the case with correlated observation noise.

The talk is based on joint work with S. Ertel.

Analysis of a two-layer (stochastic) energy balance climate model

Cristina Urbani (Universitas Mercatorum)

Monday, July 29, 14:30–15:00, IRC Seminar Room 3

Abstract: A simple yet extremely valuable approach to the study of the climate system comes from the use of Energy Balance Models (EBM), which had originally been introduced in the sixties independently by Budyko and Sellers. Such models describe in a simplified yet effective way the evolution of the zonally averaged temperature on the Earth's surface, thus reducing the problem to a single 1D field. The classical EBM can be improved by increasing the vertical resolution. In this talk I will present a two-layer energy balance model, that allows for vertical exchanges between a surface layer and the atmosphere. I will analyse stability, long time behaviour of solutions and the sensitivity of our deterministic model with respect to parameters which are partly related to the greenhouse effect, and finally I will introduce its stochastic version.

Variational techniques for a one-dimensional energy balance model

Gianmarco Del Sarto (Scuola Normale Superiore, Pisa), Jochen Bröcker (University of Reading), Franco Flandoli (Scuola Normale Superiore, Pisa), Tobias Kuna (Università degli Studi dell'Aquila)
Monday, July 29, 15:00–15:30, IRC Seminar Room 3

Abstract: A one-dimensional climate energy balance model (1D EBM) is a simplified climate model for the zonally averaged global temperature profile, based on the Earth's energy budget. We examine a class of 1D EBMs which emerges as the parabolic equation corresponding to the Euler–Lagrange equations of an associated variational problem, covering spatially inhomogeneous models such as with latitude-dependent albedo. Sufficient conditions are provided for the existence of at least three steady-state solutions in the form of two local minima and one saddle, that is, of coexisting “cold”, “warm” and unstable “intermediate” climates. We also give an interpretation of minimizers as “typical” or “likely” solutions of time-dependent and stochastic 1D EBMs.

We then examine connections between the value function, which represents the minimum value (across all temperature profiles) of the objective functional, regarded as a function of greenhouse gas concentration, and the global mean temperature (also as a function of greenhouse gas concentration, i.e. the bifurcation diagram).

Specifically, the global mean temperature varies continuously as long as there is a unique minimizing temperature profile, but coexisting minimizers must have different global mean temperatures. Furthermore, global mean temperature is non-decreasing with respect to greenhouse gas concentration, and its jumps must necessarily be upward.

Applicability of our findings to more general spatially heterogeneous reaction–diffusion models is also discussed, as are physical interpretations of our results.

Filtering Dynamical Systems Using Observations of Statistics

Eviatar Bach, Tim Colonius, Isabel Scherl, Andrew Stuart
Monday, July 29, 17:00–17:30, IRC Seminar Room 3

Abstract: We consider the problem of filtering dynamical systems, possibly stochastic, using observations of statistics. Thus, the computational task is to estimate a time-evolving density $\rho(v, t)$ given noisy observations of the true density ρ^\dagger ; this contrasts with the standard filtering problem based on observations of the state v . The task is naturally formulated as an infinite-dimensional filtering problem in the space of densities ρ . However, for the purposes of tractability, we seek algorithms in state space; specifically, we introduce a mean-field state-space model, and using interacting particle system approximations to this model, we propose an ensemble method. We refer to the resulting methodology as the ensemble Fokker–Planck filter (EnFPF).

Under certain restrictive assumptions, we show that the EnFPF approximates the Kalman–Bucy filter for the Fokker–Planck equation, which is the exact solution to the infinite-dimensional filtering problem. Furthermore, our numerical experiments show that the methodology is useful beyond this restrictive setting. Specifically, the experiments show that the EnFPF is able to correct ensemble statistics, to accelerate convergence to the invariant density for autonomous systems, and to accelerate convergence to time-dependent invariant densities for non-autonomous systems. We discuss possible applications of the EnFPF to climate ensembles and to turbulence modeling.

Dynamical reconstruction of unknown forces in quasigeostrophic and transport equations

Jochen Bröcker (University of Reading), Tobias Kuna, Vincent Ryan Martinez, Giulia Carigi
Monday, July 29, 17:30–18:00, IRC Seminar Room 3

Abstract: The term data assimilation from the geosciences refers to reconstructing the current state of a dynamical system from current and past observations of the system (which are typically polluted by stochastic noise). In this contribution, we will analyse simple data assimilation schemes that not only estimate the underlying states of a dynamical system but simultaneously reconstruct unknown components of the dynamics. Specifically, we focus on quasigeostrophic and transport-diffusion equations (for instance for

atmospheric aerosols or tracer gases) and reconstruct forcings to these equations (which may represent surfac fluxes) along with the underlying dynamical states. We are able to prove that provided the observations are sufficiently “rich” (although still finite dimensional), the algorithm will recover the unknown forcings and states with asymptotically vanishing error. Although the methods we consider are conceptually very simple (and not optimal in any way), the analysis is complicated by the fact that the considered systems are infinite dimensional.

Minisymposium on Synchronization, pattern formation and symmetries in distributed dynamical systems

Distributed Oscillators and Clocks via Random Interactions

Peter Kling (University of Hamburg)

Tuesday, July 30, 13:30–14:00, IRC Seminar Room 2

Abstract: We study the dynamics of huge systems of very simple, autonomous agents under erratic communication. Such systems find applications in, for example, the study of chemical reaction networks, the analysis of opinion dynamics in social networks, or the deployment of sensor networks to track animal populations. The standard model assumes n agents, each modeled as a simple finite state machine. Agents interact in small groups that are composed of (typically) randomly chosen agents. Upon such an interaction, each agent updates its state according to a common transition function that maps its own state and that of its interaction partners to a new state.

A major challenge in such models is to create synchronization primitives that leverage the randomized group selection to simulate some form of global signal, which allows agents to perform their respective roles in a timed fashion. We will outline the state of the art for such primitives and dive into a recent construction that seems particularly promising with respect to efficiency and provided synchronization guarantees.

Forming Large Patterns with Local Robots in the OBLLOT Model

Jonas Harbig (Paderborn University)

Tuesday, July 30, 14:00–14:30, IRC Seminar Room 2

Abstract: In the arbitrary pattern formation problem, n autonomous, mobile robots must form an arbitrary pattern $P \subset \mathbb{R}^2$. The (deterministic) robots are typically assumed to be indistinguishable, disoriented, and unable to communicate. An important distinction is whether robots have memory and/or a limited viewing range. Previous work managed to form P under a natural symmetry condition if robots have no memory but an unlimited viewing range [Yamashita, TCS 2010] or if robots have a limited viewing range but memory [Yamauchi, Sirocco 2013]. In the latter case, P is only formed in a shrunk version that has constant diameter.

Without memory and with limited viewing range, forming arbitrary patterns remains an open problem. We provide a partial solution by showing that P can be formed under the same symmetry condition if the robots’ initial diameter is ≤ 1 . Our protocol partitions P into rotation-symmetric components and exploits the initial mutual visibility to form one cluster per component. Using a careful placement of the clusters and their robots, we show that a cluster can move in a coordinated way through its component while “drawing” P by dropping one robot per pattern coordinate.

Patterns, bifurcations, and chaos in networks of oscillators

Riccardo Bonetto (University of Groningen)

Tuesday, July 30, 14:30–15:00, IRC Seminar Room 2

Abstract: Coupled oscillators are undoubtedly relevant models of real-world systems. From neuronal networks and biological systems to chemical and physical applications, networked oscillators have proven to be very effective in describing complex behaviour. The present talk aims to disclose the interplay between the network structure and the rich dynamics of systems consisting, for example, of coupled pendula or harmonic oscillators. We consider different forms of coupling within the Hamiltonian framework. We show a detailed description of bifurcations and perturbations, that are intimately related to the network structure.

The theory developed allow us to seek and understand the emergence of chaotic behaviour in complex systems with symmetries.

On the Dynamical Hierarchy in Circulant Gathering Protocols

Raphael Gerlach (Paderborn University), **Sören von der Gracht** (Paderborn University), **Michael Dellnitz** (Paderborn University)

Tuesday, July 30, 15:00–15:30, IRC Seminar Room 2

Abstract: In this talk we investigate the convergence behavior of gathering protocols with fixed circulant topologies using tools from dynamical systems. Given a fixed number of mobile entities moving in the Euclidean plane, we model a gathering protocol as a system of ordinary differential equations whose equilibria are exactly all possible gathering points. Then, we find necessary and sufficient conditions for the structure of the underlying interaction graph such that the protocol is stable and converging, i.e., gathering, in the distributive computing sense by using tools from dynamical systems. Moreover, these tools allow for a more fine grained analysis in terms of speed of convergence in the dynamical systems sense. In fact, we derive a decomposition of the state space into stable invariant subspaces with different convergence rates. In particular, this decomposition is identical for every (linear) circulant gathering protocol, whereas only the convergence rates depend on the weights in interaction graph itself.

Minisymposium on Teichmüller Dynamics

The mapping class group action on Euler class zero $SL(2, \mathbb{R})$ representations

Peter Smillie, James Farre, Martin Bobb

Wednesday, July 31, 11:00–11:30, East Hall 8

Abstract: Let $X(G)$ be the character variety of representations of a surface group into a Lie group G , so that the mapping class group acts on $X(G)$. When G is compact, this action is ergodic by work of Goldman and Pickrell-Xia, and a well-known conjecture of Goldman is that for $G = \mathrm{PSL}(2, \mathbb{R})$ (and genus at least 3), the action is ergodic on each non-Fuchsian topological component of $X(G)$. This turns out to be essentially equivalent to a conjecture of Bowditch that every non-elementary non-Fuchsian representation in $X(\mathrm{PSL}(2, \mathbb{R}))$ sends some simple closed curve loop to a non-hyperbolic element. By studying the action of the mapping class group on the tangent cone to the subvariety of nontrivial diagonal representations, we prove that Bowditch's condition holds in a neighborhood of the nontrivial diagonal representations in the Euler number zero component. This is joint work with James Farre and Martin Bobb.

Stationary measures for Fuchsian groups

Vaibhav Gadre (University of Glasgow)

Wednesday, July 31, 11:30–12:00, East Hall 8

Abstract: A random walk on a Fuchsian group (that is, a lattice in $SL(2, \mathbb{R})$) gives a random walk in the hyperbolic plane. Under mild conditions, a typical sample path for such a walk converges to the circle at infinity. The distribution of the limits of sample paths define a stationary measure on the circle. For a large class of random walks, the stationary measures are singular with respect to the Lebesgue class. This talk will survey the landscape of results related to the study of such stationary measures on the circle and in analogous contexts.

Applications of smooth curves in the moduli space of translation surfaces

Krzysztof Fraczek (Nicolaus Copernicus University)

Wednesday, July 31, 13:30–14:00, East Hall 8

Abstract: I plan to review of problems and recent results regarding smooth curves in the moduli space of translation surfaces and Teichmüller positive semi-orbits starting from such curves. The main part of the talk are applications that motivate abstract results. Two main applications relate to billiards on nibbled ellipses and Hamiltonian systems with reflections.

Quantitative Equidistribution on the Unit Square

Christian Weiß (Ruhr West University of Applied Sciences)

Wednesday, July 31, 14:00–14:30, East Hall 8

Abstract: One method to quantify equidistribution of the translation flow on the unit square is to compare the actual time that the translation flow spends in a given set with the expected time. General error bounds for measurable sets were established by J. Beck. Later, Grepstad and Larcher realized that these bounds can be significantly strengthened for geometrically nice sets like polygons. In this talk, we consider the error term for the algebra generated by convex sets which may be considered as an intermediate case between the two mentioned results. We also discuss open questions and potential generalizations. This is joint work with M. Goering.

A Central Limit Theorem for the Kontsevich-Zorich cocycle

Hamid Al-Saqban

Wednesday, July 31, 14:30–15:00, East Hall 8

Abstract: The Kontsevich-Zorich (KZ) cocycle is a key dynamical system that is closely related to the derivative cocycle of the Teichmüller geodesic flow. We will state and sketch the proof of a central limit theorem that establishes the existence of large fluctuations for the asymptotic growth of the Hodge norm of the KZ cocycle. Our work is joint with Giovanni Forni.

Skew-product systems over infinite interval exchange transformations

Olga Lukina (Leiden University)

Wednesday, July 31, 16:00–16:30, East Hall 8

Abstract: A rotated odometer is an infinite interval exchange transformation (IIET) obtained by pre-composing the von Neumann-Kakutani map with an exchange of a finite number of intervals of equal length. Such IIETs model the first return maps of flows on a certain translation surface with a wild singularity. In this talk, we consider the ergodic properties of lifts of such flows along an infinite-to-one covering map of the surface. Our main tools are theory of essential values, and the symbolic dynamical representations of the rotated odometers. Joint work with Henk Bruin (University of Vienna).

Translation Surfaces with Finite Area and Strongly Mixing Billiard Flow

Erick Gordillo (Heidelberg Universität)

Wednesday, July 31, 16:30–17:00, East Hall 8

Abstract: We construct a family of translation surfaces characterized by infinite genus and finite area, demonstrating that their vertical billiard flow is strongly mixing. In the 1980s, A. Katok established that strongly mixing billiard flows could not exist on finite-type translation surfaces. However, in 2014, Lindsey and Treviño showed that this limitation does not apply to translation surfaces of infinite type, although they did not provide explicit examples. In this talk, we present a family of such examples derived from certain suspension flows over strongly mixing staircase transformations.

Minisymposium on Turbulence, nonequilibrium, and wind energy

Active grid turbulence generated by geometrical and temporal excitation

Michael Hölling (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), **Lars Neuhaus** (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), **Joachim Peinke** (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research)

Monday, July 29, 11:00–11:30, Research III Lecture Hall

Abstract: The generation of user-defined flows is not only of interest for basic turbulence research but also

for applied problems such as mixing processes or in the field of wind energy. The latter has several goals for wind field generation in wind tunnel investigations. On the one hand, temporally distinct coherent structures are used to measure the reaction dynamics of model turbines or to further develop control strategies. On the other hand, the interaction of wind turbines with turbulent flows and their influence on the power output, the resulting forces on rotor blades and other turbine components, the development of the wake, etc. is important for the design of turbines as well as the layout of wind farms. With the help of active grids these two areas can be covered. We present results from wind tunnel experiments carried out in the large turbulent wind tunnel at the University of Oldenburg. The wind tunnel has an outlet of $3 \times 3\text{m}^2$ and a total length of 30m in a closed measurement section. The active grid consists of 80 shafts, on which square flaps with an edge length of about 10cm are mounted. The motion of each shaft can be controlled individually by means of DC motors. Specially developed motion protocols for the motors (temporal excitation) can be used to generate temporal events such as IEC wind gusts spatially coherent over a pre-defined area behind the grid [1]. Alternatively, for geometrical excitation, the generated turbulent flows develop behind the grid to e.g. eventually exhibit a $-5/3$ behaviour over an exceptionally long range in the spectrum [2] or show regions with increasing turbulence intensity and length scales. The related methods additionally offers the possibility to predict to some extent the expected statistical properties such as turbulence intensity and higher moments of the flow on the computer and thus to develop user-defined wind fields.

[1] Neuhaus et al., Experiments in Fluids 62(6), 1-12 (2021).

[2] Neuhaus et al., Physical Review Letters 125(15), 154503 (2020).

Flow, motion and power dynamics of floating wind farms

Raúl Bayoán Cal

Monday, July 29, 11:30–12:00, Research III Lecture Hall

Abstract: Wind and water tunnel experiments of turbulent wakes in a scaled floating wind farm are performed. Scaling of a floating wind farm with a scaling ratio of 1:400 is made possible by relaxing geometric scaling of the turbine platform system to correctly match dynamic response but relax Froude scaling such that the Reynolds number can be kept large enough. The response and performance of a single turbine scaled model are characterized for different wind and wave conditions. Subsequently, a wind farm experiment is performed with twelve floating turbine models, organized in four rows and three columns.

Coherent structures and transition to turbulence in the wake of a moving wind turbine

Thomas Messmer (Carl von Ossietzky Universität Oldenburg), **Michael Hölling** (Carl von Ossietzky Universität Oldenburg), **Joachim Peinke** (Carl von Ossietzky Universität Oldenburg)

Monday, July 29, 12:00–12:30, Research III Lecture Hall

Abstract: Floating offshore wind turbines (FOWT) are playing an increasingly important role in future offshore projects. These turbines, mounted on floating substructures, are free to move in response to turbulent wind and ocean waves. Their movements cover different frequency and amplitude regimes, and their impact on aerodynamics and the wake they generate remains a rather open question.

Motivated by this new topic, we have designed a set-up consisting of a model wind turbine (with a diameter $D=0.6$ m) mounted on a motorised platform that we are using to mimic the movement of a FOWT in a wind tunnel. We focus on the development of the wake under platform motion for idealised cases, i.e. low turbulent wind and harmonic platform motion.

Our results show that the wake is significantly impacted by platform motions, even at a low amplitude of motion of $A=0.01D$. A relevant non-dimensional number is the platform Strouhal number, $St = f_p D / U$ (f_p the frequency of the motions and U the mean wind speed). From our wake measurements carried out with a set of 19 hot-wires aligned horizontally at hub height, recording the streamwise wind speed for x in $[2D, 10D]$ we saw that wake recovery is enhanced by platform motion for St in $[0.2, 1.0]$. Both sideways and for-aft motions enable a higher wake recovery and faster transition to far-wake (Messmer et al. 2024).

The analysis outlined that the wake of the moving turbine features large-scale coherent structures of meandering (sideways) and pulsing (similar to vortex rings) that are forming through non-linear dynamics

driven by the motions. These structures contain much more energy than their drivers (the platform motion) and enable a shortcut to the far-wake. In the conference, we will detail the development of these structures and their role in the transition to the far-wake.

Intermittency and dissipation in turbulent wakes

Martin Obligado (Centrale Lille Institut), Felix Schmitt (Oldenburg University), André Fuchs (Oldenburg University), Konstantinos Steiros (Imperial College London), John Christos Vassilicos (Centrale Lille Institut), Joachim Peinke (Oldenburg University)

Monday, July 29, 12:30–13:00, Research III Lecture Hall

Abstract: Recently, several advances have been achieved in terms of describing the energy cascade within a turbulent wake. Nevertheless, little attention has been paid to how large- and small-scale intermittency are affected by the geometry of the wake generator nor in terms of its spatial evolution. We present a series of experiments, performed in different wind tunnels, where the turbulent wake generated by different bodies is characterised up to 50 diameters downstream of them. Hot-wire anemometry allowed us to access both the turbulent dissipation and to characterise the intermittency of the flow. While some features of the former can be described using the dissipation constant, the intermittency will be studied using two different quantities:

- The intermittency constant, first introduced by Kolmogorov's 1962 theory, and later redefined (Castaing et al., *Physica D* 46 (1990)). Globally, it quantifies departures from self-similarity within the inertial range.
- The Rice constant, defined as the ratio between the Taylor length scale and the average distance between the zero crossings of the streamwise fluctuating velocity. This quantity provides a scalar quantification of the departures of the velocity time derivatives probability density function from a Gaussian distribution.

Our results include a set of axisymmetric bluff bodies, with both fractal and regular peripheries. Furthermore, different circular cylinders were also tested, allowing us to cover both planar and axisymmetric turbulent wakes.

Our large experimental dataset allows to understand the development of intermittency in turbulent wakes in two different ways. First, we will show how the degree of small-scale intermittency is directly controlled by large scale structures. Moreover, a universal relation between the dissipation and the intermittency constants can be found, as one evolves as the inverse of the other.

Short- & long-term forecast of extreme events in atmospheric turbulence

André Fuchs (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), Finn Köhne (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), Jan Friedrich (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), Matthias Wächter (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), Kerstin Avila (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), **Joachim Peinke** (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research)

Monday, July 29, 13:30–14:00, Research III Lecture Hall

Abstract: This contribution introduces a new approach based on precursory dynamics for statistically accurate short-term & long-term forecasts of the occurrence probability of extreme events in atmospheric turbulence. We will demonstrate that the introduced stochastic approach is suitable for investigating many dynamical effects of the wind energy conversion process and wind turbine operations. In addition, this method might pave the way towards wind turbine design standards embedding a better representation of extreme events.

Inspired by a Markov process in the scale domain, our approach is based on joint multipoint statistics incorporating a stochastic scale process governed by a family of Fokker–Planck equations. These equations are derived self-consistently from given historical data measured at the onshore Hamburg weather mast, using a 3D ultrasonic anemometer with a sampling frequency of 20Hz ¹.

We will demonstrate that the derived description of the underlying stochastic process retains the statistical characteristics of the complex dynamics of one-dimensional atmospheric wind speed fluctuation along different scales. This is even true on short timescales of seconds to minutes, which are within the range of Kolmogorov-like turbulence and which are the typical response times of wind turbines and their control systems. The excellent agreement between the generated stochastically equivalent surrogate statistics and measured fluctuations confirms that the multipoint statistics approach effectively captures the short-term dynamics in general and extreme events such as gusts in particular, in a statistical sense.

By incorporating the information derived from the underlying stochastic process of the complex dynamics of atmospheric turbulence into the large deviations theory and nonequilibrium thermodynamics, it enables access to a mathematically rigorous, generalized long-term forecast theory for the occurrence of very rare wind gusts that were not measured during the measurement period. Additionally, this approach enables a detailed analysis of the measured data in terms of thermodynamic quantities, such as the identification of entropy for each wind fluctuation and the assessment of the validity of fluctuation theorems known from stochastic thermodynamics.

The proposed methodology could, therefore, provide a stochastic alternative to the deterministic extreme operating gust description used in wind turbine design standards².

Extracting similarity from data

Nikolaos Bempedelis (Queen Mary University of London), Luca Magri (Imperial College London), **Konstantinos Steiros** (Imperial College London)

Monday, July 29, 14:00–14:30, Research III Lecture Hall

Abstract: The identification of self-similarity is an indispensable tool for understanding and modelling problems in non-linear physics. Unfortunately, this is not always possible to perform formally in highly complex problems such as those that appear in turbulence and wind energy applications. We propose a methodology to extract the similarity variables of a self-similar physical process directly from data, without prior knowledge of the governing equations or boundary conditions, based on an optimization problem and symbolic regression. We explore the accuracy and robustness of our method in four problems which have been influential in fluid mechanics research: a laminar boundary layer, Burger’s equation, a turbulent wake (using measurements from a wind tunnel experiment), and a collapsing cavity (as predicted by numerical simulations).

Systematic construction of velocity gradient models for turbulence

Maurizio Carbone (University of Bayreuth), Vincent Peterhans (University of Göttingen), Alexander Ecker (University of Göttingen), **Michael Wilczek** (University of Bayreuth)

Monday, July 29, 14:30–15:00, Research III Lecture Hall

Abstract: Velocity gradients give insights into many aspects of the small scales of turbulence. Low-dimensional models for the velocity gradients aim at capturing the statistics and dynamics along Lagrangian particle trajectories, which requires formulating closures for nonlocal pressure contributions and viscous effects [1,2]. Here, we construct an evolution equation for the single-particle PDF for the velocity gradient, whose solution yields a given velocity gradient PDF by design [3]. In this data-driven approach, we use the normalizing flow machine learning approach [4] to first estimate the full eight-dimensional velocity gradient PDF from direct numerical simulation (DNS) data. Based on this, we then construct the drift term for the PDF equation using a neural network. The drift term, which contains contributions from the pressure, the viscous diffusion, and the large-scale forcing, can be gauged to achieve realistic Lagrangian

¹Lange, I., Meteorological Institute of the University of Hamburg, wind data from the weather mast for the period 2010 to 2023

²International Electrotechnical Commission, International standard IEC 61400-1. Wind turbines–Part, 1, 22-23, (2005)

time correlations. The PDF equation corresponds to a deterministic model for velocity gradients along Lagrangian trajectories, which we use to generate time series and gather statistics. We characterize the model performance by comparing several statistics, including the PDF of the principal invariants of the velocity gradient, dissipation and enstrophy PDFs, and strain and vorticity correlations.

- [1] Meneveau, *Annu. Rev. Fluid Mech.* 43, 219 (2011)
- [2] Johnson & Wilczek, *Annu. Rev. Fluid Mech.* 56, 463 (2024)
- [3] Carbone, Peterhans, Ecker, Wilczek, arXiv:2402.19158 (2024)
- [4] Tabak & Vanden-Eijnden, *Commun. Math. Sci.* 8, 217 (2010)

Stochastic analysis of jump noise along with Langevin noise in real-world data sets

Matthias Wächter (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics), Pyei Phyo Lin (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics), M. Reza Rahimi Tabar (Sharif University of Technology), Joachim Peinke (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics)
Monday, July 29, 15:00–15:30, Research III Lecture Hall

Abstract: Analysis and modeling of real-world systems by Langevin-type stochastic differential equations (SDE) has proven to be powerful for many applications. However, many systems are driven not only by a strictly normal-distributed, delta-correlated Langevin noise, but additionally show deviating noise preproperties. In many cases those systems can be better described by including a jump noise term in the SDE, accounting for non-Gaussian noise characteristics. We demonstrate the methodology at field data examples of snow hardness [1] and wind energy systems [2] where the inclusion of jump noise leads to improved results and additional insights.

- [1] Lin PP, Peinke J, Hagemuller P, Wächter M, Tabar MRR, Peinke J, *The Cryosphere* 16, 2022
- [2] Lin PP, Wächter M, Tabar MRR, Peinke J, *PRX Energy* 2, 2023

Curious observations in the wake interactions of dissimilar porous disks

Ingrid Neunaber (Norwegian University of Science and Technology), R. Jason Hearst (Norwegian University of Science and Technology)
Tuesday, July 30, 11:00–11:30, Research III Lecture Hall

Abstract: The wake interaction of transversely spaced wake-generating objects, such as wind turbines (e.g., Maus et al. (2022)), solid discs (Obligado et al. (2022)), or porous discs, is a research topic of interest both for the turbulence and the wind energy community.

In this study, we continue the analysis of wake interaction with hot-wire experiments in the low-speed wind tunnel at NTNU. Two different types of porous discs were used, namely a disc with non-uniform blockage (cf. Vinnes et al. (2022, 2023); inspired by Camp et al. (2016)), referred to as type A, and a mesh disc with uniform blockage (cf., Aubrun et al. (2013)), referred to as type B. We investigated the interaction of wakes generated by two discs of type A (case AA), two discs of type B (case BB), and a mixed case (case AB). Both discs have a diameter of $D=200$ mm, and three different spacings, measured from center-to-center, were investigated, $1.5D$, $2D$, and $3D$. The inflow velocity was 10ms^{-1} , and the turbulence intensity is below 0.3%. Both span-wise profiles at $8D$ and $30D$ downstream, and the centerline streamwise evolution between $0.5D$ and $40D$ downstream were measured using a single wire with a sampling frequency of 75 kHz that had a hardware low-pass filter with a cut-off frequency of 30kHz. The sampling duration was 180 s.

Our investigations show several interesting results: 1. As the wakes generated by type A discs expand faster than those of type B, they also meet farther upstream, and the mixing generates higher turbulence levels for the AA case than for the BB case at the centerline. In the AB case, turbulence production is enhanced compared to both the AA and the BB case, and the location of maximum turbulence intensity is biased towards the uniform disc. 2. From previous studies, it is known that single discs of type A exhibit vortex shedding, while type B discs do not. Consistently, a shedding peak is seen in the AA case but not in the BB case. Curiously, in the AB case, in the span-wise profiles at $8D$, we identify a peak typically associated with shedding both downstream of disc A and, even stronger, downstream of disc B. The oscillation present

in the wake of disc A therefore induces an oscillation of the wake of disc B. 3. When comparing the centerline streamwise evolution of turbulence for the two-disc cases to that of the respective single discs, we find that the maximum centerline turbulence intensity is reduced in the two-disc cases. It is also interesting to remark that the shape parameter, that describes the internal intermittency of turbulence, indicates a more intense turbulence interaction of the shear layers in the two-disc cases.

In conclusion, the investigation of the interaction of wakes generated by different porous discs shows some interesting features that will need further exploration.

Wind fields from atmospheric turbulence measurements

Jan Friedrich, Daniela Moreno, André Fuchs, Matthias Wächter, Carsten Schubert, Joachim Peinke
Tuesday, July 30, 11:30–12:00, Research III Lecture Hall

Abstract: Wind field models play a pivotal role in wind energy science, e.g., during the design process of individual turbines and related estimations of loads on rotors and blades. For a better understanding of the interactions between inflow turbulence and turbine, it is desirable to obtain site-specific and highly-resolved wind fields from rotor diameter down to a few centimeters. However, current Gaussian wind field models do not account for the empirically observed occurrence of extreme small-scale wind fluctuations referred to as intermittency. Here, we present a generalized stochastic wind field model that emulates the effects of higher-order statistics and reproduces observed intermittency properties with a high level of accuracy and at considerably low computational cost. Our method is based on a multipoint statistical description of turbulent velocity fields that consists of a superposition of multivariate Gaussian statistics with fluctuating covariances and can thus be considered a generalization of the commonly used Mann or Kaimal wind field models. Furthermore, we explicitly reconstruct these "superstatistical" wind fields based on real-world measurement data from a meteorological mast array or LIDAR measurements providing an exact stochastic interpolation of the data set. We present the first results on the relevance of such superstatistical wind fields in the context of fatigue loads on wind turbines by aeroelastic simulation using a superstatistical extension of the Kaimal model.

Mixing, boundary layers, and interfaces: what does turbulence do to them?

R. Jason Hearst (Norwegian University of Science and Technology (NTNU))
Tuesday, July 30, 12:00–12:30, Research III Lecture Hall

Abstract: Understanding how mixing occurs and is driven in wall-bounded flows is of significant importance both for our fundamental understanding of the turbulent motion of fluids, but also for numerous real-world applications. An example is the mixing processes present in the atmospheric boundary layers that interact with wind turbines. An experimental study was designed to assess how freestream turbulence above a boundary layer impacts the mixing of a passive scalar into the boundary layer. In a water channel, freestream turbulence was generated with an active turbulence grid at the inlet. Four different homogeneous (in transverse planes) turbulence conditions were generated and measurements of the interaction between the freestream turbulence and the turbulent boundary layer evolving on the facility floor were made at two downstream stations. The measurements consisted of simultaneous particle image velocimetry in the streamwise-wall-normal plane and planar laser-induced fluorescence where Rhodamine-6G was used as a passive scalar. The mean statistics demonstrate that an increase in the scale (intensity and length scale) of the incoming turbulence promotes more rapid mixing of the scalar towards the wall. Analysis of the instantaneous velocity and scalar fields was then undertaken to identify the mechanisms responsible for this transport. Instantaneous isocontours of the momentum and vorticity were compared to the local scalar concentration to identify the connection between these three quantities. It was found that the interfaces of momentum and vorticity in these turbulent-turbulent flows were drastically different from each other, which contrasts with similar experiments performed for the turbulent-non-turbulent-interface (TNTI) separating irrotational fluid from rotational fluid. In the TNTI case, these isocontours instantaneously lie in roughly the same area although phenomenologically the interface is typically associated with vorticity or enstrophy. However, in the case where turbulence is present on both sides of the interface, the lines are independent. In

particular, the vorticity contour cuts through uniform momentum structures in the wall-bounded region and conditional averages of the scalar concentration on the location of the momentum and vorticity interfaces show no preference for the scalar concentration to lie on either of these interfaces, i.e., the scalar just passes through them. This is attributed to differences in the transport equations for these three quantities. For the TNTI case, the transport equation for a scalar and vorticity are essentially surrogates, however, when turbulence is present on both sides of the interface the vortex stretching term in the vorticity transport equation becomes significant but is not present in the scalar equation. In addition, there is significant diffusion of the scalar everywhere in the flow when turbulence is present everywhere, and the facilitation of this process by the turbulent fluctuations is the significant driver of the scalar transport. Thus, there does not appear to be a singular interface that describes all transport processes in these turbulent-turbulent systems, but instead there are multiple interfaces that all have their own significance.

Energy exchanges taking place between coherent modes in the near wake of a wind turbine

Oliver Buxton

Tuesday, July 30, 12:30–13:00, Research III Lecture Hall

Abstract: In this work we investigate the spatio-temporal nature of various coherent modes present in a rotor wake using a combination of new time-resolved particle image velocimetry (PIV) experiments and existing data from Biswas and Buxton (2024). A multi-scale triple decomposition is applied in which the velocity field is decomposed into the mean, a sum of coherent (periodic) modes, and the residual/stochastic fluctuation that is representative of the incoherent/non-periodic turbulence. Various coherent modes are identified to exist within the near-wake region. For example, in the tip vortex system modes with characteristic frequencies corresponding to the blade passing frequency, the turbine's rotational frequency and their harmonics are identified. In addition to this lower frequency modes are also identified corresponding to the vortex shedding from the nacelle and the tower, and wake meandering. Thereafter, the energy exchanges between the various coherent modes are studied using the the multi-scale triple-decomposed coherent kinetic energy budgets developed by Baj and Buxton (2017). The different constituent frequencies forming the tip vortex system are found to be energised by different source terms, such as production from the mean flow or non-linear triadic interaction or both, similar to the primary, secondary or the mixed modes discussed in Biswas et al. (2022). In fact, the tip vortex system forms a complex network of non-linear triadic energy transfers, the nature and the magnitudes of which depend on the tip speed ratio (TSR). On the other hand, the modes associated with the vortex sheddings from the nacelle or tower and the wake meandering are found to be primarily energised by the mean flow. We show that the tip vortex system exchanges energy with the mean flow primarily through the turbine's rotational frequency. In fact, the system transfers energy back to the mean flow through the turbine's rotational frequency at some distance downstream marking the onset location of wake recovery. This wake recovery onset location is shown to reduce with TSR due to stronger interaction and earlier merging of the tip vortices at a higher TSR.

Minisymposium on Vegetation pattern formation

The roles of front instabilities in desertification.

Michel Ferré (Ben Gurion University of the Negev)

Wednesday, July 31, 13:30–14:00, IRC Seminar Room 3

Abstract: Desertification fronts are ecological processes involving biodiversity loss and ecosystem degradation. They emerge due to changes in environmental conditions, affecting the ecosystem's shaping mechanisms hierarchy and implying a reorganization of vegetation and resources. Recent studies suggested that transverse front instabilities can reverse ecosystem degradation by growing fingers back into degraded areas; nonetheless, the mechanism responsible is still unclear. To bring some light into this issue, we study a two-component reaction-diffusion equation that accounts for the vegetation biomass and soil-water content dynamics in a drylands context. Based on a singular perturbation approach, we uncover curvature-velocity relations, which help us distinguish between different front dynamical regimes, desertification fronts, recovery fronts, and finger instabilities.

A data driven approach to detect vegetation patterns - from individuals to plant group behaviour using citizen science data

Karin Mora (Leipzig University), Michael Rzanny (Max Planck Institute for Biogeochemistry), Jana Wäldchen (Max Planck Institute for Biogeochemistry), Hannes Feilhauer (Leipzig University), Teja Kattenborn (University of Freiburg), Guido Kraemer (Leipzig University), Patrick Mäder (Technische Universität Ilmenau), Daria Svidzinska (Leipzig University), Sophie Wolf (Leipzig University), Miguel D. Mahecha (Leipzig University)

Wednesday, July 31, 14:00–14:30, IRC Seminar Room 3

Abstract: Understanding the impact of climate change on ecosystems necessitates monitoring phenological changes across numerous plant species. While mobile applications featuring automated species identification provide valuable crowd-sourced spatio-temporal data, existing studies often focus on limited individual species. Here, we present an innovative spatio-temporal machine learning methodology to quantify group dynamics of plants in space and time, particularly focusing on vegetation dynamics and pattern formation.

Our approach utilises synchronised plant occurrences as indicators of observable phenological states. By linking individual phenological responses across thousands of species and geographical locations using a similarity measure, we analyse nearly ten million plant observations collected via the AI-based plant identification app Flora Incognita in Germany from 2018 to 2021.

Our findings reveal that the synchronised behaviour of thousands of species can be encapsulated by a few characteristic macrophenological patterns. Spatio-temporal changes are quantified using a data compressibility measure, highlighting nonlinear and seasonal variations throughout the annual cycle. We observe a reduction in synchronisation and macrophenological patterns outside the growing season. Furthermore, we investigate the relationship between climate drivers and phenology, along with the entanglement between phenological stages and human data collection behaviour.

Despite biases and uncertainties inherent in crowd-sourced data, our study demonstrates the feasibility of deriving meaningful indicators for monitoring plant macrophenology across large geographical scales while retaining a species-specific approach. As crowd-sourced databases continue to expand, our approach offers promise in studying climate-induced phenological shifts and feedback loops.

Mathematical modeling of below-ground plant interactions: from competition to facilitation.

Ricardo Martinez-Garcia (Helmholtz Zentrum Dresden Rossendorf e.V.)

Wednesday, July 31, 14:30–15:00, IRC Seminar Room 3

Abstract: Below-ground plant interactions are a key driver of vegetation pattern formation. However, many existing models rely on functional forms for these interactions that frequently lack empirical support. This gap between models and data stems primarily from our limited understanding of below-ground plant growth processes. Unlike aboveground shoot competition, the study of below-ground plant growth is hampered by our inability to observe roots. We have few observations of intact root systems in soil and lack a comprehensive theory for root system responses to their environment.

In this presentation, I will first review previous theoretical efforts to explain plant below-ground competition and discuss how they lead to seemingly contradictory predictions. Then, I will introduce our recent theoretical and experimental work and show how it resolves existing controversy and provides a unifying framework to study below-ground plant interactions, both competitive and facilitative. I will conclude by discussing future research lines that depart from our results, including extensions to larger spatial scales and applying this new modeling approach to vegetation patterns.

Traveling vegetation-herbivore waves help to sustain ecosystems threatened by droughts and population growth

Joydeep Singha (Ben Gurion University of the Negev), Hannes Uecker (Carl von Ossietzky University Oldenburg), Ehud Meron (Ben-Gurion University of the Negev)

Wednesday, July 31, 15:00–15:30, IRC Seminar Room 3

Abstract: Dryland vegetation exhibits spatial patterns as an adaptation to water stress. These patterns likely

arise as a response to the non-uniform distribution of water and other resources in these environments. While vegetation patterns help plants survive water stress, herbivore grazing poses an additional threat. Grazing can exacerbate the risk of desertification by damaging the vegetation cover, leading to soil erosion and loss of habitat.

We introduce a novel model that integrates both vegetation patterning and herbivore grazing dynamics with the aim of capturing the feedback loops between these two factors and how they influence each other over time. The model incorporates the behavior of herbivores, including their exploitation strategy, which involves factors such as foraging behavior, movement patterns, and preferences for certain types of vegetation. Using numerical continuation methods, we analyze the different types of solutions that emerge from the model, including uniform and patterned vegetation-herbivore dynamics.

One significant finding is the emergence of traveling waves in the vegetation-herbivore dynamics. These waves represent a dynamic pattern where both vegetation and herbivores propagate across the landscape over time. Within these traveling waves, the distribution of herbivores is asymmetric, with higher density on one side of each vegetation patch. This asymmetry results in varying levels of grazing stress across the landscape. Surprisingly, the formation of traveling waves leads to increased sustainability in the face of grazing stress. This is because the uneven distribution of herbivores results in a more balanced impact on vegetation, compared to uniform grazing patterns.

We conclude that understanding and harnessing these vegetation-herbivore dynamics is crucial for securing food production in dryland regions. By maintaining herbivore populations and preserving vegetation cover, these dynamics contribute to food security in areas threatened by droughts and population growth.

Topological defects law for migrating banded vegetation patterns in arid climates

David Pinto-Ramos, Marcel Clerc, Mustapha Tlidi

Wednesday, July 31, 16:00–16:30, IRC Seminar Room 3

Abstract: Self-organization and pattern formation are ubiquitous processes in nature. We study the properties of migrating banded vegetation patterns in arid landscapes, usually presenting dislocation topological defects. Vegetation patterns with dislocations are investigated in three different ecosystems. We show through remote sensing data analysis and theoretical modeling that the number of dislocations $N(x)$ decreases in space according to the law $N \sim \log(x/B)/x$, where x is the coordinate in the opposite direction to the water flow and B is a suitable constant. A sloped topography explains the origin of banded vegetation patterns with permanent dislocations. Theoretically, we considered well-established approaches to describe vegetation patterns. All the models support the law. This contrasts with the common belief that the dynamics of dislocations are transient. In addition, regimes with a constant distribution of defects in space are predicted. We analyze the different regimes depending on the aridity level and water flow speed. The reported decay law of defects can warn of imminent ecosystem collapse.

Spatiotemporal complexity in seagrasses

Damià Gomila (IFISC (CSIC-UIB)), Pablo Moreno Spiegelberg (IFISC (CSIC-UIB)), Max Rietkerk (Utrecht University)

Wednesday, July 31, 16:30–17:00, IRC Seminar Room 3

Abstract: Spatial self-organization is considered a resilience mechanism through which ecosystems optimize positive versus negative feedbacks, allowing them to persist in adverse situations. This phenomenon has often been studied in systems presenting stationary patterns or patterns moving along slopes due to gradients, but much less is known in the case of systems presenting persistent self-sustained oscillations or excitable dynamics, as observed, for example, in *Posidonia oceanica* meadows.

In this talk we discuss how self-organization in systems displaying complex temporal dynamics can increase ecosystems resilience. We consider a simple model that includes negative feedback mediated by an inhibitor and direct positive feedback. A similar model that has been applied to *Posidonia oceanica* meadows, where positive and negative feedbacks are well documented, and there is empirical evidence of the role of sulfide accumulation, toxic for the plant, in driving complex spatiotemporal dynamics.

We describe the formation of turbulent regimes, coherent periodic structures such as wave trains, target patterns and spirals, and traveling pulses or expanding rings. Some of these solutions can coexist for different ranges of parameters, but a general progressive transition from turbulence to traveling pulses, passing through coherent periodic states, is observed increasing the mortality rate. The formation of spatiotemporal structures allows vegetation to survive beyond the tipping point of a homogeneous vegetation cover, increasing the resilience of the system.

The model used in this paper is general and the results can be applied to other plant-soil spatial extended systems, regardless of the mechanism behind the negative and positive feedback.

Delayed loss of stability of periodic travelling waves affects wavelength changes in a model for vegetation patterns

Lukas Eigentler (University of Warwick, Mathematics Institute)

Wednesday, July 31, 17:00–17:30, IRC Seminar Room 3

Abstract: Many patterned ecosystems, such as dryland vegetation patterns and intertidal mussel beds can be described by PDEs admitting periodic travelling waves (PTWs). Under a changing environment that increases stress, such systems undergo a cascade of wavelength changes before an extinction event occurs. Classically, wavelength changes have been predicted by identifying the intersection of a PTW's wavelength contour with a stability boundary in the system's Busse balloon. In this talk, I highlight that this information is often insufficient because of a delayed loss of stability phenomenon. I show that PTWs can persist as transients for ecologically significant times after the crossing of a stability boundary in the Busse balloon. I present a method that can predict the order of magnitude of the time delay between the crossing of a stability boundary and the occurrence of a wavelength change by linking the delay to features of the essential spectra of the PTWs.

Self-organised pattern formation in network-organised metacommunities supports autotroph functional diversity

Christian Guill (University of Potsdam), **Louica Philipp** (University of Potsdam), **Toni Klauschie** (University of Potsdam)

Wednesday, July 31, 17:30–18:00, IRC Seminar Room 3

Abstract: Many natural landscapes host so-called metacommunities, where multiple ecological communities on distinct habitat patches are coupled by dispersal. This connects the patches to often large, irregular habitat networks. Turing instabilities leading to self-organised pattern formation are also possible in such network-organised systems, and the mathematical methods for their treatment are well developed. We use a small ecosystem model including explicit dynamics of a limiting nutrient, a potentially diverse autotroph community (plants or algae) and a single heterotrophic consumer species to study how the spatial and temporal habitat heterogeneity created by pattern formation affects the diversity of the autotrophs. The latter is described by a continuous distribution of a functional trait that affects both autotroph growth and their defence against the heterotroph. On isolated patches or in absence of pattern formation diversity is always lost over time due to stabilising selection. We derive conditions for a wave instability that leads to the emergence of complex spatio-temporal patterns with large, asynchronous oscillations in the species' abundances. Using numerical simulations, we illustrate how these patterns enhance local functional diversity through a spatial eco-evolutionary feedback loop involving a combination of strengthened source-sink dynamics and weakened stabilising local selection. We further show that anthropogenic habitat change that decreases network connectance by disrupting dispersal pathways leads to patterns with smaller amplitude, which weakens the feedback loop. This substantially reduces local functional diversity of the autotroph communities and diminishes their capability to adapt to changing conditions.

Travelling pulses on three spatial scales in a Klausmeier-type vegetation-autotoxicity model

Paul Carter (University of California, Irvine), **Arjen Doelman** (Leiden University), **Annalisa Iuorio** (University of Naples "Parthenope"), **Frits Veerman** (Leiden University)

Wednesday, July 31, 18:00–18:30, IRC Seminar Room 3

Abstract: Reaction-diffusion models describing interactions between vegetation and water reveal the emergence of several types of patterns and travelling wave solutions corresponding to structures observed in real-life. Increasing their accuracy by also considering the ecological factor known as autotoxicity has led to more involved models supporting the existence of complex dynamic patterns. In this work, we include an additional carrying capacity for the biomass in a Klausmeier-type vegetation-water-autotoxicity model, which induces the presence of two asymptotically small parameters: ε , representing the usual scale separation in vegetation-water models, and δ , directly linked to autotoxicity. We construct three separate types of homoclinic travelling pulse solutions based on two different scaling regimes involving ε and δ , with and without a so-called superslow plateau. The relative ordering of the small parameters significantly influences the phase space geometry underlying the construction of the pulse solutions. We complement the analysis by numerical continuation of the constructed pulse solutions, and demonstrate their existence (and stability) by direct numerical simulation of the full PDE model. [arXiv:2312.12277]

Contributed Sessions

Session on Data-driven dynamical systems

Learning effective coarse PDEs from agent-based simulations of crowd dynamics

Hector Vargas Alvarez (Scuola Superiore Meridionale, Naples, Italy), **Dimitrios Patsatzis** (Scuola Superiore Meridionale, Naples, Italy), **Constantinos Siettos** (University of Naples "Federico II")

Wednesday, July 31, 13:30–14:00, SCC Conference Hall

Abstract: In this work, we address a data-driven approach based on Machine Learning (PIML), in particular based on Feed Forward Networks (FNN) and Convolutional Neural Networks (CNN) to learn the effective coarse Partial Differential Equations (PDEs) [1,2,3] that models the emergent-collective behavior exhibited by the continuous flow of a crowd which moves in a corridor avoiding an obstacle. At the fine micro-scale level, we exploit the so-called social force model [4] to generate synthetic data. Subsequently, we exploit manifold learning to discover the appropriate macroscopic variables that describe best the emergent behavior and from these we construct the effective PDEs using machine learning.

- [1] Zichao Long, Yiping Lu, Bin Dong, PDE-Net 2.0: Learning PDEs from data with a numeric-symbolic hybrid deep network, *Journal of Computational Physics*, Volume 399, (2019).
- [2] Lee, S., Psarellis, Y.M., Siettos, C.I. et al. Learning black- and gray-box chemotactic PDEs/closures from agent based Monte Carlo simulation data. *J. Math. Biol.* 87, 15 (2023).
- [3] Galaris, E., Fabiani, G., Gallos, I. et al. Numerical Bifurcation Analysis of PDEs From Lattice Boltzmann Model Simulations: a Parsimonious Machine Learning Approach. *J Sci Comput* 92, 34 (2022).
- [4] Helbing, Dirk and Molnar, Peter Social force model for pedestrian dynamics, *Phys. Rev. E*, 4282-4286 (1995).

Complex and 3-dimensional RNA random walks: comparison and application to sequence data across biological taxa

Jack Mortimer (University of Birmingham, UK), **Jens Christian Claussen** (University of Birmingham, UK)

Wednesday, July 31, 14:00–14:30, SCC Conference Hall

Abstract: The DNA random walk is a classical attempt to grasp long-range features of DNA (or RNA) sequences by mapping pairs of amino acids to ± 1 steps of a random walk, and interpret the resulting “time series” by scaling analysis [1]. But as four letters C,G,A,T comprise the DNA alphabet it is a straightforward idea to utilize complex numbers to exploit this information (rather than ignoring it). This direction has been investigated also elsewhere [2] but different definitions were used, and it is not yet conclusive how far biological data can be differentiated.

In this contribution, we attempt a comparison of different complex RW definitions together with a 3D RW, discuss their relations between each other, and apply them to a wide range of DNA sequences. While the various DNA RW’s seem not to be directly discriminatory for each species, we find that they provide a

wide spread across the datasets. In conclusion, complex and higher-dimensional DNA random walks are a promising tool to extract long-range features from DNA, although the biological interpretation of this method remains to be investigated.

- [1] Peng, Buldyrev, Goldberger et al., Nature 356,168 (1992)
- [2] Cattani, in: Bioinf Res Dev, Springer, p. 528 (2008)

Stochastic parameterisation: the importance of nonlocality and memory

Martin Brolly (University of Edinburgh)

Wednesday, July 31, 14:30–15:00, SCC Conference Hall

Abstract: Simulations of fluid turbulence, such as in the atmosphere or ocean, rarely resolve the dynamics of all relevant scales. Typically the relevant partial differential equations are solved numerically at finite resolution. Since the effect of the unresolved small-scale dynamics on the resolved dynamics is often important, simulations must account for this through so-called parameterisations, or closures. A variety of types of parameterisations have been proposed and applied in various fields, some informed by physical intuition, others by statistical assumptions and data. In this talk we discuss stochastic parameterisation and the need to go beyond common assumptions of Markovianity and locality in space. We demonstrate these issues in experiments with the Lorenz models and two-dimensional turbulence. Our parameterisations are constructed using probabilistic machine learning models, trained using data from limited higher resolution simulations.

The relative prevalence of wave-packets and coherent structures in the inertial and kinetic ranges of turbulence as seen by Solar Orbiter

Alina Bendt (University of Warwick, UK), Sandra Chapman (University of Warwick, UK), Thierry Dudok de Wit (International Space Science Institute (ISSI))

Wednesday, July 31, 15:00–15:30, SCC Conference Hall

Abstract: The Solar Orbiter (SO) mission provides a unique opportunity to study the evolution of turbulence in the solar wind across different distances from the sun and different plasma conditions. We use SO observations of extended intervals of homogeneous solar wind turbulence to investigate under what conditions the turbulent cascade in the solar wind is supported by either, or both of two distinct phenomenologies, (i) wave-wave interactions and (ii) coherent structure formation and interaction.

We identify nine Solar Orbiter observations of extended intervals of homogeneous solar wind turbulence where each interval is over 10 hours long without current-sheet crossings and other large events. We perform a systematic scale-by-scale decomposition of the observed magnetic field using two wavelets that are known to discriminate between wave-packets and discontinuities, the Daubechies 10 (Db10) and Haar respectively.

A characteristic of turbulence is that the probability distributions (pdfs) of fluctuations obtained on small scales exhibit extended supra-Gaussian tails, and as the scale is increased, the moments decrease and there is ultimately a cross-over to Gaussian pdfs at the outer scale of the turbulence. Using quantile quantile plots, we directly compare the fluctuations pdfs obtained from Haar and Db10 decompositions. This reveals three distinct regimes of behaviour. On larger scales, deep within the inertial range (IR), both the Haar and Db10 decompositions give essentially the same fluctuation pdfs. On the smallest scales, deep within the kinetic range (KR), the pdfs are distinct in that the Haar wavelet fluctuations have a much broader distribution, and the largest fluctuations are associated with coherent structures. On intermediate scales, that span the IR-KR scale break identified from the power spectra, the pdf is composed of two populations, a core with a common pdf functional form for the Haar and Db10 fluctuations, and extended tails where the Haar fluctuations dominate. This establishes a cross-over between wave-packet dominated phenomenology in the IR, to coherent structure dominated phenomenology in the KR. We find that the intermediate range of scales is quite narrow around 0.9 au so that the crossover from wave-packet to coherent structure dominated phenomenology is quite abrupt. At around 0.3au, the crossover occurs over a broader range of scales extending down to the 0.25s scale and up to 4s.

As coherent structures and wave-wave interactions have been proposed as candidates to mediate the turbulent cascade, these results offer new insights into the distinct physics of the IR and KR.

Data-driven modeling of crowd dynamics via Convolutional Neural Networks

Gianmaria Viola (Consiglio Nazionale delle Ricerche (CNR), Naples, Italy), **Alessandro Della Pia** (Scuola Superiore Meridionale, Naples, Italy), **Lucia Russo** (Consiglio Nazionale delle Ricerche (CNR), Naples, Italy), **Constantinos Siettos** (University of Naples "Federico II")

Wednesday, July 31, 16:00–16:30, SCC Conference Hall

Abstract: A data-driven black-box macroscopic model for crowd (pedestrian) dynamics is here identified through Convolutional Neural Networks. Training data are generated by numerical simulation of the Hughes model, a seminal first-order partial-differential-equation (PDE) model of crowd dynamics inspired by Navier-Stokes fluid-dynamic conservation laws, which is employed to investigate the pedestrian motion in a corridor in the presence of an obstacle. The independent variables of the model are the crowd density and the potential function, which depends on the domain topology, e.g. on the shape and position of the obstacle. The numerical data are employed to train a Convolutional Neural Network (CNN), which takes as inputs the two variables and gives the time-derivative of the density as output. The trained CNN model is therefore tested through numerical integration over time of the learned evolution operator, for new (unseen) initial conditions. Its forecasting performance is then evaluated by comparing the results against ground truth simulation data, which originate from the same initial condition as the predicted simulation and are generated using identical methods to those employed for training data production. The potential of the CNN-based PDE learning workflow for future applications is finally pointed out, discussing possibilities of its application to experimental measurements and high-order simulation data (i.e. by agent-based microscopic models) of crowd dynamics.

Impact of weak generalized synchronization and attractor bubbling on the optimal time series prediction of reservoir computing

Hiromichi Suetani (Faculty of Science and Technology, Oita University), **Ulrich Parlitz** (Max Planck Institute for Dynamics and Self-Organization)

Wednesday, July 31, 16:30–17:00, SCC Conference Hall

Abstract: In the fields of computational neuroscience and artificial neural networks, the concept of the so-called "edge of chaos" is often used as one of the neural criticality hypothesis [1]. For example, there is a study that supports the neural criticality hypothesis, as the highest performance was observed at the edge of chaos for a classification task using reservoir computing (RC) [2]. On the other hand, the concept of the edge of chaos has been also criticized and questioned by a number of researchers [3,4,5]. In general, neural networks that engage in information processing are non-autonomous dynamical systems because they typically receive inputs from external sources. In a drive (X) -response system (Y), if we calculate the Lyapunov exponent using only a trajectory and the corresponding Jacobian matrices of Y with ignoring the existence of X, it is not the Lyapunov exponent of the entire system (X-Y), but the conditional Lyapunov exponent that measures the exponential separation rate between two trajectories of Y starting from slightly different initial conditions driven by the same input starting from a fixed initial condition of X. As already pointed out in [6, 7], what is commonly referred to as the edge of chaos should be more accurately described as the "edge of conditional stability" when considering the influence of external inputs. In this study, we explore the relationship between the edge of conditional stability associated with the transition to generalized synchronization (GS) and the ability of RCs to process information. We examined this in the context of time series forecasting and show the existence of a gap between the parameter that gives the best forecasting performance and the edge of conditional stability. We further discuss the dynamical system causes of this gap, which we link to the bubbling transition in GS [8].

[1] M.M. Nuñez, "Criticality and dynamical scaling in living systems," *Reviews of Modern Physics* 90, 031001 (2018).

[2] R. Legenstein and W. Maass, "Edge of chaos and prediction of computational performance for neural circuit models," *Neural networks* 20, 323–334 (2007).

- [3] T. L. Carroll, "Do reservoir computers work best at the edge of chaos?" *Chaos: An Interdisciplinary Journal of Nonlinear Science* 30, 121109 (2020).
- [4] J. Wilting and V. Priesemann, "25 years of criticality in neuroscience established results, open controversies, novel concepts," *Current Opinion in Neurobiology* 58, 105–111 (2019).
- [5] C. Teuscher, "Revisiting the edge of chaos: Again?" *Biosystems* 218, 104693 (2022).
- [6] I. B. Yildiz, H. Jaeger, and S. J. Kiebel, "Re-visiting the echo state property," *Neural Networks* 35, 1–9 (2012).
- [7] T. Lyburn, A. Khor, T. Stemler, D. C. Corrêa, M. Small, and T. Jüngling, "Consistency in echo-state networks," *Chaos* 29 023118 (2019).
- [8] N. Takahashi and S. Tsugawa, "Role of unstable periodic orbits in bubbling weak generalized synchronization," *Physica D* 414, 132678 (2020).

Dynamic and stochastic modelling of cybersecurity caching systems

Pierce Ryan (University College Cork), Sorchá Healy (Microsoft), Andreas Amann (University College Cork)
 Wednesday, July 31, 17:00–17:30, SCC Conference Hall

Abstract: Internet-enabled information management systems make use of various caching mechanisms to efficiently share data between servers, where information is gathered and stored permanently, and endpoints, where data is used and stored temporarily after being requested from the servers. Time-To-Live (TTL) caching is often used in applications such as cybersecurity where the data on the server may change unpredictably. The TTL mechanism ensures that data cached on an endpoint cannot remain there indefinitely while the server data changes, minimising loss of consistency between servers and endpoints. While data usage and data change can both be seen as stochastic processes, data caching is purely deterministic, leading to interesting stochastic/dynamic interactions.

In this talk, we develop a model of server/endpoint interaction which enables us to study both stochastic and dynamic aspects of this system. For the simplest case of a constant rate of data usage, we derive an optimal choice of caching period. We consider how data sampling impacts our ability to estimate the rate of data usage. Finally, we demonstrate how the combination of non-constant rates of data usage and TTL caching gives rise to a circle map in the deterministic limit. This circle map features a rich pattern of periodic solutions whose structure can be analysed through a symbolic representation.

Stochastic gradient descent based Koopman operator approximation

Mohammad Tabish (Maxwell Institute for Mathematical Sciences, University of Edinburgh and Heriot-Watt University, Edinburgh), Stefan Klus (Heriot-Watt University, Edinburgh), Neil Chada (Heriot-Watt University, Edinburgh)

Wednesday, July 31, 17:30–18:00, SCC Conference Hall

Abstract: The Koopman operator plays a crucial role in understanding the global behavior of dynamical systems. We present novel stochastic optimization algorithms for learning the Koopman operator from time series data. The techniques are based on well-known gradient-based optimization algorithms. We apply these techniques to the cost function of the EDMD algorithm. The selection of a fixed set of the basis functions is required to apply EDMD to the given data. We also aim to extend the use of these algorithms to cases where we have a set of parametric basis functions. We present results on various numerical examples, such as the Ornstein-Uhlenbeck (OU) process, the triple-well potential, and protein-folding, to illustrate the performance of our proposed methods.

Functional connectivity inference for coupled dynamical systems based on the timing of distinct events

Reik Donner (Hochschule Magdeburg - Stendal), Yong Zou (East China Normal University)

Wednesday, July 31, 18:00–18:30, SCC Conference Hall

Abstract: Obtaining information on the functional connectivity of networked dynamical systems often suffers from various practical challenges, including but not being limited to the following examples: First, we may not have access to the actual dynamics of the relevant state variables but rather observe lower-dimensional projections, nonlinear transformations or simply noisy versions thereof. Second, the recorded

time series may not necessarily represent variables with optimal observability, but such that are less characteristic for the overall dynamics of the networked system under study. Third, in some cases, we may be faced with data that do not directly represent continuously varying quantities, but rather discrete events in time. Finally, inferring functional connectivity from multivariate time series or event sequences is commonly performed by resorting to pairwise unconditional similarity measures, which do not allow distinguishing “true” direct statistical linkages from apparent, yet actually indirect links originating from strong statistical similarity emerging due to the action of common drivers or additional mediating variables.

This talk presents an overview on recent developments and related open challenges associated with functional connectivity inference from the timing of observed events. We first discuss two widely used similarity measures suitable for this purpose, event synchronization strength and event coincidence rate, along with their similarities and differences, and present numerical as well as applied examples highlighting differences in their practical utility when considering events that are (partially) clustered in time. In order to reduce the number of false positive links inferred by simply thresholding the matrix of bivariate similarity measures, we revisit the partialization trick originating in the concept of partial correlations and previously being successfully applied to partial phase coherence/synchronization analysis. Applying this idea to defining partial event coincidence rates, we obtain a new framework contributing to better distinguishing direct from indirect connectivity in empirically derived functional networks. We show that for a minimalistic example of coupled Rössler oscillators, the corresponding connectivity inference based on threshold crossings in the rather erratic z component (with poor observability characteristics) works at least comparatively well as existing methods based on the better observable continuous dynamics of the x and y components.

Finally, our new partial event coincidence rates are applied to studying multi-channel EEG recordings to investigate possible differences in coordinated alpha band activity among macroscopic brain regions in resting states with eyes open (EO) and closed (EC) conditions. Our approach leads to a significant reduction in the number of indirect connections and thereby potentially contributes to a better understanding of the alpha band desynchronization phenomenon in the EO state.

On the weight dynamics of learning networks

Nahal Sharafi (Hamburg University for Applied Sciences), Christoph Martin, Sarah Hallerberg (Hamburg University for Applied Sciences)

Thursday, August 1, 13:30–14:00, SCC Conference Hall

Abstract: Neural networks are widely employed for tackling problems in machine learning and artificial intelligence. In this contribution we use the mathematical framework of local stability analysis to gain a deeper understanding of the learning dynamics of feed forward neural networks. We derive equations for the tangent operator of the learning dynamics of three-layer networks learning regression tasks. We then compute indicators of local stability, i.e. finite-time Lyapunov exponents and covariant Lyapunov vectors during the training process. We explain how the stability indicators relate to the final training-loss. We demonstrate how loss of transverse stability can lead to non-optimal training results. Moreover we demonstrate that it is possible to predict the final training loss, by monitoring finite-time Lyapunov exponents or covariant Lyapunov vectors during the training process.

Stochastic modelling of the provision of rotational energy by wind turbines

Martin Wagner (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), Christian Wiedemann (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), Matthias Wächter (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research), Joachim Peinke (Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, ForWind - Center for Wind Energy Research)

Thursday, August 1, 14:00–14:30, SCC Conference Hall

Abstract: Due to the turbulent wind inflow, the power output of wind turbines may change by hundreds of kilowatts within several seconds. This can have destabilizing effects on the connected power grid. One approach to mitigate this effect is to use the rotational energy of wind turbines as a short time storage. Typically, this rotational energy is controlled by changing the torque on the generator of a turbine. In our work, we model the assessment of the rotational energy by means of Langevin equations, which are estimated from temporally highly resolved numerical wind turbine data. We obtain a stochastic model of the frequency dynamics of a turbine by integrating the Langevin equations, with a given wind speed time series as input. The stochastically modelled time series is compared to equivalent numerical time series, where the control strategy of the generator torque is changed at the same points in time. We see that our stochastic approach nicely reproduces the dynamics of the rotational frequency that it is a suitable tool to model the provision of rotational energy. In comparison to the equivalent numerical model it has the advantage of being much simpler and computationally cheaper. The application of our Langevin model appears particularly promising if the provision of rotational energy by multiple turbines and the related synergy effects are to be investigated in the future.

Quantification of synchrony in globally coupled populations with the Wiener order parameter

Michael Rosenblum (University of Potsdam), **Arkady Pikovsky** (University of Potsdam)

Thursday, August 1, 14:30–15:00, SCC Conference Hall

Abstract: Coupling many oscillating systems may result in a synchronization transition, at which macroscopic collective motion appears. We concentrate on the quantification of synchrony in large globally coupled populations and tackle the nontrivial case when the phases are unavailable or undefined. Furthermore, we treat the problem of incomplete observations when the population mean field is unavailable, but only a small subset of units is observed. We introduce a new order parameter and demonstrate its efficiency for quantifying synchrony via monitoring general observables, regardless of whether the oscillations can be characterized in terms of the phases. Under the condition of a significant irregularity in the dynamics of the coupled units, this order parameter provides a unified description of synchrony in populations of units of various complexity. The primary examples include noise-induced oscillations, coupled strongly chaotic systems, and noisy periodic oscillations. Furthermore, we explore how this parameter works for the standard Kuramoto model of coupled regular phase oscillators. The most significant advantage of our approach is its ability to infer and quantify synchrony from the observation of a small percentage of the units and even from a single unit, provided the observations are sufficiently long. Next, we discuss the finite-size effects. Finally, we extend the approach to treat spike time series; the case is relevant for neuroscience.

Prediction of dynamical systems from time-delayed embeddings with self-intersections

Adam Śpiewak (Institute of Mathematics of the Polish Academy of Sciences)

Thursday, August 1, 15:00–15:30, SCC Conference Hall

Abstract: Takens' embedding theorem deals with reconstructing a dynamical system $T: X \rightarrow X$ from observations. It states that if $k > 2 \dim(X)$, then a typical observable on X can be used to faithfully reconstruct the dynamics via its model in \mathbb{R}^k , which is obtained by the time-delayed embedding $x \mapsto (h(x), h(Tx), \dots, h(T^{k-1}x))$. Takens' theorem is regarded as a mathematical justification for validity of dynamical models constructed from observed time series. I will present basics of a probabilistic version of this theory, which guarantees a reconstruction in a lower dimension, at the cost of allowing intersections to occur along sets of measure zero (with respect to a given reference measure). This approach allows for applications to the convergence of prediction algorithms and related conjectures posed by Schroer, Sauer, Ott and Yorke. The talk is based on joint works with Krzysztof Barański and Jonatan Gutman.

Fluid mixing analysis by means of Lagrangian trajectories and diffusion maps

Anna Klünker (Leuphana University Lüneburg), **Kathrin Padberg-Gehle** (Leuphana University Lüneburg)

Thursday, August 1, 15:30–16:00, SCC Conference Hall

Abstract: The analysis and control of mixing in fluid flows is relevant in many applications of natural and

technical systems. For given particle trajectories from a numerical simulation or from particle tracking (e.g., 4D-PTV), we aim to describe the evolution of a scalar field to study mixing in closed and open flows. To this end, we introduce a computational framework inspired by a diffusion map approach in trajectory space [1] to model the evolution of a scalar quantity under advection and diffusion in the purely data-based setting. The method can be extended to deal with open flows, where particles enter and leave the system and thus not all trajectories are available at any given time. In more detail: A passive scalar quantity evolves according to an advection-diffusion equation, that includes a diffusion constant and the underlying velocity field. For a positive diffusion constant, the variance of the scalar field decays and mixing can be quantified in terms of the variance of the scalar field. In a Lagrangian framework, the simple advection-diffusion can be written as a system of two independent equations, where the first equation describes the motion of the passive tracers, and the second equation describes the change of the scalar along the trajectory due to diffusion. This splitting is also the basis of weighted particle methods. However, in our case only particle trajectories are given, and the description and mixing quantification of a scalar quantity requires dealing with the Laplacian in a data-based framework. For this purpose we adapt a diffusion map approach, where at each time instance an instantaneous diffusion map matrix is computed based on distances between particles. Properties of fluid packages encoded in a scalar field can be modeled by a vector, which is then evolved by the time-dependent family of diffusion map matrices.

We apply the framework to several example systems. A simple autonomous cellular flow serves as a first testbed, where our results can be compared with those of a highly resolved numerical solution of the corresponding advection-diffusion equation. We also consider the data-based evolution of two differently colored fluids in a three-dimensional model of a stirred tank reactor [2] as well as in a mixing device with in- and outflow.

The authors acknowledge funding by the Deutsche Forschungsgemeinschaft (SFB1615-503850735-TP B05).

- [1] Banisch, R., Koltai, P., Understanding the geometry of transport: diffusion maps for Lagrangian trajectory data unravel coherent sets, *Chaos*, 2017, vol. 27, no. 3, p. 035804.
- [2] Hofmann, S., Weiland, C., Fitschen, J., von Kameke, A., Hoffmann, M. Schlüter, M., Lagrangian sensors in a stirred tank reactor: Comparing trajectories from 4D-Particle Tracking Velocimetry and Lattice-Boltzmann simulations, *Chem. Eng. J.*, 2022, vol. 449, p. 137549.

Modal error analysis and prediction compensation for Earth system models

Sean McGowan (University of Adelaide), Sanjeeva Balasuriya (University of Adelaide), William Robertson (University of Adelaide), Nicole Jones (University of Western Australia)

Friday, August 2, 11:00–11:30, SCC Conference Hall

Abstract: Predicting Earth systems is an important yet challenging problem resulting from the high dimensionality, chaotic behaviour, and coupled dynamics of the ocean, atmosphere, and other subsystems of the Earth. The numerical models derived to predict these systems will ultimately contain model error due to limited knowledge, capabilities of representation, and unresolved processes due to spatial resolution. Methods for combining a numerical model with observations such as with data assimilation can successfully reduce the effects of parameter misspecification and observational error; however, errors in model structure are much more challenging to address. Deriving data-driven representations of dynamics from historical observations may achieve higher predictive skill than models in scenarios of significant model error but lack the interpretability provided by a model.

Instead, hybrid modelling by pairing a model with a data-driven component has shown promise in outperforming both purely model and data-driven approaches in predicting complex systems. Many developed hybrid schemes place their confidence in the predictive power of the data-driven component by forcing the data-driven prediction with the model dynamics. This approach may increase predictive accuracy but fails to provide additional insight into the unmodelled dynamics. The presented work highlights a hybrid method that additively combines a model with a data-driven component that may be modally decomposed to give insight into model error or used to compensate a model during prediction.

Two methods are used to decompose and predict the model error - a Koopman based approach, and

an approach involving empirical orthogonal functions evolved by a reservoir computer. The Koopman perspective extracts spatially complex linearly evolving modes, while the reservoir computing perspective nonlinearly evolves modes of maximum variance. The applicability of these techniques is demonstrated and compared on two Earth system variables: tides and sea surface temperature. The connections between the two approaches, particularly the resulting spatial modes and temporal representations, are also investigated. The applications of this work are in improving models during model development, presenting novel methods for data assimilation, and enhancing predictive accuracy when available models have significant structural error. This is joint work in collaboration with Sanjeeva Balasuriya, William Robertson, and Nicole Jones.

Dynamics of Financial Markets: Collectivity and Large Behavior Relative to it

Anton J. Heckens (Universität Duisburg-Essen), Sebastian M. Krause (Universität Duisburg-Essen), Thomas Guhr (Universität Duisburg-Essen)

Friday, August 2, 11:30–12:00, SCC Conference Hall

Abstract: The measured correlations of financial time series in subsequent epochs change considerably as a function of time. When studying correlation matrices, quasi-stationary patterns, referred to as market states, are seen by applying clustering methods. They emerge, disappear or reemerge, but they are dominated by the collective motion of all stocks. To extract more refined information, we present an approach by clustering correlation matrices which are free from the collective market motion [1]. The resulting dynamics is remarkably different and the corresponding market states are quasi-stationary over a long period of time. Furthermore, we show pieces of evidence for precursors for critical events derived from observables free from the collective market motion [2].

[1] A. J. Heckens, S. M. Krause, T. Guhr, Uncovering the Dynamics of Correlation Structures Relative to the Collective Market Motion *J. Stat. Mech.* 103402 (2020), preprint: arXiv:2004.12336

[2] A. J. Heckens, T. Guhr, A New Attempt to Identify Long-term Precursors for Financial Crises in the Market Correlation Structures *J. Stat. Mech.* 043401 (2022), preprint: arXiv:2107.09048

Performance of permutation entropy-based methods for distinguishing brain states

Juan Gancio (Universitat Politècnica de Catalunya), Giulio Tirabassi (Universitat de Girona), Cristina Masoller (Universitat Politècnica de Catalunya)

Friday, August 2, 12:00–12:30, SCC Conference Hall

Abstract: Developing reliable methodologies to decode brain state information from electroencephalogram (EEG) signals is an open challenge, crucial to implementing EEG-based brain-computer interfaces (BCIs). For example, signal processing methods that identify brain states could allow motor-impaired patients to communicate via non-invasive, EEG-based BCIs. In this work, we focus on the problem of distinguishing between the states of eyes closed (EC) and eyes open (EO), employing quantities based on permutation entropy (PE). An advantage of PE analysis is that it uses symbols (ordinal patterns) defined by the ordering of the data points (disregarding the actual values), hence providing robustness to noise and outliers due to motion artifacts. However, we show that for the analysis of multichannel EEG recordings, the performance of PE in discriminating the EO and EC states depends on the symbols' definition and how their probabilities are estimated. Here, we study the performance of PE-based features for EC/EO state classification in a dataset of $N=107$ subjects with one-minute 64-channel EEG recordings in each state. We analyze features obtained from patterns encoding temporal or spatial information, and we compare different approaches to estimate their probabilities (by averaging over time, over channels, or by "pooling"). We find that some PE-based features provide about 75% classification accuracy, comparable to the performance of features extracted with other statistical analysis techniques. Our work highlights the limitations of PE methods in distinguishing the eyes' state, but, at the same time, it points to the possibility that subject-specific training could overcome these limitations.

Consistent spectral approximation of Koopman operators using resolvent compactification

Claire Valva (Courant Institute of Mathematical Sciences, NYU), Dimitrios Giannakis (Dartmouth College)

Friday, August 2, 12:30–13:00, SCC Conference Hall

Abstract: Koopman operators and transfer operators transform nonlinear dynamics in phase space to linear dynamics on vector spaces of functions, enabling the use of spectral techniques without modeling constraints such as linearity. The extraction of approximate Koopman eigenfunctions (and the associated eigenfrequencies) from an unknown system is nontrivial, particularly if the system has mixed or continuous spectrum. We discuss a spectrally-accurate approach to approximate the Koopman operator from data via a “compactification” of the resolvent of the Koopman generator. This approach employs kernel integral operators to approximate the skew-adjoint generator in measure-preserving systems by a family of skew-adjoint operators with compact resolvent, whose spectral measures converge in a suitable asymptotic limit, and whose eigenfunctions are approximately periodic. We explore implementations of this technique using data from example systems on tori with pure point spectra and the Lorenz 63 as an example with mixing dynamics.

Distinguish between data dynamics and noise type in your data with Consistency Measure.

Jozef Jakubik

Friday, August 2, 13:30–14:00, SCC Conference Hall

Abstract: Choosing the right approach to detecting causality can be challenging. This contribution introduces a new measure, consistency, to distinguish effectively between data from dynamical systems, autoregressive models, and pure noise. Current methods, such as False Nearest Neighbor (FNN), struggle to differentiate between data with inherent structure (AR models) and pure randomness (noise).

Consistency evaluates the coherence of reconstructed manifolds, analysing the structure of a reconstructed data manifold reveals how “consistently” neighboring points progress. Dynamical systems have a high level of consistency because of their well-defined structure.

By comparing the consistency of a data set with that of a randomly permuted version, we determine the most appropriate approach to causality detection. Our results demonstrate that consistency can effectively differentiate between autoregressive, dynamical and random data. This contribution offers insights on selecting suitable methods for detecting causality. It is important to choose the right method to ensure accurate results.

An Operator Acting on the Brain Network and Provoking Disease: A Conceptual Model and a First Data-Based Application

Maria Mannone (Universität Potsdam, Germany; Università Ca’ Foscari di Venice, Italy; PIK Potsdam Institute for Climate Impact Research, Germany), Peppino Fazio (Ca’ Foscari Università’ di Venezia, Italy; VSB, Technical University of Ostrava, Czechia), Norbert Marwan (Potsdam Institute for Climate Impact Research (PIK); Universität Potsdam, Germany)

Friday, August 2, 14:00–14:30, SCC Conference Hall

Abstract: The complexity of our brains can be described as a multilayer network, from neurons, to the neural agglomerates, to lobes. Neurological diseases are often related to malfunctions in the brain network. We propose a conceptual model of the brain, where the disease can be modeled as the result of an operator affecting and disrupting the brain-network organization, called “K-operator” (from “Krankheit,” German for “disease”). In our approach, the network channel model is adapted from telecommunications, where the action of the K-operator corresponds to an alteration of the healthy communication-structure between neuronal agglomerates. The potential of this novel approach is tested by quantitatively modelling the operator with real-data considering the Parkinson disease. We use data from the dataset of Parkinson’s Progression Markers Initiative (PPMI) upon concession by the University of Southern California. The networks are reconstructed from fMRI analysis, resulting in a matrix acting on the healthy brain and giving as output the diseased brain. We finally decompose the K-operator into the tensor product of its submatrices and we are able to assess its action on each one of the regions of interest (ROI) characterizing the brain for the specific considered samples. More interestingly, this application confirms the feasibility of the proposed analytic technique. Further research development can compare operators for different patients and for different

diseases, looking for commonalities and aiming to develop a comprehensive theoretical approach.
This research was funded by Next Generation EU - Age-It (PE0000015).

Detection of symmetric chaos using Optimal Transport theory

Jaime Cisternas (Universidad de los Andes)

Friday, August 2, 14:30–15:00, SCC Conference Hall

Abstract: Symmetric dynamical systems can show complex phenomena that only reveal their symmetries (the full symmetry group of the system or a smaller subgroup) when considering long time intervals. The classification problem is relevant for situations where the structure of the chaotic attractor changes, as in crises and other bifurcations. Although there are well established methods for the classification, most notably the ‘symmetry detectives’ developed by Barany & Golubitsky (1993), recent advances in machine learning offer promising new approaches. In this presentation we will show how the theory and algorithms of Optimal Transport can efficiently quantify the distance between invariant measures (either long sequences of points or multi-dimensional histograms) and assess the presence of a given subgroup. We assess the potentialities of the new method using examples with S4 and D3 symmetries.

Session on Dynamics in Neuroscience and Physiology

Neuron modeling via one-dimensional dynamics

Piotr Bartłomiejczyk (Gdańsk University of Technology, Faculty of Applied Physics and Mathematics, Institute of Applied Mathematics)

Tuesday, July 30, 13:30–14:00, Research III Lecture Hall

Abstract: Map-based models form an essential class of models describing the dynamics of a single nerve cell, which effectively complement well-known ODE-based ones. Although these models might seem abstract from the biological point of view, their electrophysiological relevance is in many cases satisfactory and the relative low computational complexity enables to employ them successfully in larger scale simulations of neuronal circuits motivated by biological or clinical issues. However before examination of large collections of coupled neurons, it seems to be desirable to understand the dynamical mechanisms behind the phenomena observed in the chosen single neuron models. In our talk we study two discrete models of neuronal dynamics. The first model was introduced by Chialvo in 1995 and the second one by Courbage, Nekorkin and Vdovin in 2007. We show that their reduced one-dimensional versions can be treated as independent simple models of neural activity, which still display very rich and varied dynamics. We carry out a detailed analysis of both periodic and chaotic behaviour of the models using recent advances in the theory of S-unimodal (Chialvo model) and Lorenz (CNV model) maps. This is joint work with Frank Llovera Trujillo and Justyna Signerska-Rynkowska.

Cardiac dynamics prior to the onset and termination of atrial fibrillation

Boon Leong Lan (Monash University), Yew Wai Liew, Mikito Toda, Suraya Kamsani

Tuesday, July 30, 14:00–14:30, Research III Lecture Hall

Abstract: Complex dynamical systems can shift abruptly from a stable state to an alternative stable state at a tipping point. Before the critical transition, the system either slows down in its recovery rate or flickers between the basins of attraction of the alternative stable states. Whether the heart critically slows down or flickers before it transitions into and out of paroxysmal atrial fibrillation (PAF) is still an open question. To address this issue, we propose a novel definition of cardiac states based on beat-to-beat (RR) interval fluctuations derived from electrocardiogram data. Our results show the cardiac state flickers before PAF onset and termination. Prior to PAF onset, flickering is caused by the interplay between the natural and the abnormal pacemakers, and it may also be caused by an abnormal modulation of the natural pacemaker (sinus node) by the autonomic nervous system. This abnormal autonomic modulation may be the sole cause of flickering prior to PAF termination. Flickering of the cardiac state could potentially be used as part of an early warning or screening system for PAF and guide the development of new methods to prevent or

terminate PAF. The method we have developed to define system states and use them to detect flickering can be adapted to study critical transition in other complex systems.

Seizure generation mechanisms beyond bifurcations: New perspectives on critical transitions and early warning signals

Andrew Flynn (University College Cork), Sebastian Wieczorek (University College Cork)

Tuesday, July 30, 14:30–15:00, Research III Lecture Hall

Abstract: The framework of critical transitions (CTs) and early warning signals (EWSs) can enhance our understanding of how seizures occur. The challenge is that there is no consensus on what type of CTs are responsible for the onset of seizures. During this talk we address this challenge in the following three steps. Firstly, we identify the Bautin normal form as a canonical dynamical system that exhibits seizure-like CTs which can be either bifurcation-induced, noise-induced, or a combination of both. Secondly, we establish what EWSs exist prior to such seizure-like CTs, this extends on previous studies which focused solely on bifurcation-induced CTs. Thirdly, we use this same approach to analyse different EEG recordings of seizures. This enables us to infer that the majority of the analysed seizures resemble noise-induced CTs, which has been unaccounted for in the literature.

Cross-frequency coupling in the brain dynamics: Causality vs. lead-lag relationship

Milan Palus (Czech Academy of Sciences), Andreu Arinyo-i-Prats (Aarhus University), Víctor J. López-Madróna (Institut de Neurosciences des Systèmes)

Tuesday, July 30, 15:00–15:30, Research III Lecture Hall

Abstract: Cross-frequency coupling (CFC) has recently attracted a lot of attention in neuroscience, since it enriches the cooperative behavior of neuronal networks and apparently plays an important functional role in neuronal computation, communication and learning [1]. A number of methods to detect CFC have been proposed and critically evaluated [2], however, approaches for inferring its directionality are just being developed. We compare cross-frequency directionality (CFD), based on the phase slope index (PSI) with the conditional mutual information (CMI) known also as transfer entropy. The two methods give contradictory results when applied to intracranial EEG data from an animal experiment. We explain this contradiction using simulated data from unidirectionally coupled nonlinear oscillators and demonstrate that, in nonlinear systems, the lead-lag relationship is not equivalent to causality or the direction of coupling as it is understood in nonlinear dynamical systems [3]. Therefore, for constructions of mathematical models in computational neuroscience, results inferred by CMI are relevant, not the lead/lag relations, uncovered by CFD/PSI.

This study is supported by the Czech Science Foundation (grant No. GF21-14727K); and by the Czech Academy of Sciences, Praemium Academiae awarded to M. Palus.

- [1] Canolty, R. T., & Knight, R. T. (2010). The functional role of cross-frequency coupling. *Trends in Cognitive Sciences*, 14(11), 506-515.
- [2] Aru, J., Aru, J., Priesemann, V., et al. (2015). Untangling cross-frequency coupling in neuroscience. *Current Opinion in Neurobiology*, 31, 51-61.
- [3] Arinyo-i-Prats, A., Lopez-Madróna, V. J., & Palus, M. (2024). Lead/Lag directionality is not generally equivalent to causality in nonlinear systems: Comparison of phase slope index and conditional mutual information. *NeuroImage*, 120610.

TVB Inversion: Framework and Strategies for Fast Parameter Estimation of Virtual Brains

Marius Pille (Berlin Institute of Health at Charité (BIH); Department of Neurology with Experimental Neurology, Charité, Universitätsmedizin Berlin), Emilius Richter (Berlin Institute of Health at Charité (BIH); Department of Neurology with Experimental Neurology, Charité, Universitätsmedizin Berlin), Michael Schirner (Berlin Institute of Health at Charité (BIH); Department of Neurology with Experimental Neurology, Charité, Universitätsmedizin Berlin; Bernstein Focus State Dependencies of Learning and Bernstein Center for Computational Neuroscience; Einstein Center for), Dionysios Perdakis (Berlin Institute of Health at Charité (BIH); Department of Neurology with Experimental Neurology, Charité, Universitätsmedizin

Berlin), Petra Ritter (Berlin Institute of Health at Charité (BIH); Department of Neurology with Experimental Neurology, Charité, Universitätsmedizin Berlin; Bernstein Focus State Dependencies of Learning and Bernstein Center for Computational Neuroscience; Einstein Center for)
Wednesday, July 31, 11:00–11:30, Research I Lecture Hall

Abstract: Whole brain neural mass modeling and simulation, facilitated by platforms like The Virtual Brain (TVB), has significantly advanced computational neuroscience by providing a framework to study the intricate dynamics of the brain. However, traditional techniques for inferring physiological parameters from neuroimaging data (e.g., fMRI and M/EEG), including exhaustive parameter exploration and machine learning methods treating the brain model as a black box (e.g., simulation-based inference using deep neural networks), face limitations due to high computational costs and an inability to scale to a large number of parameters. This drawback can be overcome by incorporating gradient information in the model space, thereby extending the scope of hypotheses that can be explored. Recent studies have leveraged gradient information through Bayesian methods such as Hamiltonian Monte Carlo No-U-Turn Sampler (HMC NUTS). However, implementing these methods necessitates modelers to rewrite the TVB model in specific statistical programming languages (e.g., STAN) or software packages (e.g., PyMC).

In this study, we address the above limitations by introducing differentiable backends based on JAX and PyTensor for TVB simulations. By integrating these backends, we substantially enhance the efficiency of TVB simulations and facilitate the application of state-of-the-art parameter estimation techniques from machine learning, such as ADAM optimization and Bayesian methods like HMC NUTS even for the non-expert user. We demonstrate the efficacy of our approach through two use cases that illustrate the breadth of workflows now accessible with TVB.

Use Case I presents an approach to gradient-based optimization of functional connectivity and its dynamics using long-duration fMRI simulations with feedback inhibition network control, to regulate the excitation-inhibition balance. Use Case II showcases the inference of region-specific parameters from power spectral densities to reproduce the gradient in peak oscillatory frequency across the brain, typically observed in resting-state MEG data. Our results further reveal up to an order of magnitude improvement in simulation performance for TVB simulations. This enables fast, highly parallel fixed-point analysis and adiabatic scanning to investigate bifurcations in networked dynamical systems.

The introduction of differentiable simulations represents a significant methodological paradigm shift for whole brain simulations, empowering researchers to understand the brain's dynamics more effectively and efficiently. By enhancing parameter estimation techniques applied to TVB simulations, our approach opens new avenues for studying brain function and dysfunction. Furthermore, the ability to infer heterogeneous physiological parameters from neuroimaging data holds promise for advancing our understanding of brain disorders and guiding the development of targeted therapeutic interventions.

Understanding and Optimizing Learning in Spiking Neural Networks using Dynamical Systems Theory

Rainer Engelken

Wednesday, July 31, 11:30–12:00, Research I Lecture Hall

Abstract: Discovering and responding to associations between temporally distant cues is crucial for an animal's survival. Solving this temporal credit assignment problem allows organisms to bridge the gap between when a stimulus cue arrives and when its effect unfolds. Gradient-based training of recurrent neural circuit models for temporal tasks with long time horizons presents challenges, potentially leading to vanishing or exploding gradients. We have connected this issue to the Lyapunov exponents of the forward dynamics, which describe how perturbations grow or shrink in tangent space. Here, we propose to address gradient instability in recurrent spiking and firing rate networks by controlling the Lyapunov exponents of forward dynamics throughout learning.

We regularize Lyapunov exponents towards zero, ensuring that the corresponding directions in tangent space grow or shrink only slowly, for more robust propagation of learning signals over long time horizons. Our method enhances the success rate of RNNs in typical neuroscience tasks that involve bridging task events across many time steps. Furthermore, we demonstrate that applying gradient flossing during training enhances trainability for challenging temporal credit assignment tasks. We establish a connection between

the Lyapunov exponents and the dimensionality of the gradient signal in backpropagation. Additionally, we demonstrate the effectiveness of our approach both on spiking and firing rate networks. Our results suggest that dynamically controlling Lyapunov exponents can significantly improve the stability and effectiveness of RNN training. We speculate on the optimization of neural dynamics in animals over evolutionary timescales to bridge long time horizons.

Noise resilience of memory stored in low-dimensional manifolds through multiple synaptic timescales

Georg Chechelnizki (Hebrew University of Jerusalem), Nimrod Shaham (Hebrew University of Jerusalem), Alon Salhov (Hebrew University of Jerusalem), Yoram Burak (Hebrew University of Jerusalem)

Wednesday, July 31, 12:00–12:30, Research I Lecture Hall

Abstract: The reliable storage of information in short term memory is a crucial function of the brain. This function is challenged by the inherently noisy nature of neurons. Representations of continuous variables are particularly vulnerable to noise. This vulnerability arises from random drift that accumulates over time in the form of diffusive dynamics of the stored memory and can severely degrade memory accuracy. In this work we identify a neural mechanism which effectively counteracts such deterioration by employing a combination of slow excitatory and fast inhibitory synaptic connections. This yields an approximation of the temporal derivative of network activity, which can then be effectively used to stabilize it against various perturbations. This mechanism is inspired by the derivative feedback mechanism (Lim and Goldman, 2013), which was previously shown to improve stability to certain forms of parameter mistuning in noise-free networks. Furthermore, we derive a theory which accurately predicts the diffusivity in noisy continuous attractor networks (CANs). We then first examine the simple case of a linear CAN both numerically and analytically, showing that diffusivity can be made arbitrarily small if the timescale of excitation is larger than that of inhibition. We then demonstrate that similar principles generalize to a far more general class of nonlinear CANs. We successfully apply these principles to ring attractor networks, inspired by the insect head direction system. We find that our theory correctly predicts the improvement of memory stability as a function of synaptic timescale differences in such models, when endowed with the derivative feedback mechanism. This offers a plausible explanation for how the brain can stably store memories of continuous parameters, despite the ubiquity of noise. Furthermore, we identify how to engineer connectivity such that variability in neural activity perpendicular to the attractor is not affected by the stabilization mechanism. This is shown to allow for tight confinement of neural activity patterns to a one dimensional manifold, as well as the rapid relaxation of transient modes of activity back into the attractor. Insights from our theory allow us to conclude that neurons in head direction cell networks that are commonly thought to be utilized for velocity integration may also aid in stabilization against noise-driven motion.

The effect of nonlinear dynamics and axonal delays on the relationship between structural and functional brain connectivity

Helmut Schmidt (Institute of Computer Science, Czech Academy of Sciences), Stella M. Sanchez (Institute of Computer Science, Czech Academy of Sciences), Antonin Skoch (National Institute of Mental Health), Filip Spaniel (National Institute of Mental Health), David Tomecek (National Institute of Mental Health), Pavel Sanda (Institute of Computer Science, Czech Academy of Sciences), Jaroslav Hlinka (Institute of Computer Science, Czech Academy of Sciences)

Friday, August 2, 11:00–11:30, IRC Seminar Room 1

Abstract: Functional magnetic resonance imaging (fMRI) data are a standard experimental tool in investigating healthy and pathological brain activity, and are typically used to examine the functional connectivity (FC) between brain areas. In addition, MRI-based diffusion tractography can be used to uncover structural connectivity (SC) between brain areas. It has been demonstrated experimentally and theoretically that FC correlates with SC, but their exact relationship is still unknown. Here, we focus on the effect of nonlinear dynamics and axonal delays on FC. We extend the multivariate Ornstein-Uhlenbeck process by weak nonlinearities and axonal delays to derive the FC matrix analytically for a given SC matrix.

To investigate the effect of nonlinearities, we focus on the limit of weak nonlinearities in which we can

use scale separation to obtain analytical results for the FC matrix. The nonlinear term is cubic to ensure that the fixed point structure and stability of the linear part of the system is preserved. The resulting FC is the sum of the linear FC (FC in the absence of nonlinearities) and a perturbation term, which is proportional to the product of the SC and the linear FC. This in turn allows us to derive an expression for the SC given the perturbed FC, with a parameter α indicating the relative contribution of the nonlinearity. We apply the latter to empirical FC to derive the “FC-based” SC (through model inversion), and compare it with the empirical SC obtained through tractography. By varying the level of nonlinearity through α and computing the correlation between “FC-based” SC and empirical SC, we find that nonlinear dynamics are present in the empirical data, but their relative contribution to the FC is small.

Axonal delays are incorporated by dividing the tract lengths between remote brain areas (obtained from MRI) by the axonal propagation velocity. In the limit of weak connectivity and fast delays, these delays lead to a correction term in the structural connectivity. This allows us to estimate the axonal propagation velocity and the time constant of the Ornstein-Uhlenbeck process from empirical data, and we find that the highest correlation between “FC-based” SC and empirical SC occurs at physiologically plausible parameters (10ms to 40ms for time constant, 1m/s to 4m/s for axonal propagation velocity). Including axonal delays also significantly increases the match between “FC-based” SC and empirical SC.

Electromechanical memcapacitive neurons for energy-efficient spiking neural networks

Zixi Zhang (Peking University), Yuriy Pershin (University of South Carolina), Ivar Martin (Argonne National Laboratory)

Friday, August 2, 11:30–12:00, IRC Seminar Room 1

Abstract: In this conference contribution, a novel nanoscale electromechanical device called a leaky memcapacitor [1] is presented as a potential hardware solution for spiking neurons. The leaky memcapacitor functions as a movable plate capacitor that exhibits increased conductivity when the plates are in close proximity. We introduce a dynamical model of the leaky memcapacitor and demonstrate that various spiking patterns observed in biological neurons can be replicated using the leaky memcapacitor. From the point of view of circuit theory, the model of a leaky memcapacitor includes a parallel connection of memcapacitive and memristive elements. Both the resistance and capacitance in the leaky memcapacitor are dependent on a shared internal variable, namely the displacement of the movable plate. The integrated capacitive, resistive leakage, and reset features within the leaky memcapacitor streamline the development of spiking neural networks.

I. M. acknowledges funding from the Materials Sciences and Engineering Division, Basic Energy Sciences, Office of Science, US DOE. Y. V. P. was partially supported by the NSF grant EFRI-2318139.

[1] Zhang, Zixi, Yuriy V. Pershin, and Ivar Martin. *Chaos, Solitons & Fractals* 181 (2024): 114601.

Function and structure in cultured neuronal networks

Luis M. Ballesteros-Esteban (Rey Juan Carlos University; GISC; CTB-UPM), Juan A. Almendral (Rey Juan Carlos University; GISC; CTB-UPM), Inmaculada Leyva (Rey Juan Carlos University; GISC; CTB-UPM), Irene Sendina-Nadal (Rey Juan Carlos University; GISC; CTB-UPM)

Friday, August 2, 12:00–12:30, IRC Seminar Room 1

Abstract: Cultured neuronal networks (CNNs) are widely used to analyze both the topological and functional network of a nervous system, helping us to understand the complex relationship between structure and dynamics in neuronal networks, and how it changes along the development process of the culture. However, this understanding is often limited by the difficulties in resolving the structure of neural networks. Our experimental approach allows us to simultaneously study the detailed structure and dynamics of the cultured network and, therefore, to compare the topological and functional networks longitudinally. We grow CNNs from isolated neurons extracted from *Schistocerca gregaria* (locust) on top of microelectrode arrays (MEA), allowing the recording of their electrophysiological signal for 28 days. We acquire large-scale microscope images of the evolving culture from which we obtain the structural networks using a homemade image segmentation algorithm. Simultaneously, we record the electrical time series using microelectrode

arrays to detect the spikes timestamps. We analyze the spikes pair-wise synchronization events to finally produce the functional network. The results of this experiment, both structural and functional, and how they are linked, are the subject of this talk.

The dynamics of human action and perception during the coordination of a ball interception task

Prasetia Utama Putra (Konstanz University), Fumihiko Kano (Konstanz University)

Friday, August 2, 13:30–14:00, IRC Seminar Room 1

Abstract: This study aims to investigate how humans coordinate and perceive actions in a dynamic ball interception task. In this task, two individuals must continuously coordinate their actions to keep bouncing a table tennis ball towards the wall. We tracked the body and eye movements of individuals to analyze their coordination. From body and eye movement data, we extracted eye-movement and action features, such as anticipatory looks, ball pursuit duration, and kinematic energy of racket. To understand individuals' movement patterns, the Lyapunov spectrum of their racket movement was analyzed. In addition, a combination of a Hidden Markov Model and action features was employed to identify the transition from stable to semi-stable coordination states. Our preliminary findings suggested that participants' racket movements showed chaotic behavior in both short and long coordination sequences. This behavior may result from their attempts to compensate for their partner's actions or their own errors. We also observed significant differences in eye and body movements when transitioning from stable to semi-stable coordination. In the semi-stable state, the duration of pursuit became shorter, and the movement of the racket became more irregular compared to the stable state. Overall, our study offers a quantitative framework for understanding the dynamics of human movement and perception during realistic interception tasks.

Asymmetry in the distribution of unit properties is the key factor determining the efficiency of collective oscillations in FitzHugh-Nagumo networks

Stefano Scialla (National Institute of Chemical Physics and Biophysics), Marco Patriarca (National Institute of Chemical Physics and Biophysics), Els Heinsalu (National Institute of Chemical Physics and Biophysics), Marius Yamakou (Friedrich-Alexander University Erlangen-Nuremberg), Julyan Cartwright (Instituto Andaluz de Ciencias de la Tierra, CSIC-UGR)

Friday, August 2, 14:00–14:30, IRC Seminar Room 1

Abstract: The cell network structure of the pancreatic islets of Langerhans has been the subject of several experimental and theoretical studies. A long-standing dilemma is whether the collective oscillations of β -cells require the presence of specialized pacemaker cells or synchronization occurs through a “democratic” mechanism, where the collective cell network behavior is a nonlinear average of the properties of its individual elements. The topic has received increasing attention and a recent review entirely focuses on this “pacemaker or not” dilemma [1].

In a recent work [2] we mimicked the architecture of a β -cell network by a cubic lattice of heterogeneous FitzHugh-Nagumo (FHN) elements. This topology resembles the experimentally known features of a β -cell islet. We introduced heterogeneity in the network in the form of diversified external currents J_i acting on the network elements, drawn from a Gaussian distribution with standard deviation σ . We identified pacemakers with the units having J_i values corresponding to an intrinsically oscillatory state, while the units with J_i values corresponding to an excitable state were “followers”. Upon varying the distribution width σ , we found a clear “Diversity-induced Resonance” effect, co-occurring with a fraction $f \approx 5\%$ of pacemakers. This is in good agreement with experimental observations [3].

While the results of our previous study support the existence of pacemakers, they do not allow us to draw any firm conclusion on whether these elements are an emergent network property or they exist independently of the network. Trying to dig deeper into this question, here we present the results of new simulations where, using the same model, we look into the effect of introducing an asymmetry in the distribution of J_i values for both pacemakers and followers. This has some physiological basis, as several lines of evidence support a deficit in β -cell mass (β -cell apoptosis) associated with type 2 diabetes. This deficit, ranging from 20% to 65% of β -cell mass, might not equally affect all β -cell subgroups, in particular

pacemakers vs. followers, and asymmetries might arise in their ability to respond to changes in glucose levels, i.e., in the distribution of J_i values. We find that, rather than the numerical ratio between pacemakers and followers, what determines the efficiency of global network oscillations is the degree of symmetry of the overall J_i distribution around its mean value: a loss of symmetry can lead to a complete silencing of network oscillations, independently of the pacemaker/follower ratio, which was deemed to be of crucial importance in some previous works. We prove this across different network topologies, including all-to-all coupling, cubic lattice, and small-world (Newman-Watts). This sheds new light on oscillator network synchronization mechanisms and supports the hypothesis that pacemakers are more likely to be an emergent network property.

[1] B.E. Peercy, A.S. Sherman, J. Biosci. 47, 14 (2022)

[2] S. Scialla, A. Loppini, M. Patriarca, E. Heinsalu, Phys. Rev. E 103, 052211 (2021)

[3] N.R. Johnston et al., Cell Metab. 24, 389 (2016)

Session on Dynamics on graphs

A Universal Route to Explosive Phenomena

Christian Kuehn (Technical University of Munich)

Wednesday, July 31, 13:30–14:00, East Hall 3

Abstract: Critical transitions are observed in many complex systems. This includes the onset of synchronization in a network of coupled oscillators or the emergence of an epidemic state within a population. “Explosive” first-order transitions have caught particular attention in a variety of systems when classical models are generalized by incorporating additional effects. Here, we give a mathematical argument that the emergence of these first-order transitions is not surprising but rather a universally expected effect: Varying a classical model along a generic two-parameter family must lead to a change of the criticality. To illustrate our framework, we give two explicit examples of the effect in distinct physical systems: a model of adaptive epidemic dynamics and a higher-order/polyadic generalization of the Kuramoto model.

Broken detailed balance in the dynamics of directed networks

Ramón Nartallo-Kaluarachchi (University of Oxford)

Wednesday, July 31, 14:00–14:30, East Hall 3

Abstract: The structure of a complex network plays a crucial role in determining its dynamical properties. In this talk, I will show that the directed, hierarchical organisation of a network causes the system to break detailed balance, time-reversal symmetry and dictates the production of entropy through non-equilibrium dynamics. I will consider a wide range of dynamical processes including random walks, Ornstein-Uhlenbeck and Ising dynamics and show how different directed network features govern their thermodynamics. This will involve presenting a method for smoothly varying the directedness of networks. Next, using the Erdos-Renyi graph as a null model, I decouple different notions of asymmetry and show that locally evolving processes (walks) are governed by local asymmetry whilst complex systems are governed by global asymmetries and hierarchy. Thirdly, we analyse a collection of 97 empirical networks and show that strong directedness and non-equilibrium dynamics are both ubiquitous in real-world systems. Finally, we present a simple, but powerful, method for inferring broken detailed balance and directed network structure from multivariate time-series using constrained autoregression and apply our method to identify non-equilibrium and hierarchical organisation in both human neuroimaging, during task and rest, and financial time-series from the NYSE. Overall, our results shed light on the thermodynamic consequences of directed network structure, provide a clear interpretation for empirical macroscopic broken detailed balance (such as in the brain), and indicates the importance and ubiquity of hierarchical organisation and non-equilibrium dynamics in real-world complex systems.

From unbiased to maximal entropy random walks on hypergraphs

Pietro Traversa (Universidad de Zaragoza), Guilherme Ferraz de Arruda (CENTAI Institute S.p.A.), Yamir

Moreno (Universidad de Zaragoza)
Wednesday, July 31, 14:30–15:00, East Hall 3

Abstract: Random walks have been intensively studied on regular and complex networks, which are used to represent pairwise interactions. Nonetheless, recent works have demonstrated that many real-world processes are better captured by higher-order relationships, which are naturally represented by hypergraphs. Here, we study random walks on hypergraphs.

Random walks on hypergraphs are of particular interest, as they provide insights into the network structure and are widely used to characterize graphs in various practical applications. Due to the higher-order nature of these mathematical objects, one can define more than one type of walk and step. Here, by "step," we refer to the choice of the adjacency matrix representing the hypergraph, which is not a unique choice. The two most commonly used adjacencies are the ones defined by Banerjee et al. and by Battiston et al., in which the hyperedges are considered as weighted cliques of a pairwise graph. Carletti et al. proposed a whole spectrum of possible adjacency matrices, and thus random walks, using a free parameter. Under such a varied possibility of choices, in our paper, we highlight the difference between the types of random walks.

In particular, we study the unbiased and the maximal entropy random walk on hypergraphs with two types of steps, the projected and higher-order steps, emphasizing their similarities and differences. We characterize these dynamic processes by examining their stationary distributions and associated hitting times. To illustrate our findings, we present a toy example and conduct extensive analyses of artificial and real hypergraphs, providing insights into both their structural and dynamical properties. We conclude by discussing other possible steps using the bipartite projection of the hypergraph instead of the clique projection.

Overall, our work contributes to a better understanding of random walks on hypergraphs. We complement the theory about random walks on hypergraphs by allowing different steps and we expand it by generalizing the maximal entropy random walks on hypergraphs.

Master stability for traveling waves on rings and lattices

Stefan Ruschel (University of Leeds), Andrus Giraldo (Korea Institute for Advanced Study)
Wednesday, July 31, 15:00–15:30, East Hall 3

Abstract: Computing the spectrum and stability of traveling waves in discrete systems (such as rings and lattices) becomes unfeasible with increasing system size. We present a framework for effectively determining the spectrum independently of the system size by means of computing master stability curves, which comprise the full spectrum. We illustrate our method for traveling waves in coupled systems of Fitzhugh-Nagumo units, and show how the approach can be utilized to identify instabilities.

Chaotic leader-laggard cluster synchronization in laser networks for reinforcement learning

Shun Kotoku (The University of Tokyo), Takatomo Mihana (The University of Tokyo), André Röhm (The University of Tokyo), Ryoichi Horisaki (The University of Tokyo), Makoto Naruse (The University of Tokyo)
Wednesday, July 31, 16:00–16:30, East Hall 3

Abstract: We study networks of four delay-coupled lasers oscillating chaotically, where we are seeking a particular state: cluster synchronization of two lasers each, with a leader-laggard relationship between the clusters. Cluster synchronization is widely observed in delay-coupled networks, where nodes separate into several clusters, and the dynamics of nodes in one cluster perfectly synchronize. The leader-laggard relationship is similar to delay-synchronization, where one of the lasers oscillates preceding the other with an offset of the coupling delay time. However, in our case, we do not desire full delay-synchronization; instead, the temporal relation switches spontaneously under appropriate couplings.

In the four-laser networks of our interest, both types of laser interactions can coexist, enabling an innovative method to accelerate a foundational problem in reinforcement learning: the competitive multi-armed bandit (CMAB) problem. Using the properties of chaotic dynamics and network theory, we aim to pave the way for applying chaotic dynamics to machine learning.

We quantitatively analyze the stability of cluster synchronization in several four-laser networks with conditional Lyapunov exponents. In the four-laser networks, two types of synchronization solutions can exist: two-cluster synchronization, where four lasers divide into two clusters, each with two lasers, and global synchronization, where all four lasers completely synchronize. Our analysis identifies five networks with stable two-cluster synchronization and the corresponding parameter regions of couplings, while global synchronization remains unstable. Then, we observe asymmetric leader-laggard cluster synchronization, where the leader probability of one of the clusters increases while maintaining the two-cluster synchronization. The leader probability can be dynamically controlled with coupling strength, although the optical inputs into lasers in the same cluster should remain equal.

We employ these dynamics in four-laser networks to solve the CMAB problem. In CMAB, several players repeatedly select from slot machines with winning probabilities unknown and different among players, aiming to maximize the total rewards for all players. We address the most fundamental, 2-player and 2-slot CMAB problem. Our collective decision-making system employs a network comprising four lasers: Lasers 1A, 1B, 2A, and 2B. Each laser corresponds to Slot A or B selected by Player 1 or 2, with Lasers 1A and 1B (2A and 2B) allocated to Player 1 (2). Lasers 1A and 2A are mutually coupled, with Laser 1A unidirectionally coupled to Laser 1B, and similarly, Laser 2A is connected to Laser 2B.

Players select a slot corresponding to the leader at each decision point. For example, when Laser 1A is the leader, Player 1 selects Slot A. Concurrently, thanks to the cluster synchronization, Laser 2B is the leader, and Player 2 selects Slot B. Therefore, players achieve perfect conflict avoidance in CMAB, preventing missed opportunities for benefiting from the non-selected option. Additionally, players achieve exploration owing to the frequent switching of leaders.

In specific problems, a biased selection of either (1A, 2B) or (1B, 2A) is preferable for the overall benefit. This exploitation process can be achieved by dynamically adapting the laser network to unbalanced yet ordered couplings, resulting in asymmetric leader-laggard cluster synchronization. We show both numerical simulations and experimental measurements.

Analysis of the Geometric Structure of Neural Networks and Neural ODEs via Morse Functions

Sara-Viola Kuntz (Technical University of Munich), Christian Kuehn (Technical University of Munich)

Wednesday, July 31, 16:30–17:00, East Hall 3

Abstract: Besides classical feed-forward neural networks, also neural ordinary differential equations (neural ODEs) gained particular interest in recent years. Neural ODEs can be interpreted as an infinite depth limit of feed-forward or residual neural networks. In this presentation, we study the input-output dynamics of finite and infinite depth neural networks with scalar output. In the finite depth case, the input is a state associated to a finite number of nodes, which maps under multiple non-linear transformations to the state of one output node. In analogy, a neural ODE maps a linear transformation of the input to a linear transformation of its time-T map. We show that depending on the specific structure of the network, the input-output map has different properties regarding the existence and regularity of critical points. These properties can be characterized via Morse functions, which are scalar functions, where every critical point is non-degenerate. We prove that critical points cannot exist, if the dimension of the hidden layer is monotonically decreasing or the dimension of the phase space is smaller or equal to the input dimension. In the case that critical points exist, we classify their regularity depending on the specific architecture of the network. We show that each critical point is non-degenerate, if for finite depth neural networks the underlying graph has no bottleneck, and if for neural ODEs, the linear transformations used have full rank. For each type of architecture, the proven properties are comparable in the finite and in the infinite depth case. The established theorems allow us to formulate results on universal embedding, i.e. on the exact representation of maps by neural networks and neural ODEs. Our dynamical systems viewpoint on the geometric structure of the input-output map provides a fundamental understanding, why certain architectures perform better than others.

Spore Life: Dormancy promotes the persistence of populations

Daniel Henrik Nevermann (Goethe University Frankfurt am Main, Germany), Claudius Gros (Goethe University Frankfurt am Main, Germany), Jay Lennon (Indiana University Bloomington)

Wednesday, July 31, 17:00–17:30, East Hall 3

Abstract: The factors contributing to the persistence and stability of life are fundamental for solving basic and applied problems in biology. Organisms are commonly challenged by harsh and fluctuating environments that are suboptimal for growth and reproduction, which may in some instances lead to the extinction of a population. Many species contend with unfavorable and noisy conditions by entering a reversible state of reduced metabolic activity, a phenomenon known as dormancy. Throughout the tree of life, dormancy has independently arisen many times via different and occasionally complex mechanisms, which raises questions about the origins and mechanisms by which dormancy confers fitness benefits to populations. Here, we develop a cellular automaton model to investigate the effects of dormancy on population dynamics, which we call Spore Life. The model is based on Conway's Game of Life, a two-dimensional deterministic cellular automaton with two states, DEAD and ALIVE, in which the dynamics are governed by simple rules depending on an individual's neighborhood. Spore Life provides a dormant refuge for individuals that would otherwise die, which is implemented through an additional state, SPORE. These dormant cells can resuscitate when environmental conditions improve. The model includes a parameter α that controls the survival probability of spores and thereby interpolates between Game of Life and Spore Life, where the intermediate regime implies stochastic dynamics. In addition to identifying the emergence of unique periodic patterns, we found that spore survival (α) increases population survival probabilities and extinction times as well as the average number of living cells in the population. Interestingly, these emergent demographic patterns only require a comparatively small number of resuscitation events. Our approach, using a modification of a relatively simple and well-validated model, yields novel insight into the emergence of complex behavior associated with dormancy and the seed banks that they generate. Spore Life can be interactively explored at the following site: <https://itp.uni-frankfurt.de/~gros/spore-life/>.

Dynamics on directed metric graphs

Vsevolod Chernyshev (UCA)

Wednesday, July 31, 17:30–18:00, East Hall 3

Abstract: Let us consider a directed metric graph, i.e. one-dimensional cell complex. All edges are smooth regular curves and have lengths, as well as permitted direction of movement. We will consider a situation in general position and assume that all edge lengths are linearly independent over the field of rational numbers. We will also fix the vertex, which we will call the starting one. A point leaves it at the initial moment of time. At each vertex with non-zero probability, we can select an outgoing edge for further movement. Reversals on the edges are prohibited. To analyze the number of possible endpoints of such a walk, it is useful to assume that all possibilities are realized. Thus, we arrive at the following dynamical system. At the initial moment of time, points begin to move with unit speed from the starting vertex along all the edges that start from it. As soon as one of the points is at the vertex, a new point appears at each incident vertex, which begins to move towards the end of the edge, and the old one disappears. If several points simultaneously come to one vertex at once, then all of them disappear, while new points appear as if one point arrived at the vertex. Our main task is to study an asymptotics of $N(T)$, i.e. the number of moving points as time T goes to infinity on various finite compact graphs.

This dynamical system has already been studied for the case of undirected graphs. For the number of moving points, a polynomial approximation was obtained, that is, a description of a polynomial of degree $E-1$ was given, where E is the number of edges of the graph that approximates $N(T)$ up to a certain power of the logarithm. Consideration of such dynamical systems was motivated by problems of propagation of narrow wave packets on metric graphs and hybrid manifolds.

In my talk I will consider some special cases of directed metric graphs, for example directed Hamiltonian graphs.

Epidemic Dynamics on complex hypergraphs

Pia Steinmeyer (TUM)

Friday, August 2, 11:00–11:30, IRC Seminar Room 3

Abstract: When it comes to interacting particle systems (IPS) on graphs, the underlying idea is to associate to each node a state and define a dynamical system on the graph by specifying the interaction between vertices along the edges. For our purpose, we study IPS through the example of contact processes on more general geometric structures given by simplicial complexes. The classical contact process on the integer lattice is defined by two update rules, the recovery of an infected vertex at a given rate and the infection of a susceptible vertex at a rate proportional to the number of infected neighbors. On top, we will not only consider binary interactions of two vertices along an edge but interaction across higher dimensional simplices. We will study the mathematical basis of the process (in terms of markov processes), the existence of invariant measures, and the related influence of the simplicial structure on dynamics. We will develop lower and upper bounds for the critical values that mark the transition from extinction to survival (for lattice-like simplicial complexes also known as epidemic threshold). On the macroscopic level, we consider the Gillespie algorithm for k-spin systems and develop a master and mean-field equation for particular models. We study those models of social contagion on simplicial complexes, where we adapt various rules how vertices can interact across higher-dimensional simplices in order to characterize the influence of different social behavior. For this, interaction, infection as well as recovery, on different sized simplices are built-in and we compare the mean-field equations in terms of their bifurcational behavior.

Effects of time delays and graph structure on network speed

Fatihcan Atay (Bilkent University)

Friday, August 2, 11:30–12:00, IRC Seminar Room 3

Abstract: We study the speed of convergence of networks to coordinated/synchronized behavior in the presence of signal propagation delays. We use a paradigmatic linear consensus problem as the prototype dynamics and consider both directed and undirected graphs. We derive relationships between consensus speed on the one hand and the Laplacian spectrum and delay magnitudes on the other hand. We obtain exact conditions for undirected networks, whereas the case of directed networks turn out to present a considerably more challenging problem. We present analytical results for a class of directed networks.

Effect of repulsive spanning trees in attractively coupled networks

Sayantana Nag Chowdhury (Indian Statistical Institute, Kolkata)

Friday, August 2, 12:00–12:30, IRC Seminar Room 3

Abstract: Ensembles of coupled nonlinear oscillators are a popular paradigm and an ideal benchmark for analyzing complex collective behaviors. The onset of cluster synchronization is found to be at the core of various technological and biological processes. The current literature has investigated cluster synchronization by focusing mostly on the case of attractive coupling among the oscillators. However, the case of two coexisting competing interactions is of practical interest due to their relevance in diverse natural settings, including neuronal networks consisting of excitatory and inhibitory neurons, the coevolving social model with voters of opposite opinions, ecological plant communities with both facilitation and competition, to name a few. We investigate the impact of repulsive spanning trees on cluster formation within a connected network of attractively coupled limit cycle oscillators. We successfully predict which nodes belong to each cluster and the emergent frustration of the connected networks independent of the particular local dynamics at the network nodes. We also determine the local asymptotic stability of the cluster states using an approach based on the formulation of a master stability function. We additionally validate the emergence of solitary states, antiphase synchronization, and antisynchronization for some specific choices of spanning trees and networks.

Self-similar growth patterns of cellular automata on Cayley graphs and 2-dimensional von Neumann outer-totalistic elementary cellular automata

Jens Christian Claussen (University of Birmingham, UK)

Friday, August 2, 13:30–14:00, IRC Seminar Room 3

Abstract: Growth patterns of cellular automata emerging from a localized single seed initial condition have

intrigued the nonlinear dynamics community through the identification of two universality (sub)classes within the Wolfram class IV cellular automata [1], where rule 90 (Sierpinski) and rule 150 are representatives of these classes. The time series generated by the total activity of the Rule 150 remarkably follows a tensorial Fibonacci iteration [2], as a consequence of the generalized self-similarity (depending on the two previous time intervals) of the geometric pattern generated by the dynamics. However, the understanding of cellular automata patterns in the more general cases of graphs and higher-dimensional lattices remains challenging. Here we discuss two cases: Cayley graphs, and 2-dimensional ECA. For Cayley graphs (degree-regular trees) the corresponding rule 150 dynamics (for a localized initial condition) reduces to a one-dimensional ECA [2] and the total activity follows novel integer sequences [3,4]. For 2-dim cellular automata, we performed an exhaustive exploration [5] of all 2-dimensional cellular automata rules which in addition obey the criterion of outer-totalistic, i.e., the output depends on the number of active neighbour cells, but not about their spatial pattern. While being a subset of all possible 2-dim CA rules, this is the most central class of CA rules as they retain a maximum of symmetries. We identify several distinct time series related to the growth patterns, and beside the 1-dim and 2-dim versions of rule 90, we also identify a rule providing a triple replication, and generating a 2-dim spatial Sierpinski pattern. Our findings indicate that despite the large number of possible cellular automata rules, we observe only a few strictly self-similar growth patterns and associated time series.

- [1] Jan Nagler and Jens Christian Claussen, $1/f^\alpha$ spectra in elementary cellular automata and fractal signals, Phys. Rev. E 71, 067103 (2005)
- [2] Jens Christian Claussen, Time-evolution of the Rule 150 cellular automaton activity from a Fibonacci iteration, Journal of Mathematical Physics 49, 062701 (2008)
- [3] Jens Christian Claussen, Online Encyclopedia of Integer Sequences, OEIS A138276, <https://oeis.org/A138276>
- [4] Jens Christian Claussen, Online Encyclopedia of Integer Sequences, OEIS A138277, <https://oeis.org/A138277>
- [5] Kian Siadat and Jens Christian Claussen (in preparation)

Performance-dependent network evolution for enhanced computational capacity

Manish Yadav (Technische Universität Berlin), Sudeshna Sinha (Indian Institute of Science Education and Research Mohali, India), Merten Stender (Technische Universität Berlin)

Friday, August 2, 14:00–14:30, IRC Seminar Room 3

Abstract: The fundamental question of identifying network structure-function relationships has recently received significant attention in different disciplines of science. However, there is still no definitive answer as to what the optimal network architecture is, especially when it comes to complex information processing. Therefore, we address the problem of how optimal and specific network structures form to efficiently solve a given task. In this study, we introduce a novel framework of performance-dependent network evolution, leveraging reservoir computing principles. Our investigation reveals that the task-specific minimal network structures obtained through this framework consistently outperform networks generated by alternative growth strategies and Erdős-Rényi random networks. Notably, the evolved networks exhibit unexpected sparsity and adhere to scaling laws in the nodes-density space, while showcasing a distinctive asymmetry in the distribution of input and information readout nodes. As a consequential outcome, we devise a general heuristic for quantifying task complexity from evolved networks. This framework captures the intricacies of diverse information-processing tasks and provides valuable insights into the evolutionary dynamics of network structure-function relationships. Our findings not only advance the fundamental understanding of process-specific network evolution but also shed light on the design and optimization of complex information processing mechanisms across various domains including machine learning.

Session on Ergodic theory

Higher-order asymptotic expansions of pressure functionals and statistical representations of the coefficients

Haruyoshi Tanaka (Naruto University of Education)

Tuesday, July 30, 11:00–11:30, East Hall 4

Abstract: We study the higher-order asymptotic expansion of the pressure functional $\epsilon \mapsto P(\varphi + \epsilon\psi)$ with potentials φ and ψ on a countable Markov shift X :

$$P(\varphi + \epsilon\psi) = P(\varphi) + p_1\epsilon + p_2\epsilon^2 + \cdots + p_n\epsilon^n + o(\epsilon^n).$$

We show that if X is topologically mixing and has the BIP property, the potentials φ and ψ are real-valued locally Hölder continuous functions on X , and a sufficient condition for an asymptotic expansion of $\epsilon \mapsto P(\varphi + \epsilon\psi)$ is satisfied, then the 3-th coefficient of this expansion has a limit representation which looks like the asymptotic variance well:

$$p_3 = \frac{1}{3!} \lim_{m \rightarrow \infty} \frac{1}{m} \int_X \left(\sum_{k=0}^{m-1} (\psi - \int_X \psi d\mu) \circ \sigma^k \right)^3 d\mu,$$

where μ is the Gibbs measure for φ and σ is the shift transformation on X . We will also mention the 4-th and higher coefficients. In these studies, we give an explicit formula for the behavior of $\mu((\sum_{i=0}^{m-1} (\psi - \int_X \psi d\mu) \circ \sigma^i)^k)$ as $m \rightarrow \infty$. In application, we apply the convergence rate of the mean ergodic theorem in $L^k(\mu)$ with even integer k to our result. In this talk, we will also give an example that $P(\varphi + \epsilon\psi)$ has an asymptotic expansion of order n but does not have one of order $n + 1$.

Anosov properties in a symplectic map with time-reversal symmetry

Ken-ichi Okubo (Sanyo-Onoda City University), Ken Umeno (Kyoto University)

Tuesday, July 30, 11:30–12:00, East Hall 4

Abstract: We propose a two-dimensional symplectic map derived from a Hamiltonian as a model that has time-reversal symmetry on a microscopic scale while it is proven analytically that any initial density function defined almost everywhere converges to the uniform distribution in the sense of mixing (irreversible behavior) on a macroscopic scale. To show the irreversible convergence to the uniform distribution, we show that the model is an Anosov diffeomorphism in a certain parameter range on a compact torus. We also prove that such a mixing invariant measure is a unique equilibrium state, unique SRB measure, and physical measure. Moreover, we prove analytically that the Kolmogorov-Sinai entropy, which is the average gain of information per unit time, is positive by proving that Pesin's formula holds. The formula of Lyapunov exponent analytically derived by a certain assumption is consistent with numerical simulation and the critical exponent of the Lyapunov exponent is proven to be 1/2 without any assumption.

Equidistribution of cusp points of Hecke triangle groups

Laura Breitkopf (University of Bremen), Marc Keßböhrer (University of Bremen), Anke Pohl (University of Bremen)

Tuesday, July 30, 12:00–12:30, East Hall 4

Abstract: In the framework of infinite ergodic theory, we derive equidistribution results for suitable weighted sequences of cusp points of Hecke triangle groups encoded by group elements of constant word length with respect to a set of natural generators. This is a generalization of the corresponding results for the modular group whose sequence of cusp points is related to the Stern-Brocot sequence that can be constructed from the Farey mapping. In this talk, we will introduce generalizations of the Farey mapping and discuss how their dynamical properties yield the equidistribution results. We rely on advanced results from infinite ergodic theory and transfer operator techniques developed for AFN-maps.

Multivariate mean equicontinuity

Jonas Breitenbücher (Friedrich-Schiller Universität Jena), **Lino Haupt** (Friedrich-Schiller Universität Jena), Tobias Oertel-Jäger (Friedrich-Schiller Universität Jena)

Tuesday, July 30, 13:30–14:00, East Hall 4

Abstract: This is work in progress and joint work with Prof. Tobias Oertel-Jäger and Jonas Breitenbücher.

An non-trivial ergodic system with discrete spectrum has a wide range of topological models. One particular class of topological models described by the dynamical notion of "mean equicontinuity". This dynamical property is equivalent to the invertibility property that the factor map to the MEF is measure-theoretically an isomorphism.

There are many systems whose factor map to the MEF is not measure-theoretically 1:1 but finite-to-one. Examples include the Thue-Morse subshift or Toeplitz flows constructed by Iwanik and Lacroix. Clearly those systems are no topological models of systems with discrete spectrum.

In the past few years those classes of systems have been studied under the viewpoint of multivariate forms of mean sensitivity, as for example by Li, Ye, Yu (2021). We propose a notion of multivariate mean equicontinuity called "mean m-equicontinuity", which satisfies a Auslander-Yorke type dichotomy to the aforementioned multivariate mean sensitivities. We prove that a topological dynamical systems satisfies a strong form of mean m-equicontinuity if and only if the factor map to the MEF is measure theoretically m-to-1. We conjecture an equivalence between mean m-equicontinuity and this strong form of mean m-equicontinuity.

Characterising Edge States: Measures on Chaotic Non-Attracting Invariant Sets

Raphael Römer (University of Exeter, UK), Peter Ashwin (University of Exeter, UK)

Tuesday, July 30, 14:00–14:30, East Hall 4

Abstract: For continuous and discrete autonomous dynamical systems, we define an invariant probability measure on chaotic non-attracting invariant sets that is both useful from a practical point of view and that has desirable theoretical properties. Building on Sweet and Ott (2000), we formalize their ideas by defining a measure via the ratio of the Lebesgue measures of two sets and show how to sample it numerically. Properties of similar measures on the stable and unstable sets of non-attracting invariant sets are derived as well. We discuss the measures' and the sampling techniques' relevance for simple low-dimensional models as well as their usefulness and limitations for more complex and higher-dimensional models and illustrate our results using instructive examples. Knowledge about this measure on a non-attracting set (e.g. on a saddle or an edge state) provides information about the dynamics of potentially long-lived chaotic transients, and also about the set's fractal dimension. In the case of the non-attracting set being an edge state, this has implications for the geometric complexity of the basin boundary, which can be used to better quantify uncertainty close to it.

Escaping dynamics for \mathbb{Z}^d extensions

Chris Moor

Tuesday, July 30, 14:30–15:00, East Hall 4

Abstract: It is well known that in dimension greater than two, a symmetric random walk is transient with probability one. In this talk we will consider a class of generalisations of a random walk: Dynamical systems on \mathbb{R}^d , which are factors of special skew product dynamical systems with phase space \mathbb{Z}^d . We describe the transient dynamics qualitatively as α -escaping sets, where vectors α denote mean escape velocities. In this sense, recurrent points escape with velocity 0. The relative closure of possible mean escape velocities is a convex polytope known as the rotation set. For any mean escape velocity α , we will quantify the set of α -escaping points in terms of Hausdorff dimension. In particular, we will consider vectors belonging to the relative boundary of the rotation set. These results generalise previous work by Maik Gröger, Johannes Jaerisch and Marc Kesseböhmer.

Session on Fluid Dynamics

Degenerate Cauchy-Goursat problem for 2-D steady isentropic Euler system with van der Waals gas

Harsita Srivastava (Dr. B R Ambedkar National Institute of Technology Jalandhar, Punjab), M. Zafar

Thursday, August 1, 13:30–14:00, East Hall 4

Abstract: This study concerns with the existence-uniqueness of local classical sonic-supersonic solution to a

degenerate Cauchy–Goursat problem that arises in transonic phenomena. The flow is governed by 2-D steady isentropic Euler system with a polytropic van der Waals gas. The idea of characteristic decomposition has been used to convert the Euler system into a new system involving the angle variables (θ, μ) . To overcome the parabolic degeneracy caused at the sonic curve, the partial hodograph transformation and a variety of dependent-independent variables have been introduced to transform the nonlinear system into a linear one with explicit singularity–regularity structure. The uniform convergence of the sequences $(W^{(m)}, Z^{(m)})$ has been discussed by employing the mathematical induction. Eventually, the inversion of the solution from partial hodograph plane to the original plane has been established.

Effect of Reynolds Number and Thickness Tapering on the Hydrodynamic Performance of Oscillating Elastic Swimmers

Andrew Lenart (Georgia Institute of Technology), Christopher Jawetz (Georgia Institute of Technology), Alexander Alexeev (Georgia Institute of Technology)

Thursday, August 1, 14:00–14:30, East Hall 4

Abstract: Thickness tapering leads to enhanced hydrodynamic performance of oscillating elastic propulsors by drastically increasing the hydrodynamic thrust and efficiency compared to elastic propulsors with uniform thickness. Thickness tapering leads to the acoustic black hole effect at the propulsor trailing edge that minimizes the wave reflection and promotes the development of the traveling waves propagating along the propulsor length. We use fluid-structure interaction computational modeling to explore the hydrodynamic mechanism leading to the enhanced hydrodynamics of tapered propulsors. In particular, we probe the effects of the tapering geometry and the propulsor aspect ratio on the emerging flow patterns and hydrodynamic forces. Our simulations provide useful guidelines for designing efficient bio-mimetic robotic swimmers. *Support from the National Science Foundation (CBET 2217647) is gratefully acknowledged.

On the stability of a minimal nonlinear Hamiltonian dynamical system describing the interaction between vorticity waves and shear flows

Eyal Heifetz (Tel Aviv University), Erik Gengel (Tel Aviv University)

Thursday, August 1, 14:30–15:00, East Hall 4

Abstract: Here we describe a prototype of two-dimensional shear instability in terms of action at a distance between two counter-propagating Rossby waves with the effect of mutual interaction between the waves and the mean flow, where growth of the waves reduces the mean shear and vice versa. We show that a canonical action-angle Hamiltonian structure of the non-linear dynamical system is preserved even when the wave-mean flow interaction is included.

In the linearized stage, modal instability of small amplitudes is obtained when the waves are phase-locked in a way that their counterpropagation rate balances the shear imposed by the mean flow. Therefore, when reaching finite amplitudes, thus consequently reducing the mean shear, the waves' counterpropagation rate overcomes the mean shear and phases unlock. This leads to a transient dynamics by which the wave-wave interaction affects the waves' amplitudes and relative phase, as well as the magnitude of the mean shear. The change in the latter affects the waves' relative phase and consequently the waves' growth. We find that this extended mechanism gives rise to a Hamiltonian dynamics in which the waves settle into libration while their amplitudes remain finite. We close by discussing how the given dynamics relates to familiar models of Kuramoto-type phase oscillators.

Time-periodic bursting solutions on the edge to turbulence in pipes with rectangular cross-section

Markus Scherer (Institute for Water and Environment, Numerical Fluid Mechanics Group, Karlsruhe Institute of Technology), Markus Uhlmann (Institute for Water and Environment, Numerical Fluid Mechanics Group, Karlsruhe Institute of Technology), Genta Kawahara (Graduate School of Engineering Science, Osaka University)

Thursday, August 1, 15:00–15:30, East Hall 4

Abstract: The idea to study the (turbulent) motion of a fluid from a dynamical systems point of view dates

back to a seminal work by Hopf [1]: In the framework envisaged therein, a turbulent flow is interpreted as a trajectory through a high-dimensional state space, with each point corresponding to an instantaneous flow state. On its way, the trajectory visits a variety of unstable low-dimensional invariant sets including equilibria/travelling waves and (relative) periodic orbits and ‘shadows’ their dynamics while residing in their neighbourhood. Specifically for wall-bounded shear flows (e.g. the flow between two parallel plates or in a circular pipe), this geometrical state-space picture has led to the discovery of many invariant solutions and has, thereby, significantly contributed to our current understanding of the underlying mechanisms of transitional and fully-developed turbulent flows.

In the current study, we aim to identify time-recurrent invariant solutions for the flow in ducts of rectangular cross-section, for which only a handful of travelling wave solutions are known to date [2-4]. With the aid of the iterative bisection procedure of Toh and Itano [5] and a Newton-GMRES-hookstep solver, we have been able to isolate multiple invariant periodic cycles along the separatrix (‘edge’) between the laminar and turbulent basins of attraction in appropriate symmetric subspaces. While unstable w.r.t. the full state space, the detected solutions seem to be attracting within the edge manifold and reveal interesting dynamics over their period: The flow oscillates periodically between two flow states in which streamwise vorticity is predominantly residing near either of the two sets of parallel walls. ‘Switching’ events between the two states are accompanied by intermittent bursting-like high-dissipation excursions and reveal some similarity with the (chaotic) dynamics in duct turbulence at marginal Reynolds number. The talk will thus close with a comparison of the dynamics in both situations in order to assess how simple time-periodic solutions could help understanding the characteristics of duct turbulence at low Reynolds number.

- [1] E. Hopf. *Comm. Pure Appl. Math.*, 1(4), (1948)
- [2] M. Uhlmann, G. Kawahara, and A. Pinelli. *Phys. Fluids*, 22(8), (2010)
- [3] S. Okino, M. Nagata, H. Wedin, and A. Bottaro. *J. Fluid Mech.*, 657, (2010)
- [4] S. Okino and M. Nagata. *J. Fluid Mech.*, 693, (2012)
- [5] S. Toh and T. Itano. *J. Fluid Mech.*, 481, (2003)

Theory for turbulent-laminar patterns in Couette flow

Dwight Barkley (University of Warwick), Santiago Benavides (Universidad Politécnica de Madrid)
 Friday, August 2, 11:00–11:30, East Hall 4

Abstract: Transitional turbulence in wall-bounded shear flows exhibits a remarkable phenomenon: large-scale periodic patterns of alternating turbulent and laminar flow emerge spontaneously from uniform turbulence as the Reynolds number is decreased. These patterns are ubiquitous in subcritical shear flows and explaining them has been a long-standing challenge for understanding the route to turbulence. From a dynamical systems viewpoint, these patterns are fascinating because they appear in a highly fluctuating, highly nonlinear state. We will describe a recent model obtained by projecting the Navier-Stokes equations onto a few vertical modes, with closure coming from modelling Reynolds stresses and dissipation. The resulting two-dimensional PDE model is expressed in 5 fields describing the large-scale flow, and 1 or 2 fields describing the turbulent kinetic energy. The model opens new avenues to understanding the large-scale spatiotemporal dynamics observed in transitional turbulence.

Lorenz’ butterfly in a fluid experiment that is repeated 42 times by a robot

Willem van de Water (Delft University of Technology)
 Friday, August 2, 11:30–12:00, East Hall 4

Abstract: The question is whether an experiment in fluid dynamics can be repeated, especially, when the flow transitions from laminar to turbulent. Clearly, turbulent flow cannot be repeated, but how about its laminar precursor ?

In our experiment a vortical flow behind a traveling plate turns into turbulence. By exactly repeating this experiment 42 times with a robot, we study the statistics of the laminar to turbulent transition. In each realization the fate of the flow is followed over 1.7s when the plate travels with a constant velocity. It suddenly turns turbulent at a scaled travelled distance of 5.5 plate heights.

Using Particle Image Velocimetry we register the vorticity in a plane that divides the plate perpendicularly, and collect statistics over 1600 two-dimensional velocity fields. We introduce a new Lagrangian measure of variability between the experiment realizations. It quantifies the separation between two experiments that were started in the same way with robotic precision: the butterfly effect.

The Finite Time Lyapunov Exponent field (FTLE) of a single experiment predicts this variability; thus we confirm ergodicity. Apart from pointwise measures, yielding a distribution over the field of view, we study the statistics of the circulation computed over the upper and lower half of the domain. The almost perfect symmetry, both of the mean and of the fluctuations points to their origin as the fluctuating vortex ring trailing behind the plate.

During the initial phase long-time correlations exist in the flow, but they cease once the flow turns turbulent. By ordering our repeated experiments we find that extreme circulations are preceded by circulations that are larger than the median.

- [1] Transition to turbulence behind a traveling plate, Jesse Reijtenbagh, Jerry Westerweel and Willem van de Water, Phys. Rev. Fluids 8, 104601 (2023).

Deterministic and stochastic surrogate models for a slowly driven fast oscillator

Marc Aurele Tiofack Kenfack (KU Eichstätt–Ingolstadt), Marcel Oliver (KU Eichstätt–Ingolstadt)

Friday, August 2, 12:00–12:30, East Hall 4

Abstract: It has long been known that the excitation of fast motion in certain two-scale dynamical systems is linked to the singularity structure in complex time of the slow variables. We demonstrate, in the context of a fast harmonic oscillator forced by one component of the Lorenz 1963 model, that this principle can be used to construct time-discrete surrogate models by numerically extracting approximate locations and residues of complex poles via Adaptive Antoulas–Andersen (AAA) rational interpolation and feeding this information into the known “connection formula” to compute the resulting fast amplitude. Despite small but nonnegligible local errors, the surrogate model maintains excellent accuracy over very long times. In addition, we observe that the long-time behavior of fast energy offers a continuous-time analog of Gottwald and Melbourne’s 2004 “0–1 test for chaos” – the asymptotic growth rate of the energy in the oscillator can discern whether or not the forcing function is chaotic.

Data-driven nonlinear manifold learning of chaotic flow dynamics

Alessandro Della Pia (Scuola Superiore Meridionale, Naples, Italy), Dimitrios Patsatzis (Scuola Superiore Meridionale, Naples, Italy), Lucia Russo (Consiglio Nazionale delle Ricerche (CNR)), Constantinos Siettos (University of Naples “Federico II”)

Friday, August 2, 13:30–14:00, East Hall 4

Abstract: The low-dimensional manifold embedding of the fluidic pinball dynamics is identified in this work by means of a parsimonious nonlinear manifold learning algorithm based on Diffusion Maps (DMs). Two-dimensional direct numerical simulations of the incompressible Navier–Stokes equations are performed to compute the viscous wake behind the fluidic pinball, a flow configuration consisting of three cylinders of equal diameter. By variation of the flow governing parameter Re (Reynolds number), five different flow regimes are examined, spanning from steady symmetric ($Re < 18$) to fully chaotic ($Re > 115$) conditions.

In the first step of the Diffusion Maps embedding, the minimum set of DMs reduced coordinates (eigenvectors) necessary to represent the flow dynamics in all the regimes is found by projecting the high-dimensional simulation data into the reduced low-dimensional space (restriction operation). Then, the time series embedded into the manifold are lifted back to the original space by means of Geometric Harmonics (lifting operation), such as to evaluate the reconstruction error between the “ground truth” solution (i.e. high-fidelity simulation data) and the DMs “reconstruction”.

The performance of the DMs-based reconstruction is finally compared with a counterpart linear technique based on the Proper Orthogonal Decomposition (POD), which demonstrates the power of the proposed approach in parsimoniously representing the nonlinear dynamics up to the chaotic regime. The

present work intends to be the first step towards the identification of a fully data-driven DMs-based reduced-order surrogate model of the fluidic pinball configuration evolving on a nonlinear manifold.

Bifurcations of buoyancy driven spherical shell convection under differential rotation

Fred Feudel (Uni Potsdam), **Ulrike Feudel** (Carl von Ossietzky Universität Oldenburg)

Friday, August 2, 14:00–14:30, East Hall 4

Abstract: We investigate numerically bifurcation phenomena of buoyancy driven convection in a rotating spherical shell which is heated by imposing a constant temperature difference between the inner and outer spheres, and is subject to a radially directed gravity force. Along with the overall rotation of the fluid shell the influence of a shear generated by a differential rotation between both spheres on the convection pattern is the focus of this work. This configuration is an appropriate model of convection flows in geophysical and astrophysical applications, as e.g. in the outer cores of terrestrial planets.

Due to the imposed differential rotation of both spheres the dynamics for small Rayleigh numbers generates a nonzero basic flow which possesses features of the spherical Couette flow. Increasing the Rayleigh number the axial symmetry of the flow is broken in successive Hopf bifurcations generating new stable branches of rotating waves (RWs) and modulated rotating waves (MRWs), respectively, with an azimuthal mode number $m=3$. However in comparison to the configuration without differential rotation, now in addition, a new RW branch with no symmetry, $m=1$, bifurcates in a saddle node bifurcation, separated from the other branches. The stable $m=3$ MRWs and the arising stable $m=1$ RWs are coexisting along a certain interval of the Rayleigh numbers creating a region of bistability. We demonstrate that finally the stable $m=3$ MRW branch collides with an unstable RW branch in an homoclinic bifurcation, and the $m=1$ MRW branch remains in this scenario the only stable branch for larger Rayleigh numbers.

In summary, in contrast to the situation with no differential rotation in this configuration a saddle node bifurcation generates a branch with no axial subsymmetry which also enhances the heat transfer in comparison to the other branches and which forms the final attractor after the homoclinic bifurcation.

Onset of Lagrangian chaos: transition from fractal power spectrum to the absolutely continuous one

Michael Zaks (Humboldt University of Berlin), **Rafil Sagitov** (National Research Perm State University)

Friday, August 2, 14:30–15:00, East Hall 4

Abstract: We study transition to chaotic advection in the motion of the viscous fluid across a plane rectangular domain with periodic boundary conditions. The motion, induced by a combination of time-independent force with constant pumping in both spatial directions, is equivalent to dynamics on the surface of a 2-torus. At sufficiently high amplitude of the driving force, the stationary flow pattern includes, along with the streamlines corresponding to the global drift around the torus, localized vortices enclosed by separatrices of stagnation points. If the rotation number on the torus, as controlled by pumping intensities, is irrational, and the Eulerian velocity field remains time-independent, then the power spectrum of the Lagrangian observables is singular continuous (fractal) and the pattern of autocorrelation function possesses non-decaying peaks at arbitrarily large times. As the further increase of the driving force destabilizes the stationary flow pattern, the singularities in the power spectrum smoothen and the temporal autocorrelation decays.

Session on Geometry and Dynamics

Lyapunov exponents and statistical moments for instabilities in dynamical systems

Dmitry Sokoloff (Lomonosov Moscow State University, Russia)

Wednesday, July 31, 11:00–11:30, East Hall 4

Abstract: Development of instabilities in random media is considered for the case of geodesic lines on a Riemannian manifold with random curvature. The instability is described using geodesic deviation and Jacobi equation. Exponential growth of Jacobi field due to curvature fluctuations is demonstrated. Estimates of Lyapunov exponent and growth rates of statistical moments are presented. Role of the long term memory effects and fluctuations of the memory time are discussed. We discuss how to rescale the results obtained

on similar more physical problems, say, magnetic field grows in random flows. The last problem known as hydromagnetic dynamo is relevant for magnetic field generation in many celestial bodies. Presentation is based on the papers PRE, 104, 05214, 2021 and PRE 107, 044110, 2023

Darboux's Theorem, Lie series and the standardization of the Salerno and Ablowitz - Ladik models

Tiziano Penati (University of Milan, Milan)

Wednesday, July 31, 11:30–12:00, East Hall 4

Abstract: We revisit the proof of Moser-Darboux's Theorem, which provides a general and constructive scheme to deal with Hamiltonian models with non-standard symplectic structures. We take as a guiding example the Salerno and Ablowitz-Ladik (AL) models: we justify the form of a well-known change of coordinates, by showing that it comes out in a natural way within the general strategy outlined in the proof. Moreover, the full or truncated Lie-series technique in the extended phase-space reveals the leading order normal forms of the Salerno model. Applications to the Lotka-Volterra model might be added.

Stochastic geometric wave mean flow interactions

Ruiao Hu, Darryl D. Holm, Oliver D. Street

Wednesday, July 31, 12:00–12:30, East Hall 4

Abstract: Given the recent development of structure-preserving stochastic fluid models, we consider the case of structure-preserving stochastic perturbations for wave mean flow interactions. The class of wave mean flow interactions under consideration here follows from the Euler-Poincaré variational principle where the configuration Lie group is enlarged to include wave variables. We show that under these constructions, we obtain two-way interactions between the mean flow dynamics and the wave dynamics. To introduce stochasticity to the WMFI models, two frameworks for introducing stochastic perturbations are considered. They are the Stochastic Advection by Lie Transport (SALT) and the Stochastic Forcing by Lie Transport (SFLT) frameworks which preserve the Poisson structure and the Hamiltonian of the unperturbed systems respectively. Extending these stochastic frameworks for application to the WMFI models, we construct distinct classes of stochasticity which are introduced to the wave and mean flow part of the dynamics separately. We show that the new classes of stochastic perturbations are true extensions of the SALT and SFLT frameworks as they preserve the Poisson structure and Hamiltonian of the deterministic WMFI models respectively. Lastly, we present numerical simulations of the stochastic WMFI models for the example whereby an inhomogeneous 2D Euler flow is coupled to the nonlinear Schrödinger equation.

Semimartingale driven mechanics and reduction by symmetry for stochastic and dissipative dynamical systems

Oliver Street (Imperial College London), So Takao (California Institute of Technology (Caltech))

Wednesday, July 31, 12:30–13:00, East Hall 4

Abstract: The recent interest in structure preserving stochastic Lagrangian and Hamiltonian systems raises questions regarding how such models are to be understood and the principles through which they are to be derived. By considering a mathematically sound extension of the Hamilton-Pontryagin principle, we derive a stochastic analogue of the Euler-Lagrange equations, driven by independent semimartingales. Using this as a starting point, we can apply symmetry reduction carefully to derive non-canonical stochastic Lagrangian / Hamiltonian systems, including the stochastic Euler-Poincaré / Lie-Poisson equations, studied extensively in the literature. Furthermore, we develop a framework to include dissipation that balances the structure-preserving noise in such a way that the overall stochastic dynamics preserves the Gibbs measure on the symplectic manifold, where the dynamics effectively takes place. In particular, this leads to a new derivation of double-bracket dissipation by considering Lie group invariant stochastic dissipative dynamics, taking place on the cotangent bundle of the group.

Session on Mathematical and Theoretical Physics

Vibration problems in the Moore-Gibson-Thompson thermoelasticity for porous materials

Merab Svanadze (Ilia State University)

Monday, July 29, 17:00–17:30, East Hall 8

Abstract: In this work, the linear model of Moore-Gibson-Thompson (MGT) thermoelasticity for porous materials is proposed in which the combination of the deformation of elastic material, the concepts of Darcy's law for fluid flow in pore network and the MGT law of heat conduction is considered. The basis equations of motion of this model is presented. Then, the basic boundary value problems (BVP) of steady vibrations are investigated. Namely, the fundamental solution of the system of steady vibration equations is constructed. Green's identities are obtained and by virtue of these identities the uniqueness theorems for the classical solutions of the BVPs are proved. The surface (single-layer and double-layer) and volume potentials are constructed and the BVPs of steady vibrations are reduced to the always solvable singular integral equations. Finally, the existence theorems for classical solutions of the BVPs are proved with the help of the potential method and the theory of singular integral equations.

Acknowledgements: This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [Grant No. FR-23-4905].

Quantum Chaos and Bifurcation in Billiard Systems.

Hironori Makino (Tokai University), Saneyuki Yoshida (Tokai University)

Monday, July 29, 17:30–18:00, East Hall 8

Abstract: Bifurcation is a characteristic phenomenon of dynamical systems where periodic orbits are created or destroyed by coalescence, and this process provides a very important perspective to the Gutzwiller semiclassical periodic-orbit theory that connects the fluctuation property of the quantum energy spectrum to the periodic-orbits sum of the classical dynamical system. In several investigations, it has been revealed that the semiclassical contribution from the bifurcating orbit in the periodic-orbits sum becomes relatively large in the semiclassical limit, and its creation or destroy through the bifurcation has a large impact on the fluctuation properties of quantum energy spectrum. In Ref.[1], it is shown that the eigenenergy levels at the bifurcation point exhibit periodic accumulation along the energy axis, whose period is well approximated by the prediction of semiclassical theory. In this presentation, we explain our recent results on the numerical study for the fluctuation property of quantum energy spectrum that is characterized by the two-point correlation function (TPCF). The TPCF at the bifurcation point is found to exhibit a remarkable spike oscillation owing to the accumulation of levels. This oscillation is analyzed by the reduced chi-squared value, which exhibits abrupt increases at bifurcation points, thereby yielding a novel detection method to the bifurcation points. Based on this result, we have attempted to estimate the bifurcation points of billiard systems by its quantum mechanical data of eigenenergy levels. The estimates are in very good agreement with the exact solution of the bifurcation parameters. We then discuss the properties of the energy eigenfunctions at the bifurcation point. It was found that the eigenfunctions are significantly amplified along the bifurcating orbits at the energy accumulation points, and hence, this phenomenon also occurs periodically along the energy axis. Using this phenomenon, we will attempt to estimate the position of a fixed point on the Poincaré surface of section. The result will be discussed in the conference presentation.

[1] H. Makino, Prog. Theor. Exp. Phys. Vol.2022, 093A02 (2022).

Universal recasting of the evolution laws of autonomous dynamical systems into a Lotka-Volterra-like format

Diego Frezzato (University of Padova)

Monday, July 29, 18:00–18:30, East Hall 8

Abstract: The search for unifying formats underlying the evolution of deterministic dynamical systems described by ordinary differential equations (ODEs) has attracted the attention of many researchers across decades. In fact, if the ODEs of disparate dynamical systems were embeddable into a unique format, the

detailed inspection of such a structure could reveal ubiquitous traits regardless of the specificity of the single cases.

A general “universal” evolution law has been recently worked out [1] for a large class of deterministic autonomous dynamical systems, on condition that the rate equations are continuous and bounded in the domain of interest. Such a universal format takes a Lotka-Volterra-like quadratic form, and is achieved by employing the Universal Approximation procedure (borrower form the Machine Learning context) to represent the original ODEs. What we ultimately get is a system of quadratic ODEs in an extended abstract space of huge (virtually infinite) dimension, from which the actual physical system’s state can in principle be retrieved by means of a backward transformation.

Despite the practical uselessness of this recasting at the computational level, it could be potentially useful to let emerge and/or characterize universal traits of the original ODEs. Strategies for the embedding of some classes of dynamical systems into quadratic ODEs are known by decades (see for instance refs. [2-5]) and proved useful, for example, to characterize the slow manifolds in chemical mass-action kinetics [6]. Now, the universal embedding could be of use to discover dynamical traits for general dynamical systems under mild assumptions. The most intriguing fact, however, is the existence of such an underlying texture in which the dynamical systems can be framed. This calls for a physical understanding beyond the mathematics, and points straight to the provocative question raised by Peschel and Mende in the subtitle of their 1986 book “The predator-prey model” [7]: Do we live in a Volterra World?

- [1] D. Frezzato, “Universal embedding of autonomous dynamical systems into a Lotka-Volterra-like format”, *Phys. Scr.* 99, 015235 (2024).
- [2] L. Brenig, A. Goriely, “Universal canonical forms for time-continuous dynamical systems”, *Phys. Rev. A* 40, 4119–22 (1989).
- [3] J. L. Gouzé, “Transformation of Polynomial Differential Systems in the Positive Orthant”, Report RR-1308 INRIA, <https://inria.hal.science/inria-00075251/en>.
- [4] B. Hernández-Bermejo, V. Fairén, “Nonpolynomial vector fields under the Lotka-Volterra normal form”, *Phys. Lett. A* 206, 31–7 (1995).
- [5] P. Nicolini, D. Frezzato, “Features in chemical kinetics. I. Signatures of self-emerging dimensional reduction from a general formal of the evolution law”, *J. Chem. Phys.* 138, 234101 (2013).
- [6] A. Ceccato, P. Nicolini, D. Frezzato, “A low-computational-cost strategy to localize points in the slow manifold proximity for isothermal chemical kinetics”, *Int. J. Chem. Kinet.* 49, 477–93 (2017).
- [7] M. Peschel, W. Mende, “The Predator-Prey Model: Do We Live in A Volterra World?” (Springer, 1986).

Fractal relationship between recurrence microstates and local densities in recurrence plots

Felipe Eduardo Lopes da Cruz (Federal University of Paraná, Brazil), Sergio Roberto Lopes (Federal University of Paraná, Brazil), Thiago de Lima Prado (Federal University of Paraná, Brazil)
Monday, July 29, 18:30–19:00, East Hall 8

Abstract: Recurrence plots (RP) offer an intriguingly simple – yet powerful – way of extracting information from time series data by mapping them into square logical matrices. Recurrence quantification analysis (RQA), which is the name given to the quantitative way of processing the internal structures of RPs, has been showing to be a very versatile method of nonlinear time series analysis. Among the many distinct complexity measures one can obtain from this method, it was recently proposed another extension that relies on local recurrent patterns within the RPs, called recurrence microstates. These microstates are small square submatrices randomly sampled from RPs, and can be used to quantify disorder through their entropy. However, it was also observed that, the relationship between these small submatrices and the density of points of certain regions of the RP, is represented by a family of matrices presenting a fractal pattern. These matrices that relate microstates with densities can also be used to relate microstates to line structures, which enables us to create a probabilistic framework for calculating even the traditional RQA measures such as the determinism (DET) and laminarity (LAM) with ease. This fact is system-independent, which makes recurrence microstates a pivotal set of internal structures in the context of RQA, showing that understanding their properties is a matter of large interest that could potentially improve several aspect of recurrence analysis.

Quantum critical growth of entanglement entropy in disordered spin chains with tunable range interactions: Strong Disorder Renormalization Group and Quantum Simulations

Stefan Kettemann (Constructor University Bremen gGmbH)

Tuesday, July 30, 14:30–15:00, East Hall 8

Abstract: The nonequilibrium dynamics of disordered many-body quantum systems after a quantum quench unveils important insights about the competition between interactions and disorder. We examine the entanglement entropy growth after a global quench in a quantum spin chain with randomly placed spins and long-range tunable interactions decaying with distance with power α . Using a dynamical version of the strong disorder renormalization group we find for $\alpha > \alpha_c$ that the entanglement entropy grows logarithmically with time as in a quantum critical glass. It is found to become smaller with larger α as $S(t) = Sp \ln(t)/(2\alpha)$. For $\alpha < \alpha_c$, we find that the entanglement entropy grows as a power law with time, $S(t) \sim t^\gamma(\alpha)$ with $0 < \gamma(\alpha) < 1$ a decaying function of the interaction exponent α .

- [1] Y. Mohdeh, J. Vahedi, S. Haas, R. N. Bhatt, S. Kettemann, Global Quench Dynamics and the Growth of Entanglement Entropy in Disordered Spin Chains with Tunable Range Interactions, Phys. Rev. B 108 L140203 (2023).

Chaos and localized phases in a two-body linear kicked rotor system

Bharathi Kannan Jeevanandam (IISER, Pune), Anjali Nambudiripad (IISER, Pune), M.S Santhanam (IISER, Pune)

Tuesday, July 30, 15:00–15:30, East Hall 8

Abstract: Despite the periodic kicks, a linear kicked rotor (LKR) remains an integrable and exactly solvable model, with the kinetic energy term linear in momentum. Recent findings have revealed that spatially interacting LKRs also maintain integrability, leading to dynamical many-body localization (DMBL) in the corresponding quantum regime. This phenomenon of localization has been observed in analogous nonintegrable models, like the coupled relativistic kicked rotors. However, DMBL phases tend to emerge predominantly in scenarios where the underlying classical phase space exhibits regular or mixed characteristics.

In this study, our objective was to probe the possibility of dynamical many-body localization (DMBL) within systems exhibiting completely chaotic classical dynamics. Employing a two-body linear kicked rotor (LKR), we unveil two pivotal outcomes. Firstly, we demonstrate the induction of chaos in the integrable linear kicked rotor via interactions between the momenta of rotors, offering an analytical estimate of its Lyapunov exponent. Secondly, we delve into the quantum dynamics of this chaotic model, uncovering a diverse spectrum of phases as we manipulate kicking and interaction strengths. These phases encompass classically induced localization, dynamical localization, subdiffusive behaviour, and diffusive phenomena. Our findings shed light on how chaos in classical systems influences quantum behaviour, leading to the emergence of localization effects. It also presents a significant example of where DMBL can emerge, even amidst chaotic classical dynamics. We highlight the signatures of these phases from the perspective of entanglement production within the system. By defining an effective Hilbert space dimension, the entanglement growth rate can be understood using appropriate random matrix averages.

Quantum-classical motion of charged particles interacting with scalar fields

Shahnaz Farhat (Constructor University)

Friday, August 2, 11:00–11:30, East Hall 8

Abstract: The goal of this work is to investigate the dynamics of semi-relativistic or non-relativistic charged particles in interaction with a scalar meson field. Our main contribution is the derivation of the classical dynamics of a particle-field system as an effective equation of the quantum microscopic Nelson model, in the classical limit where the value of the Planck constant approaches zero ($\hbar \rightarrow 0$). We use a Wigner measure approach to study the transition from quantum to classical dynamics. Then, as a consequence of this transition, we establish the global well-posedness of the classical particle-field interacting system.

Gyrokinetic limit of the 2D Hartree equation in a large magnetic field

Denis Péricé, Nicolas Rougerie

Friday, August 2, 11:30–12:00, East Hall 8

Abstract: We study the dynamics of two-dimensional interacting fermions submitted to a homogeneous transverse magnetic field. We consider a large magnetic field regime, with the gap between Landau levels set to the same order as that of potential energy contributions. Within the mean-field approximation, i.e. starting from Hartree’s equation for the first reduced density matrix, we derive a drift equation for the particle density. We use vortex coherent states and the associated Husimi function to define a semi-classical density almost satisfying the limiting equation. We then deduce convergence of the density of the true Hartree solution by a Dobrushin-type stability estimate for the limiting equation.

The quantum van der Pol oscillator and quantisation of nonlinear dynamical systems

Andy Chia (Centre for Quantum Technologies, National University of Singapore), **Wai-Keong Mok** (California Institute of Technology), **Changsuk Noh** (Kyungpook National Univ.), **Leong Chuan Kwek** (Centre for Quantum Technologies, National University of Singapore)

Friday, August 2, 13:30–14:00, East Hall 8

Abstract: Questions regarding how nonlinear effects play out in quantum mechanics have long been of interest to physicists. Better known examples are quantum chaos and quantum stochastic resonance. In spite of the broad interest in generalising classical nonlinear effects to quantum mechanics, how a general dynamical system can be modelled quantum mechanically, i.e. quantised, remains unresolved.

Traditionally, quantisation has relied on the structural similarity between quantum theory and the variational formulation of classical mechanics by the likes of Lagrange and Hamilton. As such, a classical Hamiltonian function is required as input to the quantisation procedure. However, stable nonlinear phenomena typically require both dissipative and amplifying forces and are thus non-Hamiltonian. The limit cycle is one such example, for which the van der Pol oscillator has been a paradigm. Thus far, efforts to generalise Lagrangian- or Hamiltonian-based quantisation methods have only achieved limited success [Phys. Rep. 80, 1 (1981); Phys. Rep. 362, 63 (2002)].

In this work, we wish to communicate two results aimed at generalising classical nonlinear dynamics to quantum theory:

(i) Despite being a paradigm for relaxation oscillations (marked by the coexistence of two widely separated timescales in a single period), and more broadly limit cycles, the van der Pol oscillator has never been quantised fully and exactly.³ Recent attempts to quantise the van der Pol oscillator has been either unphysical [Phys. Rev. E 92, 062927 (2015)], or approximate [Phys. Rev. Res. 3, 013130 (2021)]. Here we show how, by leveraging a powerful theorem from the theory of open quantum systems, a physically valid and exact quantisation of the van der Pol oscillator can be achieved [Phys. Rev. E 102, 042213 (2020)]. At weak nonlinearities, i.e. a vanishingly-small nonlinearity parameter, the quantum van der Pol oscillator attains a circular limit cycle in quantum phase space. This illustrates the birth of the quantum van der Pol limit cycle via a quantum Hopf bifurcation, similar to the classical van der Pol oscillator. We then show how relaxation oscillations are generalised to quantum mechanics when the van der Pol oscillator becomes increasingly nonlinear.

(ii) Having successfully quantised the van der Pol oscillator, a natural question is whether the quantisation method could be adapted to quantise other nonlinear models. We show that any two-dimensional polynomial system, i.e. $(x', y') = (f(x, y), g(x, y))$, where $f(x, y)$ and $g(x, y)$ are arbitrary finite-degree polynomials in x and y , can be quantised using our approach. Several interesting models in nonlinear dynamics are of this form and are thus subject to our quantisation technique. This permits us to explore quantum analogues of various classical mechanisms, from simple bifurcations to coherence resonance in excitable systems.

Accuracy of approximations to relativistic quantum dynamics in the non-relativistic regime

Boon Leong Lan (Monash University), **Yohashama Kumaran**

³Unfortunately the physics literature has often misattributed the much simpler Stuart–Landau oscillator, which has been quantised, to van der Pol, e.g. the paper by Lee and Sadeghpour [Phys. Rev. Lett. 111, 234101 (2013)].

Friday, August 2, 14:00–14:30, East Hall 8

Abstract: To study quantum dynamics in the non-relativistic regime, the standard practice is to use non-relativistic quantum mechanics, instead of relativistic quantum mechanics based on Dirac's equation, because it is expected that the approximate non-relativistic result will always be close to the relativistic one. However, our recent published analytical work shows that the agreement between the predictions of the two quantum theories for wave packet dynamics will, in general, eventually break down, and our numerical work (for the free rotor, kicked rotor and hydrogen atom) shows that this can happen quickly. The breakdown time can also be estimated. Our results suggest that some published quantum non-relativistic (QNR) wave packet dynamics in the non-relativistic regime may not be entirely correct (in comparison to the benchmark quantum relativistic (QR) wave packet dynamics) for the whole calculation time. Alternatively, a classical relativistic (CR) ensemble could be used to approximate the QR wave packet. Our numerical study, for the free rotor (which is classically non-chaotic) and kicked rotor (which could be classically chaotic), shows the CR ensemble can approximate the QR wave packet for a much longer time than the QNR wave packet. This longer than expected accuracy of the CR ensemble, which should also occur for other quantum systems, could be verified experimentally using a universal digital quantum simulator to simulate the QR free or kicked rotor.

Algebraic outcomes to assessing entropy of recurrence microstates

João Vitor Vieira Flauzino (Federal University of Paraná), Felipe Eduardo Lopes da Cruz (Federal University of Paraná), Sergio Roberto Lopes (Federal University of Paraná), Thiago de Lima Prado (Federal University of Paraná)

Friday, August 2, 14:30–15:00, East Hall 8

Abstract: Since the inception of recurrence quantification analysis (RQA), interest has surged across various domains in leveraging recursive techniques to investigate physical systems. Notably, within RQA, the concept of recurrence microstates has emerged, delineated as small blocks derived from a recurrence plot. These microstates serve as foundational elements for computing several RQA metrics, with their probabilities defining an information entropy, termed as the entropy of recurrence microstates. This entropy can distinguish between correlated and uncorrelated stochastic and deterministic signals, being a fundamental quantifier of time series. This article presents algebraic constructions for computing the entropy of recurrence microstates and predicting its numerical behavior, aiming to provide a better quantification of the dynamic properties of physical processes. Particularly beneficial for quantifying limited datasets where exhaustive sampling is impractical, our analytical approach provides precise results when the probability distribution function is known and offers approximations for other scenarios. The theoretical framework to obtain the probabilities of recurrence microstates is general to characterize dynamical systems or even stochastic processes and facilitate optimal utilization of this quantifier with minimal memory overhead. Finally, our algebraic findings exhibit remarkable alignment with numerical simulations, affirming their reliability and applicability in real-world scenarios.

Session on Modeling with PDEs

Dynamics of the osmotic lysis of mineral protocells and its avoidance at the origins of life

Julyan Cartwright

Monday, July 29, 12:00–12:30, East Hall 4

Abstract: The osmotic rupture of a cell, its osmotic lysis or cytolysis, is a phenomenon that active biological cell volume regulation mechanisms have evolved in the cell membrane to avoid. How then, at the origin of life, did the first protocells survive prior to such active processes? The pores of alkaline hydrothermal vents in the oceans form natural nanoreactors in which osmosis across a mineral membrane plays a fundamental role. Here we discuss the dynamics of lysis and its avoidance in an abiotic system without any active mechanisms, reliant upon self-organized behaviour, similar to the first self-organized mineral membranes within which complex chemistry may have begun to evolve into metabolism. We show that such mineral

nanoreactors could function as protocells without exploding because their self-organized dynamics has a large regime in parameter space where osmotic lysis does not occur and homeostasis is possible. The beginnings of Darwinian evolution in proto-biochemistry must have involved the survival of protocells that remained within such a safe regime.

Pattern formation in multicellular systems: nonlocal models and the importance of domain geometry

Jan Rombouts

Monday, July 29, 12:30–13:00, East Hall 4

Abstract: The formation of spatial patterns is essential to the development of organisms: pattern formation leads to striking features such as the stripes on animal skins, but similar mechanisms also play a role much earlier in development. They can, for example, lead to the symmetry-breaking events that help an embryo distinguish its head from its tail. These patterns are typically the result of interactions between many different cells. These cells may change their gene expression levels as a result of signaling from other cells, or rearrange themselves in space to produce regular patterns. In recent years, such patterns have been often modeled using nonlocal PDEs of advection-diffusion type. Here, random cell motion is described by a diffusion term, and the advection of cells is due to cell-cell interactions. These cell-cell interactions are described by an interaction function, which describes how strongly cells repel or attract each other as function of the distance between them. One benefit of such a modeling approach is that one does not need to know the detailed chemical and mechanical mechanisms that produce these interactions - only the effective strength as function of distance is needed. This leads to the formulation of partial integro-differential equation models that describe the evolution of the density of different cell populations over time. Such models have been successful to describe phenomena, such as cell sorting, that were previously hard to capture using continuum models. These models are also more generally applicable: besides cellular systems, they have also been applied to study the behavior of animal herds. The majority of the literature focuses on these models on infinite or periodic domains. Yet, in real biological system the existence and shape of a boundary is often very important. In this talk, I will show the results from our studies of such models on bounded domains, obtained using different analytical and numerical techniques. This project is a close collaboration with experimentalists, and I will illustrate throughout how theory and experiment mutually inspire each other, and explain the biological questions we are trying to answer. Our main system of interest is the emergence of spatial patterns in collections of cells from a mouse embryo, that have been taken out of the embryo and then randomized. From these homogeneous conditions, patterns appear that recapitulate some of the gradients that are present in the actual mouse embryo. Moreover, it is possible to study these emerging patterns on arbitrary domain shapes using a technique called micropatterning. We use the combination of theory and experiment to approach fundamental questions about the interaction between shape, size and self-organization in developmental biology.

Thermocapillary thin films: periodic steady states and film rupture

Gabriele Brüll (Lund University), **Bastian Hilder** (Technical University of Munich (TUM)), Jonas Jansen (Lund University)

Monday, July 29, 17:00–17:30, East Hall 3

Abstract: The Bénard-Marangoni problem models a fluid film on a heated surface and features rich dynamical phenomena when the purely conductive steady state destabilizes. On the one hand, one can observe the formation of hexagonal patterns driven by a short-wave instability. On the other hand, a long-wave instability can lead to finite-time film rupture, where the surface profile touches the bottom surface. Both phenomena are driven by thermocapillary effects due to a temperature-dependent surface tension. In this talk, I discuss recent results on the existence of spatially periodic steady states in an asymptotic thermocapillary thin-film model, which can be formally derived from the full Bénard-Marangoni problem using lubrication approximation. These solutions lie on a global bifurcation branch and accumulate at a steady film-rupture solution. The main tools are analytic global bifurcation theory and the observation that the spatial dynamics formulation of the steady state problem has a Hamiltonian structure. This is joint work

with Gabriele Brüll and Jonas Jansen.

A Mathematical Model for Fluid and Solute Transport in Poroelastic Materials

Roman Cherniha (University of Nottingham)

Monday, July 29, 17:30–18:00, East Hall 3

Abstract: Fluid and solute transport in poroelastic materials (biological tissue is a typical example) is studied. Mathematical modeling of such transport is a complicated problem because the specimen volume and its form (under special conditions) might change due to swelling or shrinking and the transport processes are nonlinearly linked. The tensorial character of the variables adds also substantial complication in investigation of the fluid and solute transport in poroelastic materials (PEM). Therefore, developing and solving adequate mathematical models is an important and still open problem. Using modern foundations of the poroelastic theory (see books by Loret B and Simoes FMF (CRC Press, 2017), Coussy O. (Wiley and Sons, 2010)) the basic equations of the model were constructed. We consider PEM that consists of pores with fluid and solid material (matrix); molecules (e.g. glucose) are dissolved in fluid phase and transported through pores. We assume that fluid is incompressible and there are no internal sources/sinks. The governing equations of the model consist of continuity equations reflecting classical physical laws (the extended Darcy's law, Newtonian law). The stress-strain relationship was described by the Terzaghi effective stress tensor. The 1D version of this model was studied earlier in [Cherniha R. et al, *Symmetry* 2020 12, 396. <https://doi.org/10.3390/sym12030396>; *Commun. Nonlinear Sci. Numer. Simulat.* 2024 <https://doi.org/10.1016/j.cnsns.2024.107905>]. In this talk, a multidimensional model is presented. In the 2D space case, the model consists of six nonlinear PDEs and that is too complicated for deriving substantial analytical results. The Lie symmetry analysis is applied in order to reduce the dimensionality of the model. As a result, the radially-symmetric model is derived. An example describing fluid transport in PEM of the form of a circle is studied in detail.

This is a joint work with Joanna Stachowska-Pietka (IBIB of PAS, Warsaw) and Jacek Waniewski (IBIB of PAS, Warsaw).

Heat and mass transfer through MHD Darcy Forchheimer Casson hybrid nanofluid flow across an exponential stretching sheet

Aisha Alqahtani (Princess Nourah bint Abdulrahman University)

Monday, July 29, 18:00–18:30, East Hall 3

Abstract: Heat and mass transfer through MHD Darcy Forchheimer Casson hybrid nanofluid flow across an exponential stretching sheet The present work aims to study the energy and mass transition caused by Casson hybrid nanofluid flow across an extended stretching sheet. Thermal and velocity slip conditions, heat absorption, viscous dissipation, thermal radiation, the Darcy effect, and thermophoresis diffusion have all been considered in the study of fluid flow. Fluid flow is subjected to an inclined magnetic field to control the flow stream. Cu and Al₂O₃ NPs are added to the Casson fluid to generate a hybrid nanoliquid. The suggested model of flow dynamics is an evolving nonlinear system of PDEs, which is then reduced to a system of dimensionless ODEs using similarity proxies. The resulting set of ODEs is solved using the analytical program "HAM" for further processing. The following questions will be answered after solving the proposed model.

- How the energy and velocity profiles of hybrid nanofluid behave versus the inclusion of Cu and Al₂O₃ NPs.
- To find the analytic solution of the nonlinear system of PDEs.
- To analyze the effect of inclined magnetic field on the fluid flow across a stretching surface.
- To elaborate the significances of heat source and Darcy Forchhemier effect on the hybrid nanoliquid flow.

- How the slip conditions effect the velocity and energy propagation rate.
- To calculate the Nusselt number and skin friction.

This work aims to study the energy and mass transition caused by Casson hybrid nanofluid flow across an extended stretching sheet. Fluid flow is subjected to an inclined magnetic field to control the flow stream. Cu and Al₂O₃ NPs are added to the Casson fluid to generate a hybrid nanofluid (Blood). This model of flow dynamics is an evolving nonlinear system of PDEs, which is then reduced to a system of dimensionless ODEs using similarity proxies. The ODEs is solved using the analytical program “HAM” for further processing.

Session on Nonlinear Dynamics and Climate

Dansgaard-Oeschger Events: Challenges of Predicting Abrupt Shifts in Multiscale Systems

Bryony Hobden (University of Exeter), Paul Ritchie (University of Exeter), Peter Ashwin (University of Exeter)

Thursday, August 1, 13:30–14:00, Research V Seminar Room

Abstract: The last glacial period (110-10 kya) was highly unstable, punctuated by millennial-scale climate oscillations termed Dansgaard-Oeschger (DO) events. A DO event is characterised by a rapid increase in temperature (10-16°C in high northern latitudes) over decadal timescales, followed by cooling over extended periods of centuries to millennia. Unlike the anthropogenic climate change we are experiencing today, DO events are quite unique. While the high northern latitudes rapidly warmed, there was simultaneous cooling over large portions of the Southern Hemisphere. Understanding how Earth’s climate was able to undergo these natural ‘tipping points’ is crucial to gaining a greater knowledge of the stability of our current climate under the pressures of global warming. The precise cause of these abrupt shifts is still subject to debate. Building upon the low-dimensional model presented by Boers et al.[1], we present a stochastic nonlinear model that can induce self-sustaining oscillations through feedbacks between sea ice and the Atlantic Meridional Overturning Circulation. Here, transitions between stadial (cold) and interstadial (warm) states result from bifurcations in an underlying fast subsystem connected to sea ice extent. Changes in the subsurface water temperature in the North Atlantic modulate the duration spent in stadial conditions, with canard trajectories offering explanations for interesting behaviour for sustained stadial periods and smaller transitions that do not trigger full DO events. Our model, therefore, uses a combination of noise and bifurcation in a multiscale system and finds tipping in the fast subsystem. This approach suggests that if early warning signals are present, they may be hard to detect in the timescale of the slower subsystems.

- [1] N. Boers, M. Ghil, D.D. Rousseau, Proceedings of the National Academy of Sciences of the United States of America 115, 47 (2018)

Bifurcation analysis of a two-delay model for the Atlantic Meridional Overturning Circulation

Renzo Mancini (The University of Auckland), Bernd Krauskopf

Thursday, August 1, 14:00–14:30, Research V Seminar Room

Abstract: We perform a bifurcation study of a conceptual model for the Atlantic Meridional Overturning Circulation (AMOC) in the form of a scalar delay differential equation (DDE) with two time delays. The time delays are associated with the negative salinity feedback between the Equator and the North Pole, and the density exchange between the surface and deep water at the Pole. After rescaling, the delays are the only parameters of the model. The presented model is interesting beyond the context of the AMOC as a new type of DDE with two multiplicative delay terms, which is effectively the logistic growth equation with two independent delay times. As such, this DDE can be seen as a generalization of the Hutchinson-Wright equation, which has been introduced in two seminal papers in the rather different contexts of the distribution of primes and population dynamics.

We perform a comprehensive bifurcation study of this DDE model as a function of the two delays. In particular, the system exhibits different types of complex oscillatory behavior, which we analyze with the software package DDE-Biftool for Matlab. This enables the identification and characterization of fascinating

dynamics, including codimension-two Belyakov and Shilnikov-Hopf global bifurcations in the parameter plane of the two delays, which act as organizing centers for nearby dynamics. In particular, we discover previously unknown limiting periodic oscillations with rational ratios between the delays and the associated period, which we refer to as locking orbits. Moreover, we show where in the parameter space, nontrivial oscillations are stable and, hence, observable in the context of the AMOC. We observe that these stable solutions are organized in a repeating structure involving dynamics on invariant tori and associated bifurcations, which include torus break-up to chaotic behavior.

Analysis of a conceptual model of El Niño with implicit state-dependent delay.

Samuel Bolduc-St-Aubin (University of Auckland), **Bernd Krauskopf** (University of Auckland), **Priya Subramanian**

Thursday, August 1, 14:30–15:00, Research V Seminar Room

Abstract: Delay differential equations (DDEs) have been used in the past to model the El Niño-Southern Oscillation (ENSO) phenomenon. These conceptual models generally include a delayed negative feedback term and seasonal periodic forcing. Despite their simplicity, recent studies have demonstrated that this type of DDE models produce rich dynamical behaviour, even more so when the delays are not assumed to be constant. We study here an idealized model where the negative delayed feedback term is given by a step function, which enables us to explicitly construct solutions. Building upon prior research on this model, we extend our investigation by incorporating state-dependent delays. In particular, we demonstrate that the constant delay case serves as an accurate approximation for a range of implicitly defined state-dependent delays.

Dynamical regimes and statistics of haze and cloud droplets

Manuel Santos Gutiérrez (Weizmann Institute of Science), **Ilan Koren**, **Mickaël David Chekroun**

Thursday, August 1, 15:00–15:30, Research V Seminar Room

Abstract: Clouds play a crucial role in Earth's energy balances while constituting one of the major sources of uncertainty in climate projections. The physics of cloud droplet formation is a well-established theory, yet its application to the global description of clouds is hindered by the large number of interacting particles and their inherent nonlinear and turbulent dynamics. In this respect, finding analytical formulas for relative humidity and droplet size distributions is key to obtain useful, physics-based parameterizations of sub-grid cloud properties. We here focus on the basic equations of cloud microphysics to derive thresholds for cloud droplet activation and derive analytical expressions for droplet size distributions for clouds embedded in turbulent environments. For this, the stability and stochastic analysis of the equations is performed. These results are discussed to bridge theoretical findings with applications to develop new parametrization schemes.

Session on Nonlinear Dynamics and Oscillators

The effect of higher-order interactions on the synchronization transition

Iván León (Universidad de Cantabria/ University of Cantabria), **Riccardo Muolo** (Tokyo Institute of Technology), **Hiroya Nakao** (Tokyo Institute of Technology), **Shigefumi Hata** (Kagoshima University)

Monday, July 29, 13:30–14:00, IRC Seminar Room 1

Abstract: Synchronization is a crucial phenomenon in many sciences, from biology to engineering. Although the transition to synchrony when units are coupled through two-body interactions has been widely studied, only recently some attention has been paid to the effect of higher-order interactions, i.e. multi-body interaction, on the dynamics of oscillator's ensembles. In this talk, we present the simplest model of identical coupled phase oscillators subject to two-body and three-body interactions with permutation symmetry. This model is simple enough to allow us to perform a complete analysis of its dynamics, unraveling anomalous transitions to synchrony induced by higher-order interaction. For instance, multistability of synchronization, incoherent, and two-cluster states, or a transition to synchrony through slow-switching and clustering

was observed. Finally, we show similar scenarios are found even if small heterogeneity in the oscillators' frequency is included.

Minimization of Joule losses in ReRAM switching

Valeriy Slipko (University of Opole), Yuriy Pershin (University of South Carolina)

Monday, July 29, 14:00–14:30, IRC Seminar Room 1

Abstract: Resistive random access memory (ReRAM) cells, also known as memristive devices, represent a highly promising type of non-volatile memory technologies that have been extensively researched over the last few decades. Memristive devices are often modeled as a dynamic system driven by voltage or current. Due to their resistive characteristics, any basic operation (such as switching) involving memristors inherently incurs some energy dissipation. While memristor-based neural networks have shown remarkable energy efficiency for artificial intelligence applications [1], further enhancements could be achieved through optimizing the switching process.

Our study has explored strategies to optimize the switching behavior of memristive devices [2]. Specifically, we have explored conditions for minimizing Joule losses during the switching of ideal memristors within a specified time frame. This optimization conundrum can be tackled by the classical method of variations. Likewise, we have addressed the optimization problem of minimizing both Joule losses and switching time for ideal memristors under a linear cost function scenario. We have derived straightforward analytical expressions for the minimum Joule losses and switching time in the case of an ideal linear memristor. Our findings suggest that maintaining constant power throughout the switching process is the optimal strategy. By comparing Joule losses between optimal switching and constant voltage or current switching, we have demonstrated to 8/9 reduction in energy dissipation with the optimal control strategy.

For general memristive systems, we have formulated the essential set of equations for minimizing Joule losses, and these equations have been analytically solved for a practical threshold-type voltage-controlled memristive device model. This analysis has enabled us to pinpoint the optimal switching protocol, which has the potential for significantly reducing Joule losses compared to constant voltage or current switching methods.

In practical applications, additional considerations must be taken into account when switching memristive devices. In particular, a crucial factor is the current constraint, where the current flow through the device is restricted by a compliance current. By leveraging Pontryagin's maximum principle, we have explored the impact of current compliance on the optimal switching behavior of an ideal memristor. Our study indicates that a favorable strategy involves a combination of constant power and maximum current solutions for a certain duration.

We believe that the practical implications of our research are substantial, as the implementation of optimal switching protocols could greatly enhance the efficiency of memristor-based networks tailored for in-memory computing and neuromorphic computing applications.

- [1] Jebali, F., Majumdar, A., Turck, C. et al. Powering AI at the edge: A robust, memristor-based binarized neural network with near-memory computing and miniaturized solar cell. *Nature Communications* 15, 741 (2024).
- [2] V. A. Slipko and Y.V. Pershin, 'Reduction of Joule Losses in Memristive Switching Using Optimal Control', (submitted for publication), 2024.

Understanding dynamics of the model of Belousov–Zhabotinsky (BZ) reaction

Stevan Maćešić (University of Belgrade, Faculty of Physical Chemistry, Belgrade, Serbia), Ana Ivanović-Šašić (University of Belgrade, Institute of Chemistry, Technology and Metallurgy), Željko Čupić (University of Belgrade, Institute of Chemistry, Technology and Metallurgy,)

Monday, July 29, 14:30–15:00, IRC Seminar Room 1

Abstract: The Belousov–Zhabotinsky (BZ) [1,2] reaction is one of the most famous examples of oscillatory chemical reaction. This reaction represents decomposition of organic acid by bromate ions in acid solutions containing metallic catalysts. Extensive experimental studies carried on this reaction revealed it is capable to produce a very rich variety of dynamics ranging from simple oscillations to mixed mode oscillations and

chaos. Furthermore, experimental studies showed that this system is also capable to produce very complex spatiotemporal phenomena under different experimental environments and conditions. As result, Belousov-Zhabotinsky oscillatory reactions have found diverse and impactful applications in fields such as artificial intelligence, materials science, self-oscillating gels, polymer science, material engineering, and hydrogels. [3,4] One of the primary difficulties associated with the application of the Belousov–Zhabotinsky reaction is to find an efficient method for controlling and optimizing its dynamics for specific purposes. The regions where the required dynamics can be achieved are often narrow and difficult to identify, necessitating extensive experimental work. Modelling is crucial for understanding the behaviour of BZ reaction. Modelling has the potential to not only deepen our understanding of this complex chemical systems but also pave the way for practical applications in above mentioned areas. Having a realistic and reliable model can significantly aid in this task. Therefore, this paper examines a model [5] that has demonstrated strong agreement with experimental results so far. A crucial step toward better understanding the characteristics of this model and identifying measures for improving its optimization involves stability analysis. Performing stability analysis can be particularly challenging in cases involving models with a large number of reaction steps and intermediate species, as in our considered model. Therefore, specialized methods such as stoichiometric network analysis (SNA) [6,7] are required to efficiently carry out this task. SNA was utilized to efficiently analyse the model's behaviour by identifying different types of dynamics it can simulate, including conditions under which these dynamics can occur. Furthermore, reaction steps and intermediate species essential for the existence of these dynamics were found.

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[1] B. Belousov, *Sbornik Referatov Po Radiatsionnoi Meditsine Za* (1958) 145–149.

[2] A.M. Zhabotinsky, *Biofizika* 9 (1964) 306–311.

[3] M.-A. Tsompanas, C. Fullarton, A. Adamatzky, *Sensors and Actuators B: Chemical* 295 (2019) 194–203.

[4] (n.d.).

[5] S.M. Blagojević, S.R. Anić, Ž.D. Čupić, N.D. Pejić, L.Z. Kolar-Anić, *Phys. Chem. Chem. Phys.* 10 (2008) 6658–6664.

[6] B.L. Clarke, in: I. Prigogine, S. Rice (Eds.), *Advances in Chemical Physics*, John Wiley & Sons, Ltd, 1980, pp. 1–215.

[7] B.L. Clarke, *Cell Biophysics* 12 (1988) 237–253.

Periodic perturbation induces amplitude change and oscillation quenching in a bistable oscillator

Yusuke Kato (The University of Tokyo), **Hiroshi Kori** (The University of Tokyo)

Monday, July 29, 15:00–15:30, IRC Seminar Room 1

Abstract: There is a wide variety of oscillatory phenomena in nature and society. Among them, both normal and abnormal oscillations exist. For example, a circadian rhythm is a physiological oscillation essential for good health, whereas cardiac arrhythmias are pathologic rhythms that can harm our lives. It is also known that several abnormal oscillations (e.g., epileptic seizures) coexist with the normal constant equilibrium. Therefore, it is important to establish a theoretical framework for state-transition methods from a limit cycle to a fixed point.

In this study, we investigate the amplitude change of an oscillator by periodic forces. Although many previous studies focus on the frequency resonance of a monostable (i.e., either a fixed point or a limit cycle is stable) nonlinear oscillator near Hopf bifurcation, few studies address a periodically driven bistable (i.e., both a fixed point and a limit cycle are stable) oscillator. We use a bistable Stuart-Landau oscillator and add periodic forces. We find that, in a certain frequency range of external force, the frequency of the oscillator synchronizes with the external periodic force. An averaging approximation reveals that these synchronization transitions are induced by a SNIC bifurcation and a morphological change of limit cycle when crossing the phase singular point. In addition, a Hopf bifurcation occurs during the frequency range of synchronization, which makes the oscillator move over a wide area in the phase plane and fall within the basin of a stable fixed point of the original bistable oscillator without periodic forces. We also analyze the effect of multiplicative periodic forces and find that oscillation quenching can occur with a certain frequency

and strength of the external force.

Our study investigates numerically and analytically the novel nonlinear phenomena and bifurcations observed in a periodically-driven bistable oscillator. The study findings also suggest that periodic perturbations can annihilate an oscillation, contributing to the theoretical basis for the control of oscillations.

Session on Nonlinear Dynamics and Stability

Does an intermittent dynamical system remain chaotic after drilling in a hole?

Samuel Brevitt (Queen Mary University of London), Rainer Klages (Queen Mary University of London)
Monday, July 29, 11:00–11:30, IRC Seminar Room 2

Abstract: Chaotic dynamical systems may be characterised by a positive Lyapunov exponent, which measures the exponential rate of separation of nearby trajectories. However in a wide range of so-called weakly chaotic systems, the separation of nearby trajectories is sub-exponential, for example stretched exponential, in time; and therefore in such cases the Lyapunov exponent vanishes. When a hole is introduced in chaotic systems, the Lyapunov exponent on the system's fractal repeller can be related to the generation of entropy and the escape rate from the system via the escape rate formalism, but no suitable generalisation exists to weakly chaotic systems. In this work we show that in a paradigmatic one-dimensional weakly chaotic iterated map, the Pomeau-Manneville map, the generation of generalised Lyapunov stretching is completely suppressed in the presence of a hole. These results are based on numerical evidence and a corresponding stochastic model. The correspondence between map and model is tested via a related partially absorbing map. Our findings are shown to be in line with known mathematical results concerning the collapse of the system's density as it evolves in time. Finally we conclude that, as a result, no suitable generalisation of the escape rate formalism to weakly chaotic systems can exist.

Non-autonomous standard nontwist map

Marcos Vinicius de Moraes (Universidade de São Paulo (USP)), Yves Elskens (Aix-Marseille Université), Iberê Luiz Caldas (Universidade de São Paulo (USP))
Monday, July 29, 11:30–12:00, IRC Seminar Room 2

Abstract: We investigate the standard nontwist map (SNM) with time-dependent parameters. This map is a two-parameter area-preserving system, written as

$$y(n+1) = y(n) - b \sin[2\pi x(n)], \quad x(n+1) = x(n) + a[1 - y(n+1)/2].$$

Area-preserving nontwist maps locally violate the twist condition, giving rise to a special curve called shearless curve. This kind of curve appears in different physical contexts, for example in fusion plasmas, where a nonmonotonic current profile inside the plasma creates a shearless barrier that improves the plasma confinement.

Following [1,2], we consider the SNM with a time-dependent parameter such as $b(n) = b(0) + nv$ and we investigate the evolution of an ensemble of points on the phase space that initially lies on the shearless invariant curve for fixed parameter $b(0)$, called shearless snapshot torus. Differently from the SNM with constant parameters – where we can see different scenarios of collision/annihilation of periodic orbits leading to global transport, depending on the region in the parameter space – for the SNM with time-dependent parameters the route to chaos is not only related to the two parameters (a, b) , but also to the scenario of evolution of parameter $b(n)$.

We characterize the transition to chaos of the shearless snapshot torus by calculating the average distance between points initially very close to each other. In the beginning this distance hardly changes, but the distance increases exponentially when the process of breaking the snapshot torus starts. After this process, the dynamics of the ensemble is chaotic, leading to a saturated distance. From this calculation we obtain two scalings with respect to the parameter velocity v : the critical parameter for which the growth rate is saturated and the growth rate during the breaking up process.

Additionally, we studied transport properties of the map with time-dependent parameters, identifying regions in the phase space with different diffusive properties. Finally, we show the presence of anomalous transport due to the time-dependence of the parameter and we numerically find the scalings between the diffusion coefficients and the parameter velocity.

- [1] D. Jánosi and T. Tél, “Chaos in Hamiltonian systems subjected to parameter drift”, *Chaos AIP*, vol. 29, p. 121105 (2019).
- [2] D. Jánosi and T. Tél, “Chaos in conservative discrete-time systems subjected to parameter drift”, *Chaos AIP*, vol. 31, p. 033142 (2021).

Session on Nonlinear Dynamics and Waves

Bifurcation of the dynamically induced conformation for the bead-spring Hamiltonian system with fast and slow time scales.

Tatsuo Yanagita (Osaka Electro-Communication University), Yasushi Shimizu (Ritsumeikan University), Tetsuro Konishi (Chubu University), Yoshiyuki Y. Yamaguchi (Kyoto University), Mikito Toda (Nara Women’s University)

Monday, July 29, 13:30–14:00, IRC Seminar Room 2

Abstract: We study the dynamics of bead-spring Hamiltonian systems, where the beads represent the atoms or groups of atoms in a molecule and are connected by springs representing the bonds between them. If the springs are sufficiently stiff, the system remains in a quasi-equilibrium state for a long time, in which the kinetic energy of the beads deviates from the usual equipartition of the system. We also show that the conformation of the molecule depends on the excited vibrational mode. Changing the vibrational mode leads to a bifurcation between different conformations. These conformational changes can be important for understanding the function of molecules. (*Physical Review E*, **105**, pp. 064201–064215 (2022), *Physical Review E*, **104**, pp. 034209–034222 (2021))

Temporal localized states beyond the uniform field limit in injected Kerr resonators

Thomas Seidel, Julien Javaloyes, Svetlana Gurevich

Monday, July 29, 14:00–14:30, IRC Seminar Room 2

Abstract: We are interested in the dynamics of temporal localized states in an injected Kerr-Gires-Tournois interferometer in the normal dispersion regime and operated far from the uniform field limit, characterized by small losses and injection values. Our theoretical model is based upon time-delayed algebraic equations whose uniform validity allows for a systematic exploration of the parameter space. While in the long delay limit and in the weakly nonlinear regime the cavity response and the dynamics can be described by the Lugiato-Lefever equation, such a framework breaks down in the regimes of large detuning and injection. By using a combination of direct numerical simulations and path continuation techniques, we explore these new regimes that depart from the uniform field limit and find short, high-intensity localized states which live on a stable homogeneous background, resembling the scenario of the formation of the localized states typical for the Swift-Hohenberg equation. In this regime, the previously stable upper homogeneous steady state solution becomes unstable with respect to a Turing bifurcation. Further, we analyze the transition between the regimes of the Lugiato-Lefever equation and the Swift-Hohenberg equation.

Multistable Kuramoto splay states in a crystal of mode-locked laser pulses

Thomas Seidel, Svetlana Gurevich, Julien Javaloyes

Monday, July 29, 14:30–15:00, IRC Seminar Room 2

Abstract: In this contribution we shall answer a seemingly obvious question: Are the pulses in a harmonic mode-locked state coherent with each other? Similarly, one may ask if the harmonic mode-locked solution is unique. The answer to these questions is actually far from trivial if one considers a unidirectional ring laser or a Vertical External-Cavity Surface-Emitting Laser. In these configurations, the pulses never cross each other within the active material: How could they then exchange information regarding their relative

phase? Our theoretical analysis is based upon the nonlinear Haus master partial differential equation as well as the time-delay differential equation model. We demonstrate the existence of a multiplicity of co-existing harmonically mode-locked solutions that we link to the splay phases of the Kuramoto model with short range interactions. These splay states are multistable and the laser may wander between them under the influence of stochastic forces. Consequently, the many pulses circulating in the cavity are not necessarily coherent with each other. Further, we show that this partially disordered state for the phase of the optical field features regular train of pulses in the field intensity, a state that we term an incoherent crystal of optical pulses. We provide evidence that the notion of coherence should be interpreted by comparing the duration of the measurement time with the Kramers' escape time of each splay state.

Phase-to-trigger wave transition: The spatial coordination of mitosis

Owen Puls, **Daniel Ruiz-Reynés** (Laboratory of Dynamics in Biological Systems (DiBS) and Institute for Cross-Disciplinary Physics and Complex Systems (IFISC)), Franco Tavella, Minjun Jin, Yeonghoon Kim, Lendert Gelens, Qiong Yang

Monday, July 29, 15:00–15:30, IRC Seminar Room 2

Abstract: The activity of the main cell cycle regulator cyclin B-Cdk1 rises and falls throughout cell cycle progression, a cell-autonomous process known as mitotic oscillations. These oscillators can synchronize when spatially coupled, providing a crucial foundation for rapid synchronous divisions in large early embryos of the order of millimeters of size, like *Drosophila* and *Xenopus*. While diffusion alone cannot achieve such long-range coordination, recent studies have proposed two types of mitotic waves, phase and trigger waves, to explain the phenomena. Yet, it remains unclear how the spatiotemporal dynamics involved in the formation of these waves facilitate efficient spatial coordination of mitosis. In this work, we use *Xenopus laevis* egg extracts to reconstitute mitotic waves in vitro visualized with a novel Cdk1 FRET sensor. We observe a transition from phase waves to a trigger wave regime in an initially homogeneous cytosol. Adding nuclei accelerates such transition. Moreover, the system transitions almost immediately to this regime when externally driven from the boundary by metaphase-arrested extracts. Employing computational modeling, we pinpoint how wave nature, including speed-period-relation, depends on transient dynamics and oscillator properties, suggesting that phase waves appear transiently due to the time required for trigger waves to entrain the system and that spatial heterogeneity promotes entrainment. Therefore, we show that both waves belong to a single biological process leading to the formation of trigger waves that coordinate the cell cycle over long distances.

Chaotic dynamics in optical waveguide trimers: from active coupling to PT-symmetry

Ioanna Chitzanidi (FORTH, Institute of Electronic Structure and Laser)

Monday, July 29, 17:00–17:30, IRC Seminar Room 2

Abstract: In the context of non-Hermitian photonics we consider two cases of a nonlinear optical trimer: one with three lossy waveguides placed in a gainy medium (active trimer) and the PT-symmetric case where the gain and loss in the edge waveguides are exactly balanced. The active trimer exhibits stable stationary and oscillatory regimes in a wide range of the gain-loss parameter space. Chaotic dynamics through period-doubling is confirmed via Lyapunov exponent measurements and the underlying bifurcation structure is found via continuation. In contrast to the active trimer, the PT-symmetric system does not treat the edge waveguides equally. In the limit of zero gain/loss the total power of the system is conserved and for certain excitation scenarios Hamiltonian chaos is observed. We are interested in the effect of finite gain/loss on this chaotic dynamics and attempt to understand if the presence of exceptional points in the underlying linear system has an effect in the overall behavior of the system.

Breather solutions for semilinear wave equations

Wolfgang Reichel (Karlsruhe Institute of Technology (KIT))

Monday, July 29, 17:30–18:00, IRC Seminar Room 2

Abstract: We consider semilinear wave equations on the real line, a periodic graph or on \mathbb{R}^n . Due to non-constant coefficients the linear wave operator has a space-dependent finite speed of propagation. By a variational method we establish the existence of spatially localized, time-periodic solutions. The method relies on a careful design of the coefficients to allow spectral gaps in the linear wave operator. The spectral gaps are constructed and analyzed by ODE methods.

Stability and bifurcation analysis of a modified diffusive and delayed Brusselator model

Szilvia György (Eötvös Loránd University), Sándor Kovács (Eötvös Loránd University)

Monday, July 29, 18:00–18:30, IRC Seminar Room 2

Abstract: The classical diffusive Brusselator model has been extensively discussed even under the presence of certain types of discrete time-delays. However, models assuming delayed feedback have only been studied under certain highly restrictive conditions due to the computational complexity. In this talk, we study the stability of the unique equilibrium solution of the model in a more general case, leaving the mentioned conditions out of consideration. When this type of delay is introduced, the original system of differential equations (system without delay) also substantially changes, so that 4 different models need to be investigated and compared. First, the ordinary system is considered in the absence of both delay and diffusion, and then we examine whether the stability of the equilibrium solution changes when only delay or only diffusion is assumed. The simultaneous effects of delay and diffusion are then analysed. In the investigations the focus is mainly on the existence of Hopf bifurcation, and in case of models assuming diffusion on the occurrence of Turing bifurcation also. Finally, numerical simulations are carried out to support the theoretical findings and to illustrate the results.

Stable multi-pulses for the two-mode forced Lugiato-Lefever equation

Lukas Bengel (Karlsruhe Institute of Technology (KIT)), Björn de Rijk (Karlsruhe Institute of Technology (KIT))

Tuesday, July 30, 11:00–11:30, IRC Seminar Room 2

Abstract: We consider optical Kerr frequency combs generated in a nonlinear two-mode forced microresonator. In physical set-ups it has been observed that two-mode forcing increases several performance metrics and leads to better synchronization of the radio-frequency oscillator with the comb. Kerr frequency combs are modeled by stationary spatially localized solutions of the Lugiato-Lefever equation (LLE)

$$iu_t = -du_{xx} + icu_x + (\zeta - i)u - |u|^2u + if(x),$$

which is a damped, detuned, and driven nonlinear Schrödinger equation. We show by using the Lyapunov-Schmidt reduction method that two-mode forcing leads to localized 1-pulse solutions with oscillatory tails upon bifurcating from the bright NLS soliton. Adopting a spatial dynamics approach we then construct multi-pulses resembling well-separated multiple copies of the 1-pulses and we study their stability. We observe that the periodic background state traps the 1- and multi-pulses yielding better stability properties as opposed to 1-mode forcing. Numerical simulations with `pde2path` complement our analytical findings.

Experimenting with and analyzing reaction-diffusion waves on physicochemical fractal media.

Ivan Proskurkin (Immanuel Kant Baltic Federal University), Eugene Postnikov (Kursk State University), Iliya Malphanov (Immanuel Kant Baltic Federal University), Dmitry Safonov (Immanuel Kant Baltic Federal University), **Anastasia Lavrova** (Saint-Petersburg State Research Institute of Phthisiopulmonology of the Ministry of Healthcare of the Russian Federation)

Tuesday, July 30, 11:30–12:00, IRC Seminar Room 2

Abstract: Reaction-diffusion processes on fractal structures belong to the prototypic models of complex spatiotemporal dynamics in disordered media. In addition to mathematical issues of revealing features exhibiting anomalous transport, understanding the underlying dynamics is of high interest to physical applications, which vary from the soft matter for electronic devices to geophysics. At the same time, the

vast majority of studies in this field deal with idealized mathematical models confirmed only by computer simulations. On the contrary, the present work is primarily aimed at the search for the possibility of reproducing and exploring the spreading nonlinear dynamics on fractal objects in a real physicochemical experiment. For our study, we investigated travelling waves formed by the Belousov-Zhabotinsky (BZ) reaction in an excitable regime speeding in a thin (less than a hundred microns) layer of a gel containing the required BZ reagents. To create the required spatial structure, the process of photopolymerization was applied to a thin layer of pregel solution under the action of a strong blue LED-produced illumination with a set of masks of the target 2D topology with subsequent washing out low-molecular-weight compounds from non-illuminated regions. As a result, the spatial configuration of the explored gel figures represents a sequence of prefractals tending to the Sierpinski gasket. Starting from the base triangle, four subsequent self-similar iterations were implemented. Chemical wave excitation in the gel was achieved by touching one of the outer triangle's corners with a thin silver wire. Video records of the emerging travelling waves allowed quantification of the spatiotemporal spreading speed dynamics for each of the prefractals and, respectively, to consider the obtained dependence as the steps of the real-space renormalization process. The latter is discussed in comparison with the analytical theory of the nonlinear spreading process on the Sierpinski gasket. Besides, the effect of the wave front straightening was observed and analysed mathematically. The last fact was confirmed as related to the interdependence between the spreading speed and the effective catalyst concentration per front's length (e.g. the waves move faster in areas with lower catalyst concentrations) making it possible to use more general heterogeneous gel figures as controlling elements with prescribed desired properties for systems of chemical soft computing.

Time-periodic solutions to Maxwell's equations

Sebastian Ohrem (Karlsruhe Institute of Technology (KIT))

Tuesday, July 30, 12:00–12:30, IRC Seminar Room 2

Abstract: We consider Maxwell's equations without charges and currents

$$\begin{aligned} \nabla \cdot \mathbf{D} &= 0, & \nabla \times \mathbf{E} &= -\mathbf{B}_t, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{H} &= \mathbf{D}_t \end{aligned}$$

for Kerr-type optical materials, which can be modelled by the material relations

$$\mathbf{B} = \mu_0 \mathbf{H}, \quad \mathbf{D} = \epsilon_0 (\mathbf{E} + \mathbf{P}(\mathbf{E})), \quad \mathbf{P}(\mathbf{E}) = \chi_1(\mathbf{x})\mathbf{E} + \chi_3(\mathbf{x})|\mathbf{E}|^2 \mathbf{E}.$$

We show existence of time-periodic, real-valued, traveling, localized solutions $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}$ to Maxwell's equations, called breathers, under the assumption that the material parameters χ_1, χ_3 have very particular shapes.

We also consider material relations where the polarization $\mathbf{P}(\mathbf{E})$ depends on the past of the electric field \mathbf{E} .

Session on Nonlinear Dynamics in Biology

The spatiotemporal dynamics of coral reefs

Miguel Álvarez (IFISC (Institute for Cross-Disciplinary Physics and Complex Systems)), Damià Gomila, Manuel A. Matias

Thursday, August 1, 13:30–14:00, IRC Seminar Room 2

Abstract: In this study, we investigate the bifurcation structure underlying a mathematical model that reproduces the essential shapes and characteristics of coral reefs. We identify a region in the parameter space where stable traveling pulses emerge as solutions, closely resembling the formation and evolution of coral reefs. By characterizing the associated bifurcations, we establish a clear link between ecological influences and the observed spatial structures in these ecosystems. This systematic analysis significantly enhances our qualitative understanding of coral reef dynamics.

Phantastic phase space objects and where to find them: ghost channels and cycles in dynamical systems

Daniel Koch (Max Planck institute for neurobiology of behavior - caesar)

Thursday, August 1, 14:00–14:30, IRC Seminar Room 2

Abstract: Fixed points and attractors are central concepts in the mathematical description of natural systems. Quasi-stable dynamics or long transients followed by sequential rapid transitions, for example, are typical for many real-world systems and are often described via saddle-based phase space objects such as heteroclinic channels/cycles or as limit cycles of slow-fast systems. Recently, we have provided an alternative description that does not rely on fixed points or stable attractors, but in which the quasi-stable sequential dynamics result from non-hyperbolic and Lyapunov-unstable attracting sets of dynamical ghosts that can be connected to form ghost channels or ghost cycles. We find that these novel phase space objects are an emergent property of a broad class of systems (e.g. in models of gene-regulatory and neuronal networks, climate and other systems) and that their dynamics are both more robust to noise than e.g. heteroclinic objects, while at the same provide a surprising flexibility in response to external forcing. In this talk, I present some of the basic dynamic features of ghost channels and cycles in comparison to more conventional descriptions and discuss the implications of our findings for several applications (including biological computations and tipping cascades in climate or ecosystems).

Impossible ecologies: interaction networks and stability of coexistence in ecological communities

Yu Meng (Carl von Ossietzky University Oldenburg), Carl D. Modes, Pierre A. Haas

Thursday, August 1, 14:30–15:00, IRC Seminar Room 2

Abstract: Does an ecological community allow stable coexistence? In particular, what is the interplay between stability of coexistence and the network of competitive, mutualistic, or predator-prey interactions between the species of the community? These are fundamental questions of theoretical ecology, yet meaningful analytical progress is in most cases impossible beyond two-species communities. In this talk, I will therefore show how we addressed this problem statistically: For all non-trivial networks of interaction types of $N \leq 5$ species, we sampled Lotka–Volterra model parameters randomly and thus computed the probability of steady-state coexistence being stable and feasible with Lotka–Volterra dynamics. Surprisingly, our analysis reveals “impossible ecologies”, very rare non-trivial networks of interaction types that do not allow stable and feasible steady-state coexistence. I will classify these impossible ecologies, and then prove, somewhat conversely, that any non-trivial ecology that has a possible subecology is itself possible. This theorem highlights the “irreducible ecologies” that allow stable and feasible steady-state coexistence, but do not contain a possible subecology. I will conclude by showing the classification of all irreducible ecologies of $N \leq 5$ species. Strikingly, this indicates that the proportion of non-trivial ecologies that are irreducible decreases exponentially with the number N of species. Our results thus suggest that interaction networks and stability of coexistence are linked crucially by the very small subset of ecologies that are irreducible.

Periodicity in reaction-diffusion systems using examples from the biology

Sándor Kovács (Eötvös Loránd University), Szilvia György (Eötvös Loránd University)

Thursday, August 1, 15:00–15:30, IRC Seminar Room 2

Abstract: One of the fascinating properties of reaction-diffusion systems is the variety of special types of solutions they exhibit. Certain systems of this type can have for example travelling waves or rotating wave solutions, furthermore via bifurcation analysis one can show new class of solutions. This talk is about the technique how to prove time periodic oscillations when the kinetic system has it or doesn't have it. We demonstrate the techniques using the examples of a model to control the wild mosquito with sterile release and a system modelling disease propagation.

A physics-based model of swarming jellyfish

Erik Gengel (Tel Aviv University), Zafrir Kuplik (The Steinhardt Museum of Natural History, Tel Aviv University), Dror Angel (Haifa University), Eyal Heifetz (Tel Aviv University)

Thursday, August 1, 15:30–16:00, IRC Seminar Room 2

Abstract: Jellyfish are expected to become a dominating factor in many marine ecosystems because they thrive under the conditions emerging in a transformed ocean due to climate change. They are extremely durable as a species and can develop massive swarms that put a burden on economy, society and ecosystem functioning. Our particular model organism is the jellyfish *Rhopilema nomadica* which is regularly ranked among the most severe invasive species in the Mediterranean. It is known to form swarms of millions of individuals during the summer period and forces desalination and power plants to shut down. The emergence of these swarms is still not well understood, despite decades of extended observational and theoretical research, in the region and world wide.

We have developed the first of its kind active-matter model to describe the emergence of different swarming patterns for this species and jellyfish in a broader sense. The model is based on an active-Brownian particle simulation coupled to a fluid dynamics simulation. Jellyfish in the model are considered to be swarmalators, oscillator that are moving. This reflects the fact that many jellyfish species feature a pulsating bell. Based upon this oscillation, propulsion is achieved.

In particular, we are able to regenerate well designed experimental results for *R. nomadica*. This jellyfish is known to swim faster against the mean current. This is surprising because Galilean invariance is lacking for this and, in fact other jellyfish species as well. Moreover, our model is able to address the intriguing interplay of external drivers and behavioral responses.

Obviously, jellyfish are transported with the flow. However, they can make decisions based on a plethora of stimuli, such as flow speed, turbulence, salinity, temperature, insulation, prey distribution and other chemical or kinematic drivers. Thus, swarms of jellyfish are a natural realization of intelligent active matter.

A particular goal of our research is to separate genuine jellyfish behaviour from reactions resulting from an artificial environment. This is a particularly important problem in jellyfish research: Observations of jellyfish in the ocean are expensive and can not be done at times when it is most interesting to observe (e.g. during storms). Thus tank-based experiments are used instead. However, jellyfish react to a confinement and currents which are naturally present in tanks to provide fresh water.

Based on our theoretical findings we can give at hand better parameter constraints for jellyfish observations. Moreover, we elucidate how jellyfish use the environmental clues to their advantage while still being subject mainly to mean-flow transport.

Session on Nonlinear Dynamics in Networks

Reluctant synchrony breaking in hypernetworks

Sören von der Gracht (Paderborn University), Eddie Nijholt, Bob Rink

Monday, July 29, 11:00–11:30, East Hall 4

Abstract: Interconnected real-world systems oftentimes contain non-pairwise interactions between agents referred to as higher order interactions. Countless works in recent years have highlighted how this structural feature crucially shapes the collective behavior. The collective higher order interaction structure can be encoded by means of a hypergraph or hypernetwork. This talk will focus on dynamics of such hypernetworks. We define a class of maps that respects the higher order interaction structure, so-called admissible maps, and investigate how robust patterns of synchrony can be classified. Interestingly, these are only defined by higher degree polynomial admissible maps. This is in stark contrast to classical diadic networks where robust synchrony is governed by the linear admissible maps. Therefore, cluster synchronization on hypernetworks is truly a higher order, i.e., nonlinear effect. This observation has dramatic implications for the dynamics causing “reluctant synchrony breaking” when bifurcating solutions lie close to a non-robust synchrony space. The goal of this talk is to elucidate this staggering phenomenon.

Network Attractors in Systems close to Criticality: Computational Capabilities and Working Memory Analysis

Pezhman Ebrahimzadeh (Forschungszentrum Jülich GmbH), Michael Schiek (Forschungszentrum Jülich GmbH)

Monday, July 29, 11:30–12:00, East Hall 4

Abstract: Considering neural networks as dynamical systems allows for a rigorous analysis of some aspects of their dynamics regarding computational capabilities, the working memory, and their relation to spike patterns. Following this approach, it was stated already more than 50 years ago that biological neural networks operate near a phase transition point for optimal information processing [1]. Although this statement recently underwent a controversy discussion, experimental and numerical data still strongly support this hypothesis [2]. A continuation of this approach is the concept of the attractor network describing one of the dynamical mechanisms in which the network maintains persistent activity via creating locally stable attractor states. Based on the topological analysis of the state space of the input-driven system, computational capabilities of the attractor states and link to working memory can be analyzed. In this work, our study is based on networks of two different entities, namely Quadratic-Integrate-Fire (QIF) neurons and identical oscillators. Similar to pendula the oscillator differential equation include a term describing a constant but asymmetric external force which allows for oscillatory (analog to quiescence neurons) and rotating (analog to firing neurons) dynamics. For both systems we introduce an order parameter (complexity parameter) determining the resulting dynamical complexity. In the first system we analyze the interplay mechanism between synaptic connections as means of creation [and annihilation] of different dynamical regimes and their respective attractor states considering the QIF neurons coupled via both electrical and chemical synapses. For small size networks, the effect of [long ranged] chemical synapses are similar to that of the electrical ones and can be averaged. The interplay between excitatory and inhibitory neurons creates specific neural activity patterns that are network attractors. Regarding the ratio of excitatory to inhibitory synapses $\alpha = \mu^- / \mu^+$ as a complexity parameter we observe the transition of the system from ordered dynamics (e.g. clustered bump attractors) over chaotic transient attractors to ordered dynamics again. Tuning the network near the border of criticality ($\alpha \sim 0.4$) the network is inherently able to distinguish different inputs without any specific learning. In the second network of identical coupled pendula, we introduce the phase delay in the coupling as complexity parameter [3]. Already with only three coupled oscillators the system shows a rich variety of synchronization states including desynchronized, synchronized, chimera, and switching chaotic chimera states coexisting in a large region of the parameter space. The next step beyond feature/pattern recognition can be formulated as sequence generation using the encoded/learned patterns [4]. In the network's dynamical space, the sequence generation is mapped as attractor hopping. Our results underline the advantages of considering neural networks as dynamical systems, which e.g. allows to push the system into a specific dynamical region before training and thus improving its computational power.

[1] Wilson, H. R., & Cowan, J. D.; Biophysical journal, 12(1), 1-24, (1972).

[2] Beggs, J. M.; Frontiers in computational neuroscience, 16, 703865, (2022).

[3] Ebrahimzadeh, P., et al; Chaos: An Interdisciplinary Journal of Nonlinear Science, 32.10 (2022).

[4] Bouhadjar, Y., et al.; PLOS Computational Biology, 18.6 (2022)

Bifurcations and hysteresis of a nonlinear transport model on small networks

Hajime Koike (Tokyo Institute of Technology), Hideki Takayasu (Tokyo Institute of Technology), Misako Takayasu (Tokyo Institute of Technology)

Monday, July 29, 17:00–17:30, East Hall 4

Abstract: Population movements between cities, commuting flows inside cities, traffics inside cities, international trades between countries, and business transactions between companies can be understood as nonlinear transport phenomena on networks. In these cases, the flow on each edge between nodes typically depends nonlinearly on their sizes, with larger flows occurring towards larger nodes, exemplifying the "rich get richer" phenomenon.

Mathematical models of such phenomenon have been studied recently, where each node changes its distribution to connected nodes over time based on its own state and the state of adjacent nodes. We have studied one such model, called gravity interaction model, which incorporates dissipation proportional to the current amount of material and constant injection as interactions with the external environment.

The steady-state solutions of the model exhibit bifurcation phenomena due to the nonlinearity of transport, called diffusion-localization transition. Its dependence on the network structure has been investigated [Tamura, Takayasu, Takayasu, Sci Rep 8(5517):2018]. However, results are limited to specific networks such

as ring, fully connected graph, regular ring lattices, Bethe lattices, star graph, and Japanese business network [Tamura, Takayasu, Takayasu, Sci Rep 8(5517):2018; Koike, Takayasu, Takayasu, J Stat Phys 186(44): 2022], with behavior at small degrees of freedom not well understood systematically.

We investigated bifurcation phenomena both numerically and analytically on all three-node network motifs, the minimum degrees of freedom for nontrivial behavior. We found that only 3 out of 13 motifs show bifurcations. Two exhibit supercritical pitchfork bifurcations due to symmetry breaking of flows from a node with out-degree 2, with differences in continuity based on link directionality (i.e., presence of feedback). The third, a fully connected graph, exhibits two distinct localized solutions depending on the initial conditions and hysteresis through combination of saddle-node and transcritical bifurcations [Koike, Takayasu, Takayasu, Phys Rev Res 6(013059): 2024].

We will report these results and discuss further perspectives regarding phase transitions on large-scale networks based on the bifurcations obtained on three-node motifs as building blocks. If time allows, we will also report on self-excited oscillations found in $N=4$ and 5 due to saddle-node bifurcations in phase and amplitude directions.

The linear Lie algebraic structure of colored network dynamics

Fahimeh Mokhtari (VU Amsterdam), Jan Sanders (VU Amsterdam)

Monday, July 29, 17:30–18:00, East Hall 4

Abstract: In the computation of the normal form of a colored network vector field, following the semi-group(oid) approach [2], one would like to be able to say something about the structure of the Lie algebra of linear colored network vector fields. Unlike the purely abstract approach in [1], we describe here a concrete algorithm that gives us the Levi decomposition for these networks. We show that for N -dimensional vector fields with C -colors (different functions describing different types of cells in the network), this Lie algebra is isomorphic to the semidirect sum of a semisimple part, consisting of two simple components, which we write as a block matrix and a solvable part, consisting of two elements representing the identity and abelian algebra. The algorithmic approach presented in this paper is but a first step towards the classification of bifurcations of colored networks. Obvious areas of further research are the computation of Jordan-Chevalley decompositions with many variables, the generalization of the Jacobson-Morozov construction to the situation of colored networks, the construction of versal deformations of organizing centers and the nonlinear network normal form description.

[1] M. Golubitsky and I. Stewart. Coordinate changes for network dynamics. Dyn. Syst., 32(1):80–116, 2017

[2] B. Rink and J. Sanders. Coupled cell networks: semigroups, Lie algebras and normal forms. Transactions of the American Mathematical Society, 367(5):3509–3548, 2015.

Formation of collectives in opinion dynamics

Ulrich Krause (University Bremen)

Monday, July 29, 18:00–18:30, East Hall 4

Abstract: The formation of collectives is a fascinating phenomenon which has been observed in many dynamical networks. The notion of a collective, however, remains often vague and needs clarification. For a multi-agent system of opinion dynamics a collective is defined in the talk as a group of agents where all agents converge to the same opinion, a partial consensus. It can be proven, under mild assumptions, that such collectives do exist and that maximal collectives exhaust the set of all agents. This applies in particular to the so called model of bounded confidence in opinion dynamics. The process of collective dynamics exhibits contractive behavior as well as separation of orbits.

Spatial spread of epidemic in a system of weakly connected networks

Evgeniy Khain (Oakland University)

Monday, July 29, 18:30–19:00, East Hall 4

Abstract: A metapopulation consists of a group of spatially distanced subpopulations, each occupying a

separate patch. It is usually assumed that each localized patch is well-mixed. In this talk, we will discuss a model for the spread of an epidemic in a system of weakly connected patches, where the disease dynamics of each patch occurs on a network. The SIR dynamics in a single patch is governed by the rate of disease transmission, the disease duration, and the node degree distribution of a network. Monte-Carlo simulations of the model reveal the phenomenon of spatial disease propagation. The speed of front propagation and its dependence on the single patch parameters and on the strength of interaction between the patches was determined analytically, and a good agreement with simulation results was observed [1]. Finally, we will discuss front propagation in case of an Allee effect, where the effective transmission rate depends on the fraction of infected, and the state of no epidemic is linearly stable. We discovered [2] a novel phenomenon of front stoppage: in some regime of parameters, the front solution ceases to exist, and the propagating pulse of infection decays despite the initial outbreak.

[1] E. Khain and M. Iyengar, Phys. Rev. E 107, 034309 (2023).

[2] E. Khain, Phys. Rev. E 107, 064303 (2023).

Session on Nonlinear Dynamics with Stochasticity

Lyapunov stable solutions to ordinary and stochastic differential equations

Alexander Kalinin (University of Munich (LMU)), Thilo Meyer-Brandis (University of Munich (LMU)), Frank Proske (University of Oslo)

Monday, July 29, 17:00–17:30, Research II Lecture Hall

Abstract: We establish moment and pathwise Lyapunov stability of solutions to stochastic differential equations whose drift and diffusion coefficients may fail to be locally Lipschitz continuous or of affine growth. Our approach does not rely on the construction of Lyapunov functions and leads to explicit Lyapunov exponents. Moreover, we will focus on the particular case that the diffusion coefficient vanishes and the stochastic equation turns into an ordinary differential equation.

Enhancing Dynamical Systems Analysis through Gaussian Process Emulation: A Multi-Scale Approach

Arindam Saha (University College London (UCL)), Shakir Bilal (University of South Florida), Serge Guillas (University College London (UCL))

Monday, July 29, 17:30–18:00, Research II Lecture Hall

Abstract: Many common analytical tasks in dynamical systems such as computing bifurcation diagrams, Lyapunov spectra, and basin boundaries can be generally computationally intensive. This problem becomes even more relevant when the underlying model itself requires a long time to evaluate or has a high-dimensional parameter space. In such cases, a multi-scale approach to analyzing the problem may be necessary, where a large parameter space is scanned ‘approximately’ to identify smaller and more relevant regions of the parameter space which are then studied in greater detail using physics-based simulation.

Gaussian Process (GP) Emulators are statistical models that can help in the first step of the process. Given an appropriately chosen correlation kernel, these emulators fit a multidimensional Gaussian function over the relatively sparse simulation results. When compared to other emulation techniques, GP emulators are generally more flexible due to their non-parametric nature. They also inherently come with uncertainty bounds which help in judging the quality of emulation.

In this presentation, we demonstrate the use of GP emulators on popular dynamical systems such as the SIR disease model, the logistic map, and the Lorenz and Rossler attractors. By comparing the time required for emulation and simulation, we highlight how these GP emulators can be incorporated to assist the analysis of dynamical systems.

Practical resilience of natural systems: from definitions to optimised indicators

Daniele Proverbio (University of Trento), Rami Katz (University of Trento), Giulia Giordano (University of Trento)

Monday, July 29, 18:00–18:30, Research II Lecture Hall

Abstract: Biological systems have evolved in a way that maintains properties and functions that guarantee survival. Robustness and resilience are associated with a system's capability to maintain such functions despite intrinsic or extrinsic perturbations or uncertainties. However, due to the multidisciplinary nature of research revolving around biological systems, numerous competing definitions of these concepts coexist and often lack rigorous formulations. In control theory, structural analysis [1] aims at guaranteeing that a property is preserved by a whole family of uncertain systems independent of parameter values, while robustness analysis [2] guarantees that a property is preserved for all parameter values within a specified set. To go beyond these definitions and embrace additional cases that are typical in systems biology and other natural disciplines, we introduce possible formal definitions of practical resilience [3]. As a first motivational case study, we consider a family of ODE systems consisting of stochastic perturbations of a nominal deterministic system and quantify its ability to preserve a prescribed attractor. We show that our proposed definitions complement the notion of robustness, and we demonstrate their efficacy when applied to widely used models in biology. Finally, we show how the proposed definitions embrace the notion of indicators for resilience loss developed in the literature [4] and how to optimise them in case of distribution data [5] extracted from microbiological experiments.

- [1] Franco Blanchini and Giulia Giordano. Structural analysis in biology: A control-theoretic approach. *Automatica*, 126:109376, 2021.
- [2] B Ross Barmish. *New tools for robustness of linear systems*. Macmillan Publishing Company, 1994.
- [3] Daniele Proverbio, Rami Katz, and Giulia Giordano. Bridging robustness and resilience for dynamical systems in nature. Submitted, 2024.
- [4] Els Weinans, Rick Quax, Egbert H. van Nes, and Ingrid A. van de Leemput. Evaluating the performance of multivariate indicators of resilience loss. *Sci Rep*, 11:1–11, 2021.
- [5] Daniele Proverbio, Alexander Skupin, and Jorge Gonçalves. Systematic analysis and optimization of early warning signals for critical transitions using distribution data. *iScience*, 2023

Dual-tipping in multiscale systems: noise introduces uncertainty to rate-dependent transition scenarios

Ryan Deeley (ICBM, Carl von Ossietzky Universität Oldenburg), **Ulrike Feudel** (ICBM, Carl von Ossietzky Universität Oldenburg)

Monday, July 29, 18:30–19:00, Research II Lecture Hall

Abstract: Sudden, drastic shifts in the evolution of a dynamical system's variables describe critical transitions when they represent long-lasting, high-impact changes across the global scales of the system. Critical transitions are observed throughout nature and society, for instance with climate tipping events or financial crashes, and great advances have been made in modelling their onset using nonlinear, nonautonomous dynamical systems [Ashwin et al. 2012]. In particular, critical thresholds can be reached when varying an external input faster than the system's response rate (R-tipping), or alternatively, following a harmful accumulative series of stochastic perturbations (N-tipping). Often studies prescribe that one and only one of these mechanisms are acting in the system, yet in reality, they act in conjunction and there is not a complete decoupling between their effects. Here, we present within a hierarchy of systems (ranging from paradigmatic to multiscale) how noise introduces uncertainty to rate-dependent transition scenarios. We analyse, on the one hand, ensembles of coupled noise- and rate-driven trajectories that transition to alternative states for rates below the critical threshold associated with R-tipping; conversely, we explore cases where noise prevents a system from undertaking the rate-induced transition that in the absence of noise would be followed. The dynamical explanation for this arising uncertainty is that, with the additional inclusion of noise, the system can become "kicked across" the boundary separating the domains that i) track and ii) tip away from the moving attractor under the current rate of change to the external input. In multiscale systems, this separating boundary is partly determined by a maximal canard trajectory, which can be computed by applying geometric singular perturbation theory [Vanselow et al. 2024]. To ensure we are fairly assessing the impact of noise- and rate-dependent drivers working together, we consider noise intensities sufficiently weak such that the probability of observing a purely noise-induced transition is near zero. We assess the (non-negligible) probability that a rate-dependent tracking/tipping scenario changes under the influence of noise, by constructing and evaluating a delayed committor function. Further, we compute the theoretically

most-probable coupled noise- and rate-dependent tracking/tipping paths by solving the associated Euler-Lagrange equations [Slyman & Jones 2023], and compare their output with our ensemble of numerical simulations.

Session on Numerical Methods

Short-term predictions as a tool for denoising observables from dynamical systems

Anna Krakovska (Slovak Academy of Sciences)

Monday, July 29, 11:00–11:30, IRC Seminar Room 3

Abstract: The presentation focuses on numerical algorithms designed for noise reduction in discretely sampled observables originating from deterministic dynamical systems. The methods examined rely on exploiting the inherent dynamics of the system to distinguish between the authentic clean signal and noise interference. The effectiveness of the techniques is assessed on complex signals from chaotic systems and maps, deliberately infused with Gaussian white noise. As one of the key indicators to measure the efficiency of noise reduction, we use the improved capacity to estimate the fractal dimension of the underlying system. Among the most successful noise reduction techniques are those based on predictions or local approximations of the dynamics within the space created by time-delay reconstruction. Even when the clean signal is contaminated with as much as 100% noise, the methods exhibit considerable usefulness.

Lyapunov stability of nonlinear numerical methods

Thomas Izgin (University of Kassel), Andreas Meister, Stefan Kopecz

Monday, July 29, 11:30–12:00, IRC Seminar Room 3

Abstract: Positivity-preserving schemes are of high importance and gained more and more attention in recent years. However, they often result in nonlinear iteration schemes even when applied to a linear test problem $y' = A y$. Hence, a stability analysis becomes more complex. Moreover, the scheme often has to preserve additionally other properties of the analytic solution such as steady states and linear invariants. As a result, steady states of the test problem become non-hyperbolic fixed points of the method. In general, the stability analysis thereby turns into a case by case study.

In this talk, we present sufficient conditions for Lyapunov stability as well as local convergence, and hence, overcome this case by case study for a large class of steady state preserving schemes to which reasonable positivity-preserving schemes belong.

To illustrate the theoretical results, we consider so-called modified Patankar-type schemes that are positive and conservative numerical methods when applied to a positive and conservative production-destruction system. Applying the main theorem we are able to define stability functions which are the basis for the stability analysis. Finally, numerical experiments are presented to confirm the theoretical results.

Bridging the gap between Numerical analysis and Physics-informed Machine Learning: Stability and Theoretical Convergence for Stiff ODEs and Rough PDEs

Gianluca Fabiani (Scuola Superiore Meridionale, Naples, Italy), Lucia Russo (Consiglio Nazionale delle Ricerche (CNR), Naples, Italy), Athanasios Yannacopoulos (Athens University of Economics and Business, Greece), Constantinos Siettos (University of Naples "Federico II", Italy)

Monday, July 29, 12:00–12:30, IRC Seminar Room 3

Abstract: We introduce a novel adaptive, L-stable and convergent ([2]) physics-informed machine learning (PIML) approach based on hybrid space-time discretization with Random Projection Neural Networks (RPNNs) to address the numerical solution of nonlinear stiff ODEs, index-1 DAEs, and time-dependent rough PDEs ([1,3]). Our method efficiently handles the computational demands typically associated with PIML schemes, incorporating stability analysis and theoretical convergence considerations. We employ Gauss-Newton and sparse QR decomposition with L2 regularization for medium to large-scale systems, alongside a bias-variance trade-off decomposition to select fixed internal weights and biases ([1]). To manage stiffness and sharp gradients, we implement adaptive step-size and continuation methods to provide robust

initial guesses for Newton iterations ([1]), while using spectral convergent RPNN for the spatial discretization ([2,3]). Evaluation across many benchmark problems, including stiff ODEs, index-1 DAEs and rough PDEs demonstrates the competitiveness of our approach against (a) traditional implicit Runge Kutta solvers, and advanced adaptive schemes as ode23t and ode15s of MATLAB, for the time marching integration ([1]) and against (b) classical Finite Difference (FD) and Galerkin Finite Elements Methods (FEM) for the spatial discretization ([3]). Furthermore, comparison with the DeepXDE physics-informed deep learning library underscores the superior speed and accuracy of our numerical-assisted machine learning scheme ([1,2]).

- [1] Fabiani, G., Galaris, E., Russo, L., & Siettos, C. (2023). Parsimonious physics-informed random projection neural networks for initial value problems of ODEs and index-1 DAEs. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 33(4).
- [2] Fabiani, G. (2024). Random Projection Neural Networks of Best Approximation: Convergence theory and practical applications. arXiv preprint arXiv:2402.11397.
- [3] Fabiani, G., Calabrò, F., Russo, L., & Siettos, C. (2021). Numerical solution and bifurcation analysis of nonlinear partial differential equations with extreme learning machines. *Journal of Scientific Computing*, 89(2), 44.

A numerical method for measuring dynamical phase transitions in time series

Bulcsú Sándor (Babes-Bolyai University Cluj-Napoca), **András Rusu** (Babes-Bolyai University Cluj-Napoca), **Károly Dénes** (Babes-Bolyai University Cluj-Napoca), **Mária Ercsey-Ravasz** (Babes-Bolyai University Cluj-Napoca), **Zsolt Lázár** (Babes-Bolyai University Cluj-Napoca)

Monday, July 29, 12:30–13:00, IRC Seminar Room 3

Abstract: Detecting and predicting dynamical phase transitions in time series generated by complex systems are crucial for various scientific and practical applications. This study presents a novel numerical method for identifying phase transitions by leveraging order- q Rényi-entropy analysis of the statistics of trajectory spaces, with a specific focus on detecting discontinuities at $q = 1$, indicative of intermittency.

Based on the numerically estimated first-order Markov process model of the symbolized time series, we derive an analytic formula allowing for the direct computation of the derivative of the Rényi-entropy at $q = 1$, termed here the Lyapunov measure, from the transition probability matrix. For systems at the edge of chaos, viz at parameters close to a periodic window in the bifurcation diagram, the Rényi entropy drops abruptly for increasing q values, exhibiting hence a pronounced peak in the Lyapunov measure.

We demonstrate that computing the $q = 1$ Rényi entropy and its derivative, viz the Kolmogorov-Sinai entropy and the Lyapunov measure, not only enables us to distinguish meaningful dynamics from random noise but also enhances our ability to detect critical transitions. Through extensive simulations of discrete time dynamical systems, we showcase the efficacy and versatility of our numerical approach, implemented in the Julia package called `StateTransitionNetworks.jl`, in combination with different symbolization techniques. Finally, we discuss its extensions to continuous-time dynamical systems and multivariate time series analysis (e.g. EEG data).

This work was supported by the grant of the Romanian Ministry of Research, Innovation and Digitization, CNCS - UEFISCDI, project number PN-III-P4-PCE-2021-0408 (BS, KD, MER, ZL), PN-III-P4-ID-PCE-2020-0647 (BS, ZL), ERANET-FLAG-ERA-ModelDXConsciousness (BS, MER, ZL), within PNCDI III, and SRG-UBB 32993/23.06.2023 (BS) within UBB Starting Research Grants of the Babes-Bolyai University.

Slow Invariant Manifolds for Singularly Perturbed systems of ODEs and in general Slow-Fast Dynamical Systems via Physics-Informed Neural Networks

Gianluca Fabiani (Scuola Superiore Meridionale), **Constantinos Siettos** (University of Naples "Federico II"), **Dimitrios Patsatzis**, **Lucia Russo** (Consiglio Nazionale delle Ricerche (CNR), Naples, Italy)

Wednesday, July 31, 11:00–11:30, IRC Seminar Room 3

Abstract: We present a physics-informed neural network (PINN) framework for the discovery of slow invariant manifolds (SIMs), for singularly perturbed systems of ODEs [1] and more general fast/slow dynamical systems [2]. In contrast to other machine learning (ML) approaches that construct reduced order black box surrogate models using simple regression, and/or require a priori knowledge of the fast and slow

variables, our approach, simultaneously decomposes the vector field into fast and slow components and provides a functional of the underlying SIM in a closed form. The decomposition is achieved by finding a transformation of the state variables to the fast and slow ones, which enables the derivation of an explicit, in terms of fast variables, SIM functional. The latter is obtained by solving a PDE corresponding to the invariance equation within the Geometric Singular Perturbation Theory (GSPT) using a single-layer feedforward neural network with symbolic differentiation. The performance of the proposed physics-informed ML framework is assessed via various benchmark problems. We also provide a comparison with other GPST methods, namely the quasi steady state approximation (QSSA), the partial equilibrium approximation (PEA) and CSP with one and two iterations. We show that the proposed PINN scheme provides SIM approximations, of equivalent or even higher accuracy, than those provided by QSSA, PEA and CSP, especially close to the boundaries of the underlying SIMs. The proposed approach facilitates the construction of reduced order models (ROMs).

- [1] Patsatzis, D., Fabiani, G., Russo, L. and Siettos, C., 2024. Slow invariant manifolds of singularly perturbed systems via physics-informed machine learning. *SIAM Journal on Scientific Computing*, 46(4), pp.C297-C322.
- [2] Patsatzis, D.G., Russo, L. and Siettos, C., 2024. A physics-informed neural network method for the approximation of slow invariant manifolds for the general class of stiff systems of ODEs. arXiv preprint arXiv:2403.11591.

Inferring the connectivity of coupled oscillators from event timing analysis

Raul de Palma Aristides (Pompeu Fabra University (UPF)), Hilda Cerdeira (UNESP - São Paulo State University), Cristina Masoller (Polytechnic University of Catalonia (UPC)), Giulio Tirabassi (Universitat de Girona)

Wednesday, July 31, 11:30–12:00, IRC Seminar Room 3

Abstract: Understanding the coupling structure of interacting systems is an important open problem, and many methods have been proposed to reconstruct a network from observed data. Most require continuous observation of the nodes’ dynamics; however, in many situations, we can only monitor the times when some events occur (e.g., in neural systems, spike times). Here, we propose a method for network reconstruction based on the analysis of event times at the network’s nodes. First, from the event times, we generate phase time series. Then, we assimilate the phase time series to the Kuramoto model by using the unscented Kalman filter (UKF) that returns the inferred coupling coefficients. Finally, we use a clustering algorithm to discriminate the coupling coefficients into two groups that we associate with existing and non-existing links. We demonstrate the method with synthetic data from small networks of Izhikevich neurons, where we analyze the spike times, and with experimental data from a larger network of chaotic electronic circuits, where the events are voltage threshold-crossings. We also compare the UKF with the performance of the cross-correlation (CC), and the mutual information (MI). We show that, for neural network reconstruction, UKF often outperforms CC and MI, while for electronic network reconstruction, UKF shows similar performance to MI, and both methods outperform CC. Altogether, our results suggest that when event times are the only information available, the UKF can give a good reconstruction of small networks. However, as the network size increases, the method becomes computationally demanding.

A morphological approach to Poincaré sections to identify anomalous transport

André Farinha Bósio (Universidade de São Paulo (USP)), Iberê Luiz Caldas (Universidade de São Paulo (USP)), Ricardo Luiz Viana (Universidade Federal do Paraná), Yves Elskens (Aix-Marseille Université)

Wednesday, July 31, 12:00–12:30, IRC Seminar Room 3

Abstract: Exploring chaotic systems via Poincaré sections has proven crucial in dynamical systems, yet studying many maps is a tiresome process. Here, we propose a novel approach to identify superdiffusion. Since anomalous transport due to ballistic modes sufficient but not required we just need to verify if they exist. To do so, we apply a morphological method to extract and identify different transport regimes in each region of the phase space. This routine is approximately 10 times quicker than the conventional approach of using the mean square displacement (MSD), making this a valuable tool for an exploratory strategy. In

particular, here we applied the method to a well-known system, the standard map, given by:

$$p_{n+1} = p_n + K \sin(\theta); \theta_{n+1} = \theta_n + p_{n+1}.$$

It correctly differentiates the chaotic and periodic regions and identifies anomalous transport for values of K coherent with the literature. Since this system is discrete, usually time evolution-related studies are fast, so a better use case would be on nonintegrable continuous-time systems, which require much more computational efforts, such as the Hamiltonian that describes transport in plasma due to propagating electrostatic waves:

$$H(x, y, t) = A_1 \sin(kx) \cos(ky) + A_2 \sin(kx + \theta_x) \cos(k(y - vt)).$$

Again, implementing the same method here made it possible to identify when superdiffusion happens in an $A_2 \times \theta_x$ phase space, which would normally take much longer

Posters

Complexity characterization of temporal correlations

Andres Aragoneses (Whitman College)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We present quantifiers and techniques to classify and identify different families of chaos in a dynamical system. These are based on temporal correlations among consecutive events in a time series and explore the hidden temporal and reversible symmetries in the behaviour of the dynamical system. We apply the techniques to various mathematical and physical systems and show how they not only are simple and powerful to distinguish types of chaos, and to find signatures of universality in a broad range of chaotic systems, but they also allow to forecast transitions in dynamical regimes.

Inspired on the ordinal patterns technique introduced to compute permutation entropy, and based on experimental results from photonic neurons, we introduce a set of combinations of patterns defined to identify some temporal structures. We find that these combinations of patterns allow to identify approximate temporal symmetries. When applied to chaotic models they identify similarities and differences that allow to classify the families of chaos present in chaotic systems as we vary the control parameter, or as we move from one chaotic system to another.

Our results analyze 1D and 2D iterative chaotic systems as well as experimental data from photonic systems. We also introduce visual approach to interpreting these quantifiers.

[1] <https://www.mdpi.com/2304-6732/9/12/938>

[2] <https://www.mdpi.com/2673-8716/3/4/40>

[3] http://andresaragoneses.weebly.com/uploads/6/0/1/5/60153403/2021_spi_chak_2.pdf

DynamicalSystems.jl: fast, featurefull, open, excellently-documented software for all of nonlinear dynamics

George Datsaris (University of Exeter)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: DynamicalSystems.jl is an award-winning Julia software library for nonlinear dynamics and nonlinear timeseries analysis. Its main three goals are 1) to be a library for nonlinear dynamics in the literal sense, 2) to make the field of nonlinear dynamics accessible and reproducible, and 3) to fundamentally change the perception of the role of code in both scientific education as well as research.

The content of DynamicalSystems.jl covers the entire field of nonlinear dynamics, all in one place: delay embeddings, recurrence quantification, stability analysis, continuation, global stability and multistability, chaos, timeseries analysis, complexity measures, and the list goes on. No known software, open or closed source, comes anywhere near to the amount of content in DynamicalSystems.jl.

Perhaps most importantly, DynamicalSystems.jl is an open source community driven software. Anyone can be a developer, everyone is welcomed to contribute and put in there their new methods, making the accessible to the community instantly as runnable code when their paper is published. Extra care has been taken so that DynamicalSystems.jl is accessible: the source code is simple and concise and the programming interfaces of the software have been designed to be easily extendable. This lowers the threshold for a new contributor. Additionally, because the software is written in Julia, turning on development for it is a matter of only 1 minute (literally, no exaggerations!).

Join us in this poster to have a look at the content of DynamicalSystems.jl, its list of features, example code run live during the poster presentation and snippets of its source code, which are surprisingly readable even if you don't know Julia!

Stochastic Thermodynamics of Cost of Computing in Autonomous Nanomechanical Systems

Zabreen Nissar (NWO Institute AMOLF)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Modern computers are powerful devices that have given impetus to our abilities to make discoveries in many fields of science and technology. With their ubiquitous use, the amount of energy they consume is on the rise. Current electronic computers consume energies that are orders of magnitude higher than the theoretical predicted cost, given by the Landauer Limit. The Landauer limit states that the minimum energy required to erase a bit, an operation essential for irreversible computation, at a given temperature, depends on the entropy change that occurs in the erasure. Recent experiments have verified the Landauer Limit using time-dependent potentials. This approach does not take into account the energy required to break the detailed balance and establish an arrow of time for the erasure. In this work, a theoretical nanomechanical system, that does an autonomous bit erasure, is presented and the energy cost for the bit erasure and breaking detailed balance are discussed. The system consists of two coupled nonlinear mechanical oscillators, a bistable buckled beam that acts as the “bit” and a heavy mass, here called the “power clock”, that provides energy and sets the time scale for the erasure. The system is studied using the framework of stochastic thermodynamics. The simulation results show that the system can do a bit erasure using 250 kbT of energy ($1 \text{ kbT} = 4.1 \times 10^{-21} \text{ J}$) compared to the 3900 kbT of energy used to do a logical operation in modern electronic computers.

Transport in the Biquadratic Nontwist Map

Gabriel Grime (University of São Paulo), Marisa Roberto (Aeronautics Institute of Technology), Iberê Luiz Caldas (University of São Paulo)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Key results about transport in Hamiltonian systems rely on the assumption of a monotonic profile for the frequency of orbits, e.g. Kolmogorov-Arnold-Moser theorem and the Aubry-Mather theory. In nontwist systems, present in plasma and fluid descriptions, such a condition does not hold, leading to typical phenomena such as shearless transport barriers, which eliminate or reduce transport between chaotic regions in phase space. Recently, the so-called Biquadratic Nontwist Map was used to study nontwist systems with multiple shearless curves and their transition to chaos. However, the transport properties of partial transport barriers in these multiple barrier systems have not been analyzed yet. In this work, we use the escape time and barrier transmissivity to investigate the transport properties of the Biquadratic Nontwist Map. A sensitive dependence of such dynamical quantifiers on the parameters of the map is verified and interpreted using the turnstile mechanism. In addition, the effect of the multiple transport barriers is quantified.

Stability of nonlinear Dirac solitons by using the phase portrait

Niurka R Quintero (University of Seville, Spain)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The stability of nonlinear excitations in complex dynamical systems under the action of external

forces or potentials constitutes an important issue not only from the theoretical point of view, but also due to the applications in many fields. In particular, the Nonlinear Dirac (NLD) equation, which models, for instance, a Bose-Einstein condensate in a honeycomb optical lattice, has soliton solutions that decay in the presence of dissipation. This presentation addresses precisely the stabilization of the damped NLD solitons by using a time-periodic parametric force that supplies the required balance between energy losses and gains. This equation has two exact stationary soliton solutions under certain conditions satisfied by the parameters of the parametric force and the damping coefficient. In order to study their stability, we use a variational approach with only two degrees of freedom, which reduces the original problem to an autonomous dynamical system with two fixed points corresponding to the two stationary solutions. It is shown that one of these solutions is stable, whereas the other is unstable. These predictions are compared with the results of linear stability analysis around a stationary soliton.

Data driven approach to morphology of human respiratory patterns

Hiromichi Suetani (Faculty of Science and Technology, Oita University), Ulrich Parlitz (Max Planck Institute for Dynamics and Self-Organization), Grzegorz Graff (Faculty of Applied Physics and Mathematics, Gdańsk University of Technology), Paweł Pilarczyk (Faculty of Applied Physics and Mathematics, Gdańsk University of Technology), Maciej Torhan (Faculty of Applied Physics and Mathematics, Gdańsk University of Technology), Stefan Luther (Max Planck Institute for Dynamics and Self-Organization), Beata Graff (Medical University of Gdańsk)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Human respiratory system is known to exhibit a variety of nonlinear oscillations through interactions with the circulatory and nervous systems. Although there are several typical types of these respiratory patterns, their classification is primarily based on visual inspection by medical professionals.

In this study, we explore the morphology of respiratory patterns using a data-driven approach applied to time series collected from dozens of patients. Specifically, we will classify respiratory patterns and visualize them in a low-dimensional space by the combination of various feature extraction methods, including analytical signals through the Hilbert transformation, ordinal patterns, and persistent homology, with machine learning techniques such as reservoir computing, deep learning, and manifold learning. The results obtained from these methods will be compared to identify the most effective approach. Through this comparative study, we intend to take the initial steps towards nonlinear dynamic modeling of respiratory patterns.

Re-entrant instability in systems of weakly-incommensurate fractional differential equations

Zuhur Alqahtani (Princess Nourah bint Abdulrahman University), Mustafa El-Shahed, Nigel Mottram

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: In this paper we analyse the stability of dynamical systems consisting of weakly-incommensurate fractional differential equations. Specifically, we analytically determine the approximate stability boundary for a typical two-dimensional dynamical system for which the orders of the fractional time derivatives are approximately equal. We find that in certain parameter regimes it is possible for an equilibrium point of the system to be unstable at low and high values of a particular fractional derivative order but stable within a range of intermediate values. This re-entrant instability behaviour is confirmed numerically, as is the accuracy of the analytic approximation to the stability boundary.

The transition to synchronization of networked systems

Kirill Kovalenko (Scuola Superiore Meridionale)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We focus on the synchronization properties of a generic networked dynamical system. Under the approximation that the trajectories followed by the nodes forming a synchronized cluster do not substantially differ from those featured during synchronization of the entire network, we show that the transition to synchronization can be predicted and characterized with the only help of eigenvalues and eigenvectors

of the graph's Laplacian matrix. Indeed, under such an approximation, the transition comes out to be made of a well defined sequence of events, each of which corresponds to either the nucleation of one (or several) cluster(s) of synchronized nodes or to the merging of multiple synchronized clusters into a single one. The network's nodes involved in each of the clusters can be identified, and the value of the coupling strength at which the events are taking place can be approximately ascertained. We moreover clarify that the synchronized clusters are formed by those nodes which are indistinguishable at the eyes of any other network's vertex, and as so they receive the same dynamical input from the rest of the network. Therefore, such clusters are more general subsets of nodes than those defined by the graph's symmetry orbits, and at the same time more specific than those described by network's equitable partitions. Finally, we present large scale simulations which show the accuracy of the approximation made, and of our predictions in describing the synchronization transition of both synthetic and real-world large size networks, and we even report that the observed sequence of clusters is preserved in heterogeneous networks made of slightly non-identical systems.

A computational study of data assimilation for point vortex systems

Gakuto Kambayashi (Tokyo University of Science)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Many attempts have been made to model various phenomena as dynamical systems to understand and predict them; however, the nonlinearity of the systems often makes it difficult because of the existence of sensitivity to initial conditions, i.e. chaotic dynamics. To predict the future state in a chaotic dynamical system, data assimilation is effectively used to combine observational data into a numerical simulation. The various data assimilation methods, such as the Bayesian statistics-based methods, are currently employed for fields related to weather forecasting and geophysics, atmospheric science, and so on.

This study aims to clarify the underlying relationships between the increasing uncertainty due to the inherent chaotic dynamics in fluid systems and the decreasing uncertainty due to data assimilation methods by targeting the nudging data assimilation of 2-point vortex systems. The simplicity of both the data assimilation method and the targeted fluid system makes the underlying relationships clear.

When observing the state of the system, it is generally difficult to capture microscopic structures of the system, while macroscopic structures of the system can be captured more certainly. Such an issue is generally important in research on data assimilation and it is desired that only the macroscopic structures of the system be assimilated to reconstruct not only macroscopic structures but also microscopic structures. Hence, in this research, we assume that the location of the dominant point vortex with a larger circulation is observable. The goal of our data assimilation problem is to accurately estimate the location of the point vortex with a smaller circulation only observing the location of the dominant one.

As a result of numerical experiments, we show some well-suited conditions for successful data assimilation in terms of the proportion of circulations, the gain of nudging, and the initial locations of two vortices. In particular, speaking of the initial locations dependency, it is interesting to note that the initial location of the point vortex with a smaller circulation for successful data assimilation is distributed in a spiral-like shape in the 2-dimensional plane when the parameters other than the initial location of the point vortex with a smaller circulation are fixed. The resulting dynamics of the nudging data assimilation system can be categorized by several types of typical motion. These results could potentially be applied to multiple-point vortex systems if the dominant point vortex has very large circulations compared to one for a smaller point vortex and will provide a dynamical structure crucial for further data assimilation studies of more complex fluid systems.

Neuronal spike generation via a homoclinic orbit bifurcation increases irregularity and chaos in balanced networks

Moritz Drangmeister (Humboldt-Universität zu Berlin), **Rainer Engelken** (Columbia University), **Schleimer Jan-Hendrik** (Humboldt-Universität zu Berlin), **Schreiber Susanne** (Humboldt-Universität zu Berlin)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Both modeling studies and experiments have shown that a large class of neurons can be brought into a regime of bistable firing, by tuning variables like temperature, extracellular ion concentrations, or channel expression levels within the physiologically plausible range. In this regime, spike trains manifest burst-like irregular firing induced by stochastic switches between attractors. Despite understanding the single-neuron bifurcations underlying this burst-like activity (Hesse et al., 2022; Contreras et al., 2021; Hürkey et al., 2023; Schleimer et al., 2021; Niemeyer et al., 2021), the impact of this burst-like activity on recurrent neural network behavior remains poorly understood. Recurrent spiking networks in a balanced state often (Monteforte et al., (2010), though not always (Monteforte et al., 2012), exhibit chaotic activity and it remains unclear how stochastic burstiness on the single neuron level affects chaos and fundamental network dynamics such as attractor dimension and dynamical entropy rate.

In this study, we induce a transition in the neuron’s spiking dynamics, called the saddle-node loop (SNL) point. This transition switches the spike-onset from a saddle-node on an invariant circle (SNIC) bifurcation to a homoclinic (HOM) bifurcation in quadratic integrate-and-fire (QIF) neurons by elevating the reset voltage beyond the SNL point (Hesse et al., 2017). We observe that an increase in the reset voltage leads to stochastic burst-like spiking activity in the recurrent network. In this network state, we identify slow-frequency components within the power spectrum and a mean coefficient of variation (CV) for interspike intervals beyond one, both of which are accurately described through a self-consistent renewal approximation.

Concurrently, we find that the maximum Lyapunov exponent, Kaplan York attractor dimension, and Kolmogorov-Sinai entropy rate are enhanced in the network. We explain these observations based on the alignment of the first covariant Lyapunov vector with neurons during stochastic bursts. Our findings open avenues for studying the computational implications of enhanced chaos, particularly in the context of task optimization within recurrent networks. This study links individual neuron biophysics with collective dynamics in large recurrent circuits, thereby highlighting the computational relevance of single-cell dynamics.

Self-organization of large random ecosystems under deterministic perturbations

Frederik Jasper Thomsen (Delft University of Technology, Delft, The Netherlands), Johan Dubbeldam (Delft University of Technology), Rudolf Hanel (Medical University Vienna)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Complex biological systems exhibit remarkable stability and adaptability. However, this observation is not always immediately supported by mathematical modeling. Following May’s classical stability criterion, various mechanisms were proposed to explain its seemingly paradoxical relationship to the system dimension. Adaptability is attributed to natural selection favoring systems located at the edge of chaos, despite this typically being a set of zero measure in parameter space.

In [1], a minimally nonlinear model for gene regulatory networks was introduced, in which the edge of chaos is inflated by a finite-time extinction mechanism. In this work we formulate the model as a piecewise linear dynamical system generated by a random interaction matrix. We study the effect of low- and high-rank deterministic perturbations to interaction matrices with spectrum below the May-threshold yielding unstable outlier eigenvalues. A system trajectory then corresponds to a sequence of extinction events reducing the dimension of the system, or equivalently, a sequence of submatrices of the interaction matrix. We identify sets of initial conditions where the sequence leads to a stable submatrix and the resulting changes to the network topology. Our results give insight into how large ecosystems respond to changes in the environment and to species that threaten community stability.

[1] Stokić , Dejan, Rudolf Hanel, and Stefan Thurner. “Inflation of the edge of chaos in a simple model of gene interaction networks.” *Phys. Rev. E* (2008)

ESABO Co-Abundance Analysis: cases where the binarization threshold matters

Devi Chandran (University of Birmingham, UK), **Jens Christian Claussen** (University of Birmingham, UK)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Population dynamics including their complex interactions lead, in societies and microbial populations, to rich co-abundance patterns, and often only the (co)abundance pattern data is measured – whereby

the precise interactions remain unknown. In this context, ESABO [1] has been introduced to grasp interactions that remain unseen especially for low-abundant species. In [1], the ESABO method has recovered positive and negative interactions between agents (or species) within the population based on co-abundance data. However, in the medium abundance region, instead of a binarization threshold of 1, might it be worth to consider larger binarization thresholds? Here we investigate, based on two datasets, whether higher thresholds can lead to a higher information gain (in the sense of ESABO), and demonstrate cases of higher information gain for higher thresholds, but also confirm that the original threshold of 1 can be optimal for other datasets [2].

- [1] Jens Christian Claussen, Jurgita Skiecevičienė, Jun Wang, Philipp Rausch, Tom H. Karlsen, Wolfgang Lieb, John F. Baines, Andre Franke, Marc-Thorsten Hütt, Boolean analysis reveals systematic interactions among low-abundance species in the human gut microbiome, *PLoS Computational Biology* 13(6): e1005361 (2017)
- [2] Devi Chandran and Jens Christian Claussen (in preparation)

Nonmonotonic safety factor profile for a tokamak with ergodic magnetic limiter

Michele Mugnaine

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The plasma confined in a Tokamak with ergodic magnetic limiter can be described by a symplectic two dimensional map, the Martin-Taylor map. The map was proposed by Martin and Taylor in a scenario where the effect of the magnetic limiter is restricted to a small part of the toroidal circumference and the safety factor radial profile monotonically increases. Expanding this idea, we generalize the map and study the influence of a non-monotonic safety factor profile. The investigation of symplectic maps with nonmonotonic profiles is of great interest since the inclusion of non-monotonic profiles proved to improve the confinement of plasma in tokamaks. Therefore, we obtain the Extended Martin-Taylor map, a nontwist map. With the inclusion of a nonmonotonic profile, we obtain a nontwist map which presents the characteristic properties of degenerate systems, such as the twin islands scenario, shearless curve, and separatrix reconnection. The results obtained in this research emphasize the relation between the number of toroidal pairs of coil segments in the limiter and the period of the islands in the phase space. The magnetic islands do not have a pendular form and, for this reason, their half-width do not follow the scale rule predicted by the pendulum approximation. Lastly, our numerical simulations about the shearless curve show that its position and aspect depend on the control parameters. The extended map presents the same nontwist properties of other nontwist maps with small or no physical background. With this, the extended map is a suitable model for the study of properties and phenomena related to degenerate systems in physical applications, such as the confinement of toroidal plasmas under the effects of ergodic limiters.

Continuum limit of the Kuramoto model with adaptive coupling

Rok Cestnik (Lund University), Erik Andreas Martens

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: A growing interest has arisen for networks with adaptive coupling, where the interaction strength between the oscillators co-evolves with the nodal dynamics – inspired by neural interactions which are indeed known to change in time. From a modelling perspective, this gives rise to richer dynamics and new oscillatory states which have not yet been described. Large networks are abundant in nature and therefore of particular interest. However, since the number of connections grows quadratically with the number of nodes, a large number of network nodes makes numerical simulations impractical. Here, we study the paradigmatic Kuramoto model with adaptive coupling to address this issue. Considering the continuum limit and using an approximate dimensional reduction we derive a self-consistency equation which we can analyze to obtain the bifurcation diagram for the adaptive Kuramoto oscillator model. Moreover, we discover regions with new states and regions with dense multistability.

Multistability and bifurcations in the modified Rypdal model

Arnold Alonso Alvarez (Universidade de São Paulo - USP), Eduardo Luís Brugnago, Iberê Caldas

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The Rypdal model is a low-dimensional Lorenz-like system derived from a two-field model for transport in magnetically confined plasmas in Helimak configurations. In this work, we study the stability, bifurcations, and emergence of chaos in the Rypdal model when an exponential perturbation in the parameter related to the plasma production rate is included. This change causes a drastic modification in the structure of the attractors and the appearance of infinite self-similar shrimp-shaped domains, corresponding to periodic attractors, present in the two-dimensional parameter space, built from the spectrum of Lyapunov exponents. We implemented different methods to analyze the behavior of the periodic attractors found in these domains to schematize the organization of these.

Relating interfacial Rossby wave interaction in shear flows with Feynman's two-state coupled quantum system model for the Josephson junction

Nimrod Bratspiess (Tel Aviv University), Eyal Heifetz (Tel Aviv University), Leo Maas, Anirban Guha
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Here we show how Feynman's simplified model for the Josephson junction, as a macroscopic two-state coupled quantum system, has a one-to-one correspondence with the stable dynamics of two interfacial Rossby waves in piecewise linear shear flows. The conservation of electric charge and energy of the superconducting electron gas layers become, respectively, equivalent to the conservation of wave action and pseudoenergy of the Rossby waves. Quantumlike tunneling is enabled via action at a distance between the two Rossby waves. Furthermore, the quantumlike phenomena of avoided crossing between eigenstates, described by the Klein-Gordon equation, is obtained as well in the classical shear flow system. In the latter, it results from the inherent difference in pseudoenergy between the in-phase and anti-phased normal modes of the interfacial waves. This provides an intuitive physical meaning to the role of the wave function's phase in the quantum system. A partial analog to the quantum collapse of the wave function is also obtained due to the existence of a separatrix between normal mode regions of influence on the phase plane, describing the system's dynamics. As for two-state quantum bits (qubits), the two-Rossby wave system solutions can be represented on a Bloch sphere, where the Hadamard gate transforms the two normal modes and eigenstates into an intuitive computational basis in which only one interface is occupied by a Rossby wave. Yet, it is a classical system which lacks exact analogs to collapse and entanglement, and thus cannot be used for quantum computation, even in principle.

Desynchronization of temporal solitons in Kerr cavities with pulsed injection

Daria Dolinina (Weierstrass Institute for Applied Analysis and Stochastics, Berlin), Guillaume Huyet (Université Côte d'Azur, CNRS, Institut de Physique de Nice (INPHYNI)), Dmitry Turaev (Imperial College London, UK), Andrei G. Vladimirov (Weierstrass Institute for Applied Analysis and Stochastics, Berlin)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Optical frequency combs have had a significant impact on several fields including spectroscopy, optical ranging, metrology, exoplanet search, microwave photonics and optical communications. A conventional approach to frequency comb generation involves the use of optical microresonators. In particular, considerable attention has been paid to microcavity soliton frequency combs. These combs are characterised by the generation of temporal cavity solitons (TCSs), which are stable, periodic light pulses that maintain their shape as they propagate. In simpler setups, TCSs are generated by injecting a continuous wave (CW) laser into a microcavity. However, the use of pulsed injection can be advantageous as it allows a reduction in the TCS excitation energy and the ability to tune the TCS repetition frequency by synchronising it with the injection pulse repetition rate. On the other hand, to achieve this synchronization, the repetition frequency of the injected pulses must be close to or a multiple of the free spectral range of the cavity. Therefore, it is important to study the locking range and understand how it depends on the microcavity and external injection parameters. A standard theoretical tool for describing TCS formation in microcavities is the paradigmatic Lugiato-Lefever equation (LLE). It has proven to be a very efficient tool for describing high-finesse microcavities used for optical frequency comb generation. In this research, using the LLE, we comprehensively

investigate the bifurcation mechanisms leading to the unlocking between the repetition rates of the injection pulse and the TCS in a synchronously pumped optical microcavity. We show that for a sufficiently broad injection pulse, unlocking occurs via an Andronov-Hopf bifurcation rather than the saddle-node bifurcation responsible for the disappearance of the stationary TCS, as predicted by the TCS drift equation investigated in earlier studies on a small frequency mismatch between the injection pulse repetition rate and the cavity free spectral range (FSR). Furthermore, we introduce a simple asymptotic criterion for the occurrence of the AH bifurcation, which requires only the knowledge of the injection pulse shape and the TCS solution with homogeneous injection. This semi-analytical criterion shows excellent agreement with results derived from numerical simulations of the LLE.

Lyapunov spectra of chaotic recurrent neural networks with time-varying external input

Rainer Engelken (Columbia University), Fred Wolf (Georg-August-Universität Göttingen), Larry Abbott (Columbia University)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Recurrent networks are widely used as models of biological neural circuits and in artificial intelligence applications. Mean-field theory has been used to uncover key properties of recurrent network models such as the onset of chaos and their largest Lyapunov exponents, but quantities such as attractor dimension and Kolmogorov-Sinai entropy have thus far remained elusive. We calculate the complete Lyapunov spectrum of recurrent neural networks and show that chaos in these networks is extensive with a size-invariant Lyapunov spectrum and attractor dimensions much smaller than the number of phase space dimensions. The attractor dimension and entropy rate increase with coupling strength near the onset of chaos but decrease far from the onset, reflecting a reduction in the number of unstable directions. We analytically approximate the full Lyapunov spectrum using random matrix theory near the onset of chaos for strong coupling and discrete-time dynamics. We show that a generalized time-reversal symmetry of the network dynamics induces a point symmetry of the Lyapunov spectrum reminiscent of the symplectic structure of chaotic Hamiltonian systems. Temporally fluctuating input can drastically reduce both the entropy rate and the attractor dimension. We lay out a comprehensive set of controls for the accuracy and convergence of Lyapunov exponents. For trained recurrent networks, we find that Lyapunov spectrum analysis quantifies error propagation and stability achieved by different learning algorithms. Our methods apply to systems of arbitrary connectivity and highlight the potential of Lyapunov spectrum analysis as a diagnostic for machine learning applications of recurrent networks.

Basin entropy as a tool to study bifurcations in time-delayed systems

Juan Pedro Tarigo (Facultad de Ciencias, Universidad de la República), Arturo Martí (Facultad de Ciencias, Universidad de la República), Cecilia Stari (Facultad de Ingeniería, Universidad de la República)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Studying the predictability of multistable dynamical systems presents an open challenge due to the inability of determining to which attractor will the system evolve to. This problem becomes more apparent near bifurcations where the stability of the attractors and the structure of the basins of attraction changes. In time-delayed systems there is the additional challenge of dealing with an infinite dimension phase space. Basin entropy has been proposed to quantify the predictability of the final state of a system and has been shown to be a suitable indicator of different bifurcations in non-delayed systems. In this work we aim to study different bifurcations in time-delayed systems and the evolution of the basin entropy as we approach each type of bifurcation.

Trajectory based analysis and visualization of coherent flow structures in stirred tank reactors

Thanh Tung Thai (Leuphana University Lüneburg), Anna Klünker (Leuphana University Lüneburg), Christian Weiland (Hamburg University of Technology (TUHH)), Eike Steuwe (Hamburg University of Applied Sciences (HAW Hamburg)), Kathrin Padberg-Gehle (Leuphana University Lüneburg), Alexandra von Kameke (Hamburg University of Applied Sciences (HAW Hamburg))

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The analysis and quantification of fluid transport and mixing in chemical reactors is of great interest in order to avoid dead zones and to control heterogeneities in concentration distributions. From a Lagrangian perspective, coherent flow structures play a central role in this context. In the past few years, different computational methods have been developed to identify such finite-time coherent sets directly from trajectories of fluid particles. Such type of trajectory data is obtained via numerical simulations or lab experiments (e.g. by time-resolved particle tracking (4D-PTV) or by Lagrangian sensors).

In this contribution, we demonstrate the application of different trajectory-based approaches for the identification of coherent flow structures in stirred tank reactors. For this purpose, several recently proposed methods, such as spectral clustering of trajectories or single-trajectory diagnostics have been implemented in Python to improve performance and facilitate embedding. Furthermore, a Python-based data visualization tool has been developed to improve data visualization and to allow fine-grained filtering.

To illustrate the methods, we use trajectory data from a transient numerical simulation of a stirred tank reactor. Although numerical simulations usually provide complete and clean data, experimental data are often subject to perturbations and missing records. Additionally, obtaining experimental data can be challenging, resulting in sparse trajectory data only. As an outlook, to fill the gaps of missing trajectories appropriately, methods such as fitting interpolations or machine learning need to be utilized.

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Nonlinear dynamics and opinion formation in time varying networks

Anja Göbel (Leuphana University Lüneburg), Anna Klünker (Leuphana University Lüneburg), Kathrin Padberg-Gehle (Leuphana University Lüneburg)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The communication structures within our society can be visualized as networks, which are dynamic and change over time due to various factors. Numerous mathematical models have been developed to simulate opinion dynamics. These models are predominantly agent-based, where an opinion-forming process occurs through interactions between individual agents. The interaction processes are based on an underlying network of agents. The DeGroot model is the most well-known continuous opinion space model. According to this model, a person's opinion is derived from their previous opinion and the influence process. In addition to DeGroot-based models, there are also continuous opinion space models with bounded confidence. These are characterized by individuals ignoring ideas or opinions that are too far removed from their own, but these models do not assume an underlying interaction network, because they rather consider all possible interactions. Our objective is to model, simulate, and analyze nonlinear dynamic opinion developments in time-varying networks of different structures.

We extend the simulation of the DeGroot-Friedkin model with self-weights for the development of social influence networks to a bounded confidence model. The simulation determines the mutual influence of agents on their opinions, using random interaction matrices for weekdays and weekends. The initial opinions of the agents are generated randomly. Each agent is assigned a self-weight, which indicates their relative control of keeping the own opinion. The sensitivity of an agent's opinion change after interacting with another agent can be adjusted. The opinion of the interaction partner only affects the agent's own opinion if it falls within the predefined epsilon neighborhood. Further, we introduce special agents that can influence the interaction partners outside of the epsilon neighborhood. These special agents are therefore considered to represent social media because their opinions never change, and everyone is influenced by one of them. We simulate and analyze random opinion formations under different conditions and examine the emergence of consensus, polarization, and coexistence of different opinions.

When multiplicative noise stabilises unstable fixed points

Ewan Phillips, Holger Kantz, Benjamin Lindner

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We provide a simple framework for the study of state-dependent (multiplicative) noise, making use of scale parameters. We show that for a large class of stochastic differential equations multiplicative noise surprisingly causes the mass of the stationary probability distribution to become increasingly concentrated around fix points, whilst also exhibiting a kind of intermittent burst like jumps between these fix points. In the case that there is only one fixed point this causes on-off intermittency. Our framework relies on first term expansions, which become more accurate for larger noise intensities. In this work we show that the full width half maximum in addition to the maximum is appropriate for quantifying the stationary probability distribution (instead of the mean and variance, which are often undefined). We define a corresponding new kind of stationarity. Such results have application to extreme events and tipping points. We apply the general ideas to the a problem of synchronisation chaos, showing that noise improves synchronisation between chaotic systems/noisy oscillators.

Partial shearless transport barriers in drifted guiding-center model for magnetized plasmas

Leonardo Antonio Osorio-Quiroga (University of São Paulo), Marisa Roberto (Aeronautics Institute of Technology), Iberê Luiz Caldas (University of São Paulo)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: An $\mathbf{E} \times \mathbf{B}$ drift wave transport model was implemented to investigate chaotic transport at the edge of magnetized plasmas in tokamaks. Using this model, some works have shown that reversed-shear plasma profiles can create shearless transport barriers (STBs) capable of confining chaotic orbits inside the plasma. These barriers are identified by the extreme values of the rotation number profile, whenever they exist. However, for certain values of the control parameter, STBs break up, and partial barriers, associated with the remnants of the STB, emerge, also reducing transport. In our study, we identify such partial barriers by implementing several diagnostics, mainly the transmissivity of particles, the escape time, the finite-time Lyapunov exponent (FTLE), and the finite-time rotation number (FTRN). Through the latter, we show that the strongest partial barrier, i.e. the one that prevents most particles from crossing, is also the one that covers the largest area of maxima of the FTRN and presents fewer structures.

Anticipating critical transitions in multi-dimensional systems driven by time- and state-dependent noise

Andreas Morr, Keno Riechers, Leonardo Rydin Gorjão, Niklas Boers
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Anticipating bifurcation-induced transitions in dynamical systems has gained relevance in various fields of the natural, social, and economic sciences. Before the annihilation of a system's equilibrium point by means of a bifurcation, the system's internal feedbacks that stabilise the initial state weaken and eventually vanish; a process referred to as critical slowing down (CSD). In one-dimensional systems, this motivates the use of variance and lag-1 autocorrelation as indicators of CSD. However, the applicability of variance is limited to time- and state-independent driving noise, strongly constraining the generality of this CSD indicator. In multi-dimensional systems, the use of these indicators is often preceded by a dimension reduction in order to obtain a one-dimensional time series. Many common techniques for such an extraction of a one-dimensional time series generally incur the risk of missing present signs for CSD. Here, we propose a data-driven approach based on estimating a multi-dimensional Langevin equation to detect local stability changes and anticipate bifurcation-induced transitions in systems with generally time- and state-dependent noise. Our approach substantially generalizes the conditions under which CSD can reliably be detected as demonstrated in a suite of examples. Changes in deterministic dynamics can be clearly discriminated from changes in the driving noise using this method. This substantially reduces the risk of false and missed alarms of conventional CSD indicators in settings with time-dependent or multiplicative noise. In multi-dimensional systems, our method can greatly advance the understanding of the coupling between system components and can avoid risks of missing CSD due to dimension reduction, which existing approaches suffer from.

Analysis of dynamics of the dam body tilts and the displacement of foundation during exploitation of high arch dam; case study For Enguri dam, Georgia

Teimuraz Matcharashvili (Ivane Javakhishvili Tbilisi State University), Aleksandre Sborshchikovi (Ivane Javakhishvili Tbilisi State University), Ekaterine Mepharidze, Dimitri Tepnadze, Levan Laliashvili
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We investigated dynamical features of the behavior of the Enguri High Dam body and its foundation. We based our analysis on the corresponding data sets obtained for the period of monitoring at the Enguri Dam International Test Area (EDITA). Measurements of dam foundation and dam body tilts, deformation of foundation, dam body temperature, water level variation in the water reservoir behind high dam, etc. have been collected from the beginning of high arc dam construction (1974) to the present days. In the present research, we used data sets of dam body tiltmeter measurements (presently monitored in real-time) and installed in the dam foundation strainmeter data sets together with the water level variation data including periods of dam construction and the large hydropower plant exploitation when regular changes in water level in the artificial lake behind the dam take place.

To assess the character of changes that occurred in the behavior of the dam body and foundation, we used contemporary methods of linear and nonlinear time series analysis. Namely, complex networks visibility graph (VG), the recurrence quantitative analysis (RQA), algorithmic (Lempel-Ziv) complexity measure calculation, singular spectrum analysis (SSA), and multivariate data analysis combined with surrogate data testing methods have been used.

It was shown that the dynamics of dam foundation displacement is strongly influenced by the process of high dam construction and especially by water level change in the artificial reservoir behind Enguri High Dam during hydropower plant exploitation. It has been observed that dam body tilt sequences mainly reveal qualitative properties of low- dimensional dynamics and correlate with water level change. On the other hand, quantitative changes in the dynamics of dam body tilt generation connected with different stages of anthropogenic influence, such as the construction of the Enguri high dam and reservoir fill, have also been detected.

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Control of synchronization in two-layer power grids using higher-order interactions

Manuel Lourenco (Network Evaluation Technologies, Fraunhofer-Institut für Algorithmen und Wissenschaftliches Rechnen SCAI), Mehrnaz Anvari (Network Evaluation Technologies, Fraunhofer-Institut für Algorithmen und Wissenschaftliches Rechnen SCAI), Simona Olmi (Istituto dei Sistemi Complessi, Consiglio Nazionale delle Ricerche), Eckehard Schöll (Institut für Theoretische Physik, Technische Universität Berlin)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: In this study, we propose a two-layer network to model the dynamics of power grids, incorporating higher-order interactions within the control layer. The first layer represents the power grid with nodes symbolizing generators and loads. For the dynamics of the power grids we use the Kuramoto model with inertia. Additionally, a second layer, namely a communication layer, generates control signals for the generators. Previous work has demonstrated that this model can achieve frequency synchronization for different perturbation schemes. This outcome depends on both the communication network topology and the form of the control function employed. In this work, we introduce a novel control function to achieve frequency synchronization and investigate the impact of higher-order interactions.

Geometric decomposition and analysis of planar oscillators

Lucas Morales-Moya (Friedrich Miescher Institute for Biomedical Research), Helge Grosshans (Friedrich Miescher Institute for Biomedical Research)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Oscillations are prevalent in many systems, and they often perform important functions, including

in biological systems. However, it is unclear how a set of two variables and their interactions (like proteins in the case of a biological system) determines the dynamics, and the resulting behaviour of a limit cycle is poorly understood. Redefining a system in terms of a set of more relevant variables is often a sought transformation, as it can reveal underlying properties of the system and simplify the understanding of perturbations. Here I present an orthogonal decomposition of planar oscillators and the application of the study to biological oscillators of interest.

Rogue Waves for a Generalized Semilinear Wave Equation

Julia Henninger (KIT Karlsruhe Institute of Technology), Wolfgang Reichel
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We study the generalized semilinear wave equation

$$V(x)u_{tt} + d(t)M(x, \partial x)u - |u|^{p-1}u = 0 \quad \text{for } (x, t) \in \mathbb{R}^N \times \mathbb{R}$$

where M is elliptic and d is a positive periodic step potential. Our goal is to construct solutions which are localized in space and time (rogue waves) by means of variational methods. We present our approach with its main difficulties and discuss suitable examples for M and d .

This is a joint work with Wolfgang Reichel (KIT).

Effects of Instantaneous Time Reversal on the propagation of gravity waves in thin fluid layers.

Felipe Rinderknecht (Facultad de Ciencias, Universidad de la República), Cecilia Cabeza (Facultad de Ciencias, Universidad de la República), Mathias Fink, Carlos Negreira (Facultad de Ciencias, Universidad de la República), Javier Brum (Facultad de Ciencias, Universidad de la República)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Instantaneous Time Reversal (ITR) involves the abrupt modification of a medium’s properties, impacting wave propagation. Specifically, this is achieved by subjecting a thin layer of fluid to a strong and brief vertical impact, thereby altering effective gravity. This alteration leads to a change in the medium’s refractive index, resulting in the production of a new wave that is time-reversed and converges back toward its source. The temporal interruption creates an instantaneous time reversal mirror that operates throughout space without requiring the recording and reversal of the field. In this study, we implemented ITR and obtained results to validate the experimental setup. We observed the generation of the returning wave and analyzed its response in terms of the impact’s acceleration and frequency. We also examined how the wave and its spectrum are affected before and after the occurrence of ITR.

The Mathematics of balance

Javier Chico Vazquez (Mathematical Institute, University of Oxford)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Fluid dynamics within the vestibular system are crucial for maintaining balance in humans. Within the inner ear are semicircular canals filled with a viscous fluid called endolymph, interconnected by a voluminous cavity termed the utricle. Rotational motion generates a fluid flow within this system prompting deflection of an elastic membrane called the cupula, subsequently eliciting responses from sensory hair cells that transmit signals to the nervous system.

Historically, research has primarily focused on understanding how the cupula deflects within this system, by theorising a second order dynamical system. Recent studies, such as those by Obrist et al., have explored modelling semicircular canals with uniform widths to yield precise analytical solutions for the cupular deflection.

Our study aims to expand upon this by considering semicircular canals with non-uniform widths to better understand the role of the utricle. By deriving and asymptotically solving the equations governing fluid motion within these toroidal channels, we can account for variations in width that reflect utricular

function. We introduce new solution methods, resulting in accurate leading-order solutions up to second order in the small aspect ratio between the width of the canal and its arc length.

From the nonlinear PDEs governing fluid flow, through asymptotic analysis, the calculation is reduced to a second order non-autonomous dynamical system for the cupular deflection. Corrections to the leading order problem gives rise to a Volterra-like integral equation. Furthermore, arbitrary forcing can be accounted for with Laplace transformations.

Using our refined model, we can predict how the body responds to rotational stimuli and evaluate manoeuvres aimed at preventing or reducing instances of balance disruption. In particular, we use this model to investigate simple situations such as: after spinning around a fixed number of times, is there an optimal degree of counter-spinning that minimises the sensation of dizziness? Further, we combine our model with simple experimentation.

This research enhances our understanding of the mechanisms underlying balance regulation, with potential implications for developing therapeutic interventions or vestibular prosthetics.

Network inference for synchronous and asynchronous oscillatory systems based on the circle map

Akari Matsuki (Hokkaido university)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: For understanding the oscillatory systems, inference of coupling network from observed signals is an important task. Various methods have been developed for inferring the coupling from asynchronous oscillatory systems. However, inference of coupling between well-synchronized oscillators has remained difficult. In this study, we propose a new inference method based on the circle map. Our method allows accurate inference for a wide class of oscillatory systems, regardless of synchrony or asynchrony.

Adapting Lyapunov Functions for ODE Systems Into Functionals for DDE Systems

Julian Christopher (Wilfrid Laurier University, Waterloo, Ontario, Canada), **Connell McCluskey** (Wilfrid Laurier University, Waterloo, Ontario, Canada)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Lyapunov functions are a valuable tool for the global stability analysis of nonlinear dynamical systems. However, they are notoriously difficult to find, even for ODEs, and there is no general method for constructing them. Despite this, Lyapunov functions have been found for many dynamical systems, giving a “library” of pairs: (dynamical system, Lyapunov function). This library can be drawn upon when exploring the stability of a dynamical system that is similar to a system for which a Lyapunov function is known.

When studying a delay differential equation (DDE), there is typically an associated ODE that can be obtained by setting the delay equal to zero. If the associated ODE appears in the library, then this can give a good starting point for finding a Lyapunov function for the DDE.

In this talk, we will present a set of theorems that allow us to adapt known Lyapunov functions for ODE systems to a class of related DDE systems. Examples from mathematical epidemiology will be included.

Dynamics of an SEIR model with exogenous reinfections and quarantine control

Danchen He (Freie Universität Berlin), **Isabelle Schneider**

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Building upon the SEIR model proposed by Isaac Mwangi et al., our study delves into the dynamic behavior of the SEIR model with exogenous reinfection influenced by quarantine control and transmission rates. In addition to the known backward bifurcation behavior, we prove that the model exhibits transcritical bifurcation when the basic reproduction number $\mathfrak{R}_0 = 1$. Our findings illuminate the significant roles played by the disease transmission rate β and the exogenous reinfection rate $\tilde{\rho}$ ($\tilde{\rho}$ is the exogenous reinfection rate influenced by quarantine) in altering the qualitative dynamics. We explore the global stability conditions of endemic equilibrium under the joint influence of these two parameters. Furthermore, we delve into how our tuberculosis transmission model undergoes Hopf bifurcation under β and $\tilde{\rho}$.

High Energy of Recurrent Chaotic Trajectories in a Time-Dependent Potential Well Model

Matheus Palmero (USP, Fapesp)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: In this study, we explore the Recurrence Quantification Analysis (RQA) of chaotic trajectories, which provides a novel numerical method for detecting uncommon dynamical behaviours. By defining an ensemble of initial conditions, evolving them until a given maximum iteration time, and computing the recurrence rate of each orbit, it is possible to find particular trajectories that widely differ from the average dynamical behaviour. Based on this approach, we analyse the effects of these recurrent chaotic trajectories in a symplectic model for a time-dependent potential well. We show that for specific values of parameters and initial conditions, the chaotic system may experience rare transitory states of high energy caused by temporary but sufficiently long quasi-periodic dynamics.

Entropy in the mathematical theory of dynamical systems: a historical overview

Roland Gunesch (University of Education Vorarlberg)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: This poster presents a historical overview of various types of entropy in the mathematical theory of dynamical systems, as well as entropy in the natural sciences, including thermodynamics. The mathematical discussions of entropy include the well-known types of topological entropy and measure-theoretic entropy. There is a discussion of historical roots of the concepts of entropy in theoretical thermodynamics and connections with modern topics, such as information theory.

In the context of information theory, the classical concept of entropy has gained much importance, and so have the mathematical theories of entropy. Foremost among these is the discussion of entropy in the theory of dynamical systems, where concepts of uncertainty and distribution of ensembles have been formalized and turned into a mathematically rigorous theory.

This poster covers a long history of the use of entropy from the 1850s to the most modern applications in the mathematical treatment of Big Data in the 2020s.

Emergence of multifrequency activity in a multi-population neural mass model

Raul de Palma Aristides (Pompeu Fabra University (UPF)), Pau Clusella (Universitat Politècnica de Catalunya), Roser Sanchez-Todo (Pompeu Fabra University (UPF) / Neuroelectronics), Giulio Ruffini (Neuroelectronics), Jordi Garcia-Ojalvo (Pompeu Fabra University (UPF))

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Non-invasive measurements of brain activity such as EEG and LFP frequently capture the collective dynamics of thousands of neurons. One of the characteristic features of these measurements is the presence of oscillations at multiple frequency bands. Understanding how rhythmic patterns emerge and change can not only shed light on how the brain functions but also provide insights into spectral anomalies that are characteristic of conditions such as Alzheimer's disease, Parkinson's disease, and epilepsy.

Neural mass models (NMM) aim to trace the principles underlying such macroscopic neural activity from the intricate and multi-scale structure of the brain. Recently, Sanchez-Todo et al. proposed a Laminar NMM (LaNMM) capable of displaying coupled slow and fast oscillations (Sanchez-Todo et al., 2023), resulting from the interaction between different cortical layers. This simultaneous oscillatory activity allows for studying mechanistically different disease-related spectral dysfunctions.

Here, we analyze how oscillations emerge in the LaNMM through an extensive bifurcation analysis of its parameter space. Our results show that the model is capable of displaying periodic, quasi-periodic, and chaotic oscillations at alpha and gamma frequency ranges. This study enhances our understanding of the LaNMM and holds promise for advancing more accurate whole-brain computational models and effective neuromodulation therapies for neurological disorders.

Dynamics of buyers populations in fresh product markets

Ali Ellouze, Bastien Fernandez

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We introduce and study a discrete time model for the buyers population dynamics in a market with N competing sellers. Buyers are partly loyal to previously visited sellers and partly sensitive to sellers intrinsic attractiveness. The sellers in turn adapt their attractiveness depending of the relative volumes of clientele, introducing a negative feedback in the dynamics. We identify a sufficient condition on the system characteristics, on the seller reactivity rate in particular, for the local asymptotic stability of the fixed points. Moreover, we provide a counter example of instability when this condition is violated. We finally prove that fixed points global stability must occur under a concavity type condition, as long as seller attractiveness is not indifferent. Altogether, our analysis provides mathematical insights into the consequences of introducing feedback into buyer-seller interactions and its diversified impacts on the long term levels of clientele in the markets.

Title: Upper fractal Weyl bounds for Hecke triangle surfaces with non-unitary twist

Jan Klüver

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Abstract: Counting resonances, in the sense of upper fractal Weyl laws, of the Laplace-Beltrami operator on hyperbolic surfaces got some attention in recent decades motivated through application possibilities in physics. For Schottky surfaces, using the Selberg-Zeta function and transfer operator techniques, Guilloupe-Lin-Zworski have established an upper fractal bound which is in line with a bound conjectured by Sjöstrand. For surfaces with finite volume and cusp, i.e. $SL(2, \mathbb{Z})$, the distribution of resonances is also known. An upper bound in the case of infinite area surfaces with cusp (Hecke triangle surface) and unitary twist has been established by Naud-Pohl-Soares but the precise asymptotics of the resonance counting function aren't known yet. In this poster, we present the application of different transfer operator techniques to gain additional information about the asymptotics for the counting function in the aforementioned case of Hecke triangle surfaces while using a non-unitary twist. This involves the usage of the Lerch-Zeta function.

A time-delayed model for active mode-locking

Elias Koch (University of Münster), **Svetlana Gurevich** (University of Münster), **Julien Javaloyes** (Universitat de les Illes Balears)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We study theoretically the dynamics and bifurcations of an actively mode-locked laser (AML) by employing a delay differential equation model. The model is devoid of restriction regarding the values of the round-trip gain and losses allowing us to access the typical regimes encountered in semiconductor lasers and to perform an extended bifurcation analysis. Close to the harmonic resonances and to the lasing threshold, we recover the Hermite-Gauss solutions. However, the presence of the linewidth enhancement factor induces complex regimes in which even the fundamental solution becomes unstable. We discover a global bifurcation scenario in which a single pulse can jump, over a slow timescale, between the different minima of the modulation potential. Finally, we derive a Haus master equation close to the lasing threshold which shows a good agreement with the original time-delayed model.

On an extension of a topological conjugation between avalanche dynamics in neural networks and translations on the n -Torus

Lothar Dirks (University of Bremen)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Neuronal avalanches have developed into an omnipresent tool for describing the behaviour of large neuronal networks and have thus been an active field of research for several years. For a generalisation of the EHE model with arbitrary connection topologies Ernst, Keßböhrer, and Schünemann analysed these in 2022. In the process, the authors established a topological conjugation of their model to a simple translation dynamics on the n -Torus, which allows for an in-depth analysis of the dynamics of the model

and results in exact avalanche distributions and closed-form analytic expressions for avalanche sizes. In this poster, we demonstrate that this topological conjugation can be extended to a larger class of models and present some resulting avalanche analyses for a selection of these models.

Mixed Dirichlet-Robin problem for coupled anisotropic Darcy-Forchheimer-Brinkman equations

Andrei Gasparovici (Babes-Bolyai University, Faculty of Mathematics and Computer Science, Cluj-Napoca)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We study a mixed Dirichlet-Robin boundary value problem for a system of coupled nonlinear anisotropic Darcy-Forchheimer-Brinkman equations as a general model of viscous flows in multidisperse porous media.

First, we consider the mixed Dirichlet-Robin problem for the anisotropic Brinkman system, which is a generalization of the Brinkman system for fluids with an anisotropic viscosity tensor. Regarding this problem, we present a well-posedness result obtained using a variational technique. Then, we obtain an existence and uniqueness result concerning the mixed problem for the nonlinear anisotropic Darcy-Forchheimer-Brinkman system with sufficiently small data using the Banach fixed-point theorem.

Second, we consider the mixed Dirichlet-Robin problem for multiple coupled anisotropic Darcy-Forchheimer-Brinkman equations. Such a system generalizes models proposed in the literature for describing fluid flows in bidisperse and tridisperse porous media. Concerning this problem, we obtain an existence and uniqueness result by employing the Perov fixed-point theorem, a generalization of the Banach fixed-point theorem for operators defined on spaces endowed with vector-valued metrics.

Finally, as applications of the previously mentioned boundary value problems, we present numerical results related to viscous incompressible fluid flows in bidisperse and tridisperse porous media.

Multidimensional dust-acoustic rogue waves in electron-depleted complex magnetoplasmas

Weaam Alhejaili (Princess Nourah bint Abdulrahman University)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Modulation instability (MI) and characteristics of dust-acoustic rogue waves (DARWs) are theoretically investigated in electron-depleted dusty magnetoplasmas comprised of massive dust grains of opposite polarities and nonextensive ions by considering a three-dimensional geometry. For this purpose, the (3+1)-dimensional nonlinear Schrodinger equation is derived using the derivative expansion method. Also, the criteria of the MI are derived and discussed in detail. Numerical analysis revealed that stable and unstable domains of the modulated waves are strongly sensitive to the magnetic field, ions nonextensivity, and some relevant dust grain characteristics (mass and density). It is worth to mention that the frequency range where the MI varies as in the one-dimensional (1D) case has been also identified. The obtained results are important to understand the DARWs in some plasma environments, including interstellar medium, Jupiter's magnetosphere, upper mesosphere, comets, Saturn's rings, and Earth's atmosphere.

Mathematical Analysis of a Model of Coupled PDEs and its Application to Pedestrian Flow Analysis

Doris Bohnet (HTWG Hochschule Konstanz Technik, Wirtschaft und Gestaltung), Rebekka Axthelm (HTWG Hochschule Konstanz Technik, Wirtschaft und Gestaltung), Matthias Schmid (ZHAW School of Engineering), David Bernhardsgrütter (ZHAW School of Engineering)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We consider a continuous mathematical model to describe the pedestrian flow in evacuation scenarios - close to the model equations proposed by Hughes (2002) and consisting of the regularized eikonal equation coupled with an advection-diffusion equation and a so-called fundamental diagram which links walking speed to the local crowdedness. While the eikonal equation provides us with the walking direction of the pedestrians, the advection-diffusion equation computes the development of the density of pedestrians in time and space. To our knowledge, there exist no existence and stability results for the general case of this model, but only partial results in the one-dimensional setting. In our article, we revisit the model equations and the corresponding mathematical theory for existence and uniqueness of solutions, the a priori error

bounds for the corresponding finite element approximations and fill in missing informations. Further, we numerically confirm the convergence rates and theoretically found (in)stability results.

Application of dynamical entropies in measuring the complexity of time series

András Rusu (Babes-Bolyai University Cluj-Napoca), **Bulcsú Sándor** (Babes-Bolyai University Cluj-Napoca), **Zsolt Lázár** (Babes-Bolyai University Cluj-Napoca), **Dénes Károly** (Babes-Bolyai University Cluj-Napoca), **Mária Ercsey-Ravasz** (Babes-Bolyai University Cluj-Napoca)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The success of differentiating empirical time series based on their dynamical properties strongly depends on how accurately one can estimate time-like correlations. Recent developments on the topic include methods using artificial neural networks to assess the complexity of several data sets [1] and epsilon-machine construction to discriminate planetary complexity [2]. Here we show complexity measures that can differentiate between different dynamics even if the coarse-graining technique is rudimentary. Shannon entropies of coarse-grained states (also called symbols) of the dynamics only depend on the distribution of occurring states and thus they are insensitive to reshuffling of the symbolic time series. In addition to these, we use here the well-known Kolmogorov-Sinai (SK) entropy of symbol-space trajectories together with a related quantity, the Lyapunov measure [3]. To estimate the probability of trajectories, a Markovian approximation of the dynamics is used and both are computed from the transition probability matrix that belongs to a given symbolization scheme (partitioning into equally-sized cells and ordinal patterns were tested). An advantage of these measures is that they're general, once the time series has been symbolized, they can be computed regardless of the symbolization scheme. As an example, in sleep EEG data one can observe distinct clusters in the complexity measure space corresponding to several sleep stages as well as their surrogates. The absolute values may depend on whether we use ordinal patterns or grid partitioning but discrimination is seen in both cases wrt. these measures.

- [1] Boaretto, B.R.R., Budzinski, R.C., Rossi, K.L. et al. Discriminating chaotic and stochastic time series using permutation entropy and artificial neural networks. *Sci Rep* 11, 15789 (2021)
- [2] Bartlett, S., Li, J., Gu, L. et al. Assessing planetary complexity and potential agnostic biosignatures using epsilon machines. *Nat Astron* 6, 387–392 (2022)
- [3] Sándor, B.; Schneider, B.; Lázár, Z.I.; Ercsey-Ravasz, M. A Novel Measure Inspired by Lyapunov Exponents for the Characterization of Dynamics in State-Transition Networks. *Entropy* 2021

Self-organized traveling pulses shape seagrass meadows

Daniel Ruiz-Reynés (Laboratory of Dynamics in Biological Systems (DiBS) and Institute for Cross-Disciplinary Physics and Complex Systems (IFISC)), **Elvira Mayol**, **Tomas Sintes**, **Iris E. Hendriks**, **Emilio Hernández-García**, **Carlos M. Duarte**, **Núria Marbà**, **Damià Gomila**

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Seagrasses provide numerous ecosystem services and act as intense carbon sinks in coastal regions worldwide but are threatened by multiple anthropogenic pressures, leading to enhanced seagrass mortality that reflects in the spatial self-organization of the meadows. In general, scale-dependent feedbacks drive the dynamics that govern the spatio-temporal evolution of vegetation, leading to the emergence of spatial patterns in such different ecosystems as drylands, peatlands, salt marshes, or seagrass meadows. Spatially extended interactions between plants not only generate regular patterns in the spatial distribution of vegetation but reveal critical information to assess the health of the ecosystem. Our work reports on the formation of vegetation traveling pulses, which create complex spatiotemporal patterns and rings in the Mediterranean seagrass meadows of *Posidonia oceanica*. We demonstrate that these structures arise from an excitable behavior resulting from the coupled dynamics of vegetation and porewater hydrogen sulfide in the sediment, which is toxic to seagrass. The observed spatiotemporal patterns resemble those formed in other physical, chemical, and biological excitable media, but on a much larger scale. Based on theory, we derive a model that reproduces the observed seascapes and predicts the annihilation of these circular structures as they collide, a distinctive feature of excitable pulses. Combining aerial images taken in the field

and the empirically resolved radial profiles of vegetation density and sediment sulfide concentration across the structures, we show that the patterns are consistent with predictions from the theoretical model. Our results identify these spatial structures as a harbinger of the terminal state of the seagrass meadows prior to their collapse underscoring their diagnostic value in assessing ecosystem degradation.

A probabilistic approach to dispersal in spatially explicit meta-populations

Rajat Karnatak (Carl-von-Ossietzky Universität Oldenburg), Sabine Wollrab (IGB Leibniz-Institute of Freshwater Ecology and Inland Fisheries)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: While existing meta-population/community models have advanced our understanding regarding species persistence/biodiversity patterns, they often rely on deterministic assumptions to simplify the underlying species dispersal process. This study proposes a new framework, which implements dispersal as a probabilistic process across a spatially explicit network to describe species movement within a meta-population/community. This approach differs significantly from previous studies and reveals a more realistic representation of the system. Importantly, this network approach can also capture the transient nature of dispersal-mediated connections between habitats, thereby offering an ability to make short-term predictions about species distribution within meta-populations/communities.

Travelling fronts in a two component PDE with spatially dependent coefficients

Lara van Vianen (Leiden University), Martina Chirilus-Bruckner (Leiden University), Frits Veerman (Leiden University)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We consider a two component singularly perturbed system of reaction-diffusion equations with spatially heterogeneous coefficients. There are two stable equilibria, which depend on space due to the heterogeneity. Constructing just these background states is already a difficult challenge, since the nonautonomous system of differential equations forces to work with noncompact invariant manifolds. Consequently the theorems of Fenichel for geometric singular perturbation theory needs to be generalized to this noncompact setting. A travelling front is an entire solution of the PDE such that for each fixed time the profile is a sharp interface connecting the two background states asymptotically. Both the speed and the shape of the profile of such a front are varying in time which renders a comoving frame approach useless. We manage to prove existence of travelling fronts using an intricate spatio-temporal approach.

Physics of Electric and Magnetic Imaging of Cardiac Muscle Activity and Its Consequences

Igor Olczak (Warsaw University of Technology), Kazimierz Pęczalski (Warsaw University of Technology), Teodor Buchner (Warsaw University of Technology), Judyta Sobiech (Warsaw University of Technology), Maryla Zajdel (Warsaw University of Technology)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Advent of a new class of atomic (SERF) and quantum magnetometers has opened wide perspective, in particular to cardiac imaging. This exposed a need to develop a physical theory for their interrelation, which would bridge the gap between measured data and the clinical state of the patient. In our opinion such a theory should not be based on traditional approach, which considers only remote source, passive tissue and a magnetometer. It seems more reasonable to use the 300 years old Huyghens principle, which states, that each point of the physical medium becomes a source of elementary spherical wave. According to this principle, the physical quantity which is being measured comes from the local properties of the tissue right under the head of the magnetometer probe. We draw a sketch of the general measurement model for the Magnetocardiography (MCG) combined with Electrocardiography (ECG) and show how they complement each other in order to provide a wide perspective on the local functional state of the muscle. The submission is based on recent experimental results [1]. Specifically we show, that the resemblance between the ECG and the MCG is spurious, as one of them is based on spatial derivative, and the other on time derivative. As there quantities are interrelated, both images have similarities, but also have significant differences, that are

important on clinical level. Of significant importance is the length of the QT interval, which was previously shown to be significantly longer in MCG than in ECG [2], which we also confirm in our study [1]. It shows, that the tail of the activation wave extends longer, than what is shown by sole ECG recording. This finding seriously affects patient electrical stability against life threatening arrhythmias. It also shows importance of magnetic recordings, when cardiotoxicity of drugs or response to cardiological medications is concerned.

- [1] Pęczalski, K., Sobiech, J., Buchner, T. et al. Synchronous recording of magnetocardiographic and electrocardiographic signals. *Sci Rep* 14, 4098 (2024).
- [2] Smith, F. E. et al. Effect of averaging for the automatic measurement of QT dispersion using multichannel magnetocardiography and electrocardiography. In *Computers in Cardiology*, 2004 (IEEE, 2005).

Advanced Oceanic Particle Advection Simulation

Viacheslav Kruglov (Carl von Ossietzky University Oldenburg), Ulrike Feudel (Carl von Ossietzky University Oldenburg)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We have developed an advanced software to simulate the propagation dynamics of substantial quantities of tracer and inertial particles within interpolated oceanic velocity fields. The software is developed using C++, incorporating parallelization routines via Intel Threading Building Blocks (Intel TBB), which significantly enhances computational efficiency.

The velocity field data used in our simulations are taken from the HYCOM database, which is publicly accessible. To facilitate rapid nearest-neighbor searches, we construct a kd-tree of the grid points. This approach allows for efficient search of the grid points proximal to a given particle, enabling fast interpolation of the velocity field's eastward and northward components at the particle's location. We employ a Gaussian-shaped weight function for interpolation, which offers a robust alternative to inverse distance interpolation by avoiding singularities.

Our methodology also takes into account the spherical shape of the Earth surface, because all planar projections introduce distortions. To address this, we solve two-dimensional equations for tracer particles and the Maxey-Riley equation for inertial particles on a flat tangent plane. Subsequently, we convert the particle positions back to longitude-latitude coordinates using the azimuthal equidistant projection, thereby minimizing large-scale deviations in simulations spanning thousands of kilometers.

The software's capabilities extend to simulating the dispersal of seeds and algae by ocean currents, handling hundreds of thousands of particles under various initial conditions. It enables the reconstruction of connectivity maps between distant shores and the identification of transport barriers through the computation of finite-time Lyapunov exponents. Additionally, the software can compute derivatives of interpolated fields, including divergence, vorticity, and the Okubo-Weiss parameter, further broadening its applicability in oceanographic research.

We showcase the advanced features of our software through two illustrative examples. First, we examine the origins of particles, akin to plant seeds, and their potential pathways to Hawaii. Second, we evaluate the likelihood of harmful algae blooms reaching the Baffin Sea during peak summer temperatures.

Exploring dynamics in flocking systems in presence of noise

Harishankar Manoharan (Indian Institute of Technology Hyderabad (IITH)), **Induja Pavithran** (Ben Gurion University of the Negev, The Jacob Blaustein Institute for Desert Research), Vishnu Rajasekharan Unni (Indian Institute of Technology Hyderabad (IITH))

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: From a school of fish to a flock of birds, the seemingly synchronized movement of animals in the biological world has always fascinated humans. This collective motion results from interaction among multiple agents and is characterized by complex behaviors such as emergence, adaptation, and self-organization. A multi-agent model can describe systems undergoing collective motion, which can be numerically simulated using an agent-based approach.

Many physical systems are influenced by environmental and inherent noise. In general, noise is viewed as an undesired element. Previous studies have examined how different noise levels can affect the overall stability of different models for flocking. However, the effect of noise on the transient dynamics resulting in flocking, such as the time taken to form a flock, the formation of local clusters en route to flocking, etc., are poorly understood. Through the present work, our goal is to gain a deeper understanding of these aspects and uncover insights into the influence of noise on transient dynamics in flocking systems. We do this by defining novel network measures that uncover time-varying connectivity patterns within the flock.

For our study, we use a modified Reynolds flocking model, where each agent updates its velocity based on its tendency to align and prevent colliding with other agents in the flock and also to stay close to the center of the flock. We look at instances where the velocity information of agents is corrupted with noise. The collective motion of agents trying to reach a common consensus (in terms of direction, inter-agent distance, etc.) can be viewed as a dynamic network. Thus, the various phenomena associated with flocking, such as emergence and self-organization, can be studied by studying the underlying network dynamics. Through this work, we devise network measures that give insights into the patterns in which individuals are connected and how these connections evolve over time. The knowledge of time-varying connectivity in a network can shed light on aspects such as information flow within the network, clustering behavior etc., and thus provide us with new measures to ascertain the quality of flocking. Further, these measures can serve as a scope into the transient nature of flocking, which happens over short time scales and hence are harder to analyse. With these measures in hand, we analyze flocks that are influenced by noise. Systems with noise have shown qualitatively better flocking in some cases compared to systems without noise. This indicates potential advantages in introducing or retaining noise in systems exhibiting collective behavior.

Phase-Isostable Reduction of Oscillatory Neural Mass Networks with Delays in local dynamics and network connections

Robert Allen (University of Nottingham, United Kingdom), Rachel Nicks (University of Nottingham, United Kingdom), Stephen Coombes (University of Nottingham, United Kingdom)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Large-scale brain signals that are seen in EEG and MEG recordings are believed to derive from the combination of local synaptic activity and non-local delayed axonal interactions. Considering networks of neural mass models, two sources of delays may be present; within node delays which can induce oscillations, and delayed network interactions between nodes which can strongly influence phase-locked network states and their bifurcations.

To understand how delays influence patterns of phase-locked states we aim to derive network equations for the phase and isostable coordinates of oscillations of each node, extending the work in [1] to include delays. This requires knowledge of the phase and isostable responses of the delay-induced oscillations of the nodes to perturbations in addition to incorporating coupling delays into the phase-isostable interactions.

We show how the response functions may be computed using a harmonic balance method, providing an improvement in efficiency and accuracy on standard solvers for delay differential equations.

[1] R. Nicks, R. Allen, and S. Coombes, *Chaos* 34, 013141 (2024)

The magnetic field application during the injection of medicine on it's dispersion in a cardiac tissue

Nataliya Gulko (Ben Gurion University of the Negev)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The magnetic field application during the injection of medicine on it's dispersion in a cardiac tissue N.G. Gulko (in collaboration with I.T Selezov) The Mathematical model of the effect of the magnetic field applied during the injection of medicine on it's dispersion in a cardiac tissue is presented. We study such phenomena as diffraction and nonlinear refraction causing self-focusing and self-defocusing in slightly non linear media with sufficiently strong dispersion. For some special case the exact analytical solution based on the Laplace transform and Ephros theorem was obtained. For the magnetic field fluctuations the

distance changes interesting and informative results were obtained . The result seems to be useful for cardiac treatment .

Coupled Systems with Periodic and Aperiodic Drivings

Jin Yan (WIAS Berlin)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: In this poster we first review and study dynamical properties of coupled periodically kicked rotor systems, focusing on bifurcations and spatio-temporal patterns; then we present recent works on coupled rotors with aperiodic kicks, leading to novel phenomena such as long prethermalisation, in particular, the prethermal lifetime can be algebraically controlled by temporal-correlations in the random kicks.

Revisiting Memory Capacity of a Reservoir Computer

Swarnendu Mandal (The University of Tokyo), Kazuyuki Aihara (The University of Tokyo)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Reservoir computing (RC) has been proven to be overwhelmingly successful in handling a wide range of complex tasks in nonlinear science. The root of its exceptional capability is the utilization of natural information processing capacity of a dynamical system in its hidden layer. However, the ‘memory-nonlinearity trade-off’ of a dynamical system creates a bottle-neck in performance of a reservoir computer. We explore an alternate architecture of RC without any internal dynamics in its reservoir layer. The reservoir states are created as a non-temporal function of discretized input sequence. The memory capacity is then enhanced using a combination of multiple delayed reservoir states. The existing ways utilize delayed reservoir states by vertical stacking. This significantly increases effective dimension of the reservoir states leading to increased computational cost. We propose an efficient method to combine multiple reservoir states without increasing its size. Thus, we achieve a great enhancement in memory capacity without compromising on computational resource.

A new paradigm in power grid modeling

Jakob Niehues (Potsdam Institute for Climate Impact Research (PIK), Technische Universität Berlin), **Anna**

Büttner (Potsdam Institute for Climate Impact Research (PIK), Humboldt-Universität zu Berlin)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Renewable energy sources such as wind turbines and solar cells are connected to the power grid via inverters. As we move towards a fully renewable grid, there is a growing need for so-called grid-forming inverters (GFIs) that can contribute to grid stability and synchronization. This shift presents a significant challenge as GFIs are a relatively new and complex technology, of which there is a limited practical and theoretical understanding.

The so-called normal form of GFIs is a general formulation of power grid dynamics that encompasses the space of all plausible grid actors. Due to its analytical tractability and intermediate dimensionality, the normal form is a promising tool for obtaining general, technology neutral stability conditions, as well as efficient numerical simulations of extreme event scenarios.

First contribution: Using system identification on detailed simulation data and lab measurements, we show that the normal form can accurately capture the dynamics of real-world GFIs.

Second contribution: We present the normal form framework and recent results in its application to linear stability.

Control-based continuation in an atomic force microscope experiment

Hannes Wallner (University of Rostock), Lukas Böttcher (University of Rostock), Ingo Barke (University of Rostock), Wolfram Just (University of Rostock), Niklas Kruse (University of Rostock), Sylvia Speller (University of Rostock), **Jens Starke** (University of Rostock)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We present experimentally obtained frequency-response curves in the near resonant regime of dynamic mode atomic force microscopy including a branch of unstable periodic orbits that are obtained through a control-based continuation method. In dynamic mode atomic force microscopy a microcantilever is monoharmonically forced in proximity (nanometer regime) of a sample. Highly nonlinear interaction forces between tip and sample induce unstable periodic orbits in the displacement motion of the tip, manifesting in nonlinear frequency response curves (bifurcation diagrams) with fold bifurcations. By utilizing adaptive filters to estimate Fourier coefficients online, we are able to stabilize formerly unstable periodic orbits in lab experiments of dynamic mode atomic force microscopy by a non-invasive phase-locked loop controller and measure complete frequency-response curves. As second bifurcation parameter, we change the distance between the cantilever tip and sample and show the transition from linear to nonlinear frequency-response curves.

Chemical Reaction-Diffusion Waves Visualize Gravitational Lensing

Niklas Manz (The College of Wooster, Wooster, OH, USA), **Kiyomi Sanders** (The College of Wooster, Wooster, OH, USA), **Daniel Cohen-Cobos** (California State University, Long Beach, CA, USA), **Heather Guarnera** (The College of Wooster, Wooster, OH, USA), **John Lindner** (North Carolina State University, Raleigh, NC, USA)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We are presenting a table-top analog for the astrophysical phenomenon known as Gravitational Lensing using reaction-diffusion (RD) waves, specifically the chemical Belousov-Zhabotinsky reaction. Gravitational lensing is a general relativistic (GR) phenomenon where a massive object redirects light, deflecting, magnifying, and sometimes multiplying its source. Observed perturbations of BZ wave fronts propagating in a hollow, quasi-two-dimensional acrylic glass mold while interacting with spatial tree-dimensional obstacles, which act like massive gravitational wells for light, were reproduced using a simulation written in Python. The laws of General Relativity were applied to model diffusion obstacles, representing the velocity changes due to the path length differences, that acts as a mass in space, creating deflection angles for the chemical wave fronts similar to the angles light rays experience while passing a massive object in space.

Superstatistics approach to doubly stochastic processes

Jan A. Freund (Carl von Ossietzky Universität Oldenburg)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Superstatistics blends a family of parametric distributions in accordance with a distribution of parameters. This is clearly applicable to situations where a statistical ensemble of independent systems is underlying as, for instance, an ensemble of stochastic processes with distributed parameters. In the asymptotic regime ($t \rightarrow \infty$) stationary distributions are blended which allows to construct power law statistics via mixing Gaussian distributions. The same superstatistics approach to a doubly stochastic process, i.e. a stochastic process where a parameter is itself a stochastic process, requires a separation of time scales to interpret the blended distribution in the framework of time-averaged stationary distributions. While this connection was already pointed out as a general statement a detailed treatment in the framework of stochastic processes is, to the best of my knowledge, still missing. Here, I fill this gap by numerically simulating a bivariate stochastic process and find a crossover from a marginal power law statistics, for well-separated time scales, to a normal distribution, when time scales are similar. In addition, I discuss this crossover in the framework of the related Fokker-Planck equation.

Driven Shock in Three Dimensions: Euler Equation Versus Molecular Dynamics, and Navier-Stokes Equation

Amit Kumar (The Institute of Mathematical Sciences, Chennai), **Rajesh Ravindran** (The Institute of Mathematical Sciences, Chennai)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Isotropic and continuous localised perturbations in a stationary gas, created by an external point source, cause a spherically symmetric shock wave with the energy $E(t)$ increasing in time t as $E(t) \sim t^\delta$,

where $\delta \geq 0$. The analytical solution of the Euler equation providing the spatio-temporal behavior of the shock wave, is a classic problem in gas dynamics. The exact solution shows that the asymptotic behavior of non-dimensionalised thermodynamic quantities obey power law behavior in rescaled distance near the shock center with the exponents independent of δ . However, using Event Driven Molecular Dynamics Simulations, we find that the exact solution does not match with EDMD results, anywhere, mainly in terms of the power law exponents near the shock center. We show that this mismatch is due to ignoring the contribution of heat conduction and viscosity terms, and the mismatch between theory and numerics can be resolved by taking into account the Navier-Stokes equation. We showed that the direct numerical solution of Navier-Stokes equation captures these observed power law exponents in the EDMD simulations indicating the significance of viscosity and heat conduction in shock problems.

Synchronized firing patterns shape topology in neuronal networks

Paulo Protachevicz (University of São Paulo (USP), Brazil), Fernando S. Borges (State University of New York Downstate Health Sciences University), Antonio Marcos Batista (State University of Ponta Grossa), Murilo Baptista (University of Aberdeen), Iberê Luiz Caldas (University of São Paulo), Elbert Macau (Federal University of São Paulo), Ewanson Lameu (Hotchkiss Brain Institute)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The brain has a remarkable capacity to encode information on firing patterns and synaptic connectivity. However, a complete understanding of the relationship between activity and structure in the neuronal networks has not been achieved yet. In light of this, we investigate how different synchronized firing patterns are related to the potentiation and depression of connections in a neuronal network with plasticity. We consider four neuronal networks connected by chemical synapses equipped with spike-timing-dependent plasticity (STDP), where each neuron activity is represented by the Hodgkin-Huxley model. Internally, each neuronal network is fully connected, but without auto-connections. Externally, between the networks, a small fraction of connections allowed is considered. Both, internal and external connections consider a weak initial intensity in the synaptic conductance. We focus on three main types of synchronized neuronal firing patterns observed experimentally between the cortical areas: phase, anti-phase, and shift-phase synchronization. We observe that each one of these synchronized firing patterns generates specific configurations in the connectivity between the neuronal networks. In conclusion, the STDP rule promotes equivalence between function and structure for synchronized firing patterns, indicating that topology arises from behaviour, besides behaviour emerges from topology.

Explaining Dynamic Catalysis via Periodically Forced Networks

Julius Fischbach

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Dynamic catalysis aims to overcome the limitations of conventional catalysts by applying an oscillating surface binding energy to the catalyst surface. By tuning the oscillation parameters, optimal conditions can be created for each intermediate reaction step, enhancing the overall reaction rate and/or product selectivity beyond the optimum set by Sabatier's principle.

To better understand how to optimize these oscillation parameters, we study a prototypical dynamic catalyst, modeled via a set of polynomial differential equations. We analyze this system using chemical reaction network theory and geometric singular perturbation theory and derive general conclusions about the optimal forcing protocol.

Minimal model for spatio-temporal excitability in plant-soil systems

Pablo Moreno Spiegelberg

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The mechanisms driving the formation of spatio-temporal structures in sessile species populations, such as plants or fungi, remain poorly understood in the literature. These structures include rings, wave trains, spirals, and spacetime chaos. A possible mechanism behind this behaviour is spatio-temporal

excitability, mediated by plant-soil interactions. To explore this hypothesis, we present a minimal model incorporating only the essential elements for excitable behavior in vegetation systems: negative feedback mediated by an inhibitor and direct positive feedback. This model has been applied to *Posidonia oceanica* meadows, where positive and negative feedbacks are well studied. The model's generality allows our results to be applied to other plant-soil spatially extended systems, regardless of the specific feedback mechanisms involved.

We describe the formation of traveling pulses and wave trains using bifurcation theory. The existence of these solutions can be traced back to the local (homogeneous) dynamics. We further investigate the stability of wave trains and traveling pulses, which can be linked to the formation of spirals, spatio-temporal chaos, target patterns, and pattern wavelength selection. Notably, different populated solutions coexist for a wide range of parameters, even beyond the classical tipping point, increasing the resilience of the system.

Scalable real-time coupled metronome experiment setup with controllable time dependent coupling strength and phase-lag aiming on power grid stability analysis

Michael Schiek (Forschungszentrum Juelich GmbH), Pezhman Ebrahimzadeh (Forschungszentrum Juelich GmbH), Yuriy Maistrenko (Forschungszentrum Juelich GmbH)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The synchronization behavior of coupled oscillators has lost none of its fascination since Huygens described his observations of coupled pendula centuries ago. One of the reasons for this is the apparent universality of the phenomena like e.g. synchronization, chimera, solitary states observed in comparatively simple systems. Prominent real-world examples where these phenomena are discussed to play a crucial role are the neuronal dynamics in the brain and the stability and, in particular, instability of power grids. Although the vast majority of studies of these phenomena are based on numerical simulations, the generation of correspondingly observed complex synchronization states in systems of coupled mechanical oscillators (e.g. mechanical metronomes) continues to be a litmus test for their relevance in real systems. Many experimentally setups of coupled metronomes have been presented in the last years in which e.g. Chimera states could be observed. However, due to the restricted flexibility of the coupling terms in most of these setups it was difficult to reproduce in the physical experiments the parameter studies performed by the numerical simulations (e.g. the effect of increasing phase lag in the coupling). Recently we proposed an electromagnetic coupling approach with the coupling terms being calculated by a real time system based on the contactless measured angles of metronomes rods (1). The setup required an arrangement of 7 Hall sensors per metronome for the real-time angle determination and thus was hardly scalable. To overcome this disadvantage, the Hall sensor arrangement was replaced by a single analog light barrier which is shaded by a spiral element proportional to the angle of the metronome rod. With this approach, the system requires one analog input and two analog output channels per metronome only and can be scaled easily. Additionally, the software, based on Mathworks® SIMULINK real-time framework, was extended allowing for time dependent variation of the coupling terms strength and phase lag.

The new system setup was specifically designed to perform physical experiments focusing on the role of asymmetry in the desynchronization of power grids. In the experiments including 4 metronomes we will mimic the cutting of a link between two network nodes or removing a node as has been numerically analyzed in (2).

- [1] Ebrahimzadeh, P., Schiek, M., Jaros, P., Kapitaniak, T., van Waasen, S., & Maistrenko, Y.; Minimal chimera states in phase-lag coupled mechanical oscillators. *The European Physical Journal Special Topics* (2020).
- [2] P. Jaros, R. Levchenko, T.Kapitaniak, J. Kurths, and Y. Maistrenko. Asymmetry induces critical desynchronization of power grids. *CHAOS* (2023)

Basin entropy and the escape choice in the Revtokamap model

Pedro Haerter Pinto (Universidade Federal do Paraná), Ricardo Luiz Viana (Universidade Federal do Paraná), Miguel Sanjuán (Universidad Rey Juan Carlos)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The basins of attraction constitute a fundamental part of studying dynamical systems, although they only appear naturally in dissipative systems. For conservative systems accessing the basins of attraction requires the system to be open, and the manner in which the exits are chosen can directly influence the results outcomes. In this work, we explore the impact of opening area choices on the escape basins by using a model of particles transported by field lines in tokamaks with reversed shear. These phenomena are quantitatively evaluated using the basin entropy and the fractal dimension across various system openings. Allied with the manifolds of the system, it is possible to explain why some abrupt changes happen in the entropy and how to avoid them.

Stability of nonlinear Dirac solitons under the action of external potential

David Mellado-Alcedo, Niurka R. Quintero

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The nonlinear Dirac equation in 1+1-dimensions supports localized solitons. Theoretically, these traveling waves propagate with constant velocity, energy, momentum, and charge. However, the soliton profiles can be distorted, and eventually destroyed, due to intrinsic or numerical instabilities. The constants of motion and the initial profiles can also be modified by external potentials, which may give rise to instabilities.

In this work [1], we study the instabilities observed in numerical simulations of the Gross-Neveu equation [2] under linear and harmonic potentials. We perform an algorithm [3] based on the method of characteristics to numerically obtain the two soliton spinor components. All studied solitons are numerically stable, except the low-frequency solitons oscillating in the harmonic potential over long periods of time. These instabilities are identified by the non-conservation of both energy and charge, and can be removed by imposing absorbing boundary conditions. We find that the dynamics of the soliton is in perfect agreement with the prediction obtained using an Ansatz with only two collective coordinates. By applying the same methodology, we also demonstrate the spurious character of the reported instabilities in the Alexeeva–Barashenkov–Saxena model [4] under external potentials.

- [1] D. Mellado-Alcedo and N. R. Quintero, “Stability of nonlinear Dirac solitons under the action of external potential.” *Chaos* 34, 013140 (2024)
- [2] D. J. Gross and A. Neveu, “Dynamical symmetry breaking in asymptotically free field theories”, *Phys. Rev. D* 10, 3235 (1974)
- [3] T. Lakoba, “Numerical study of solitary wave stability in cubic nonlinear Dirac equations in 1D,” *Phys. Lett. A* 382, 300 (2018)
- [4] N. V. Alexeeva, I. V. Barashenkov, and A. Saxena, “Spinor solitons and their PT-symmetric offspring,” *Ann. Phys.* 403, 198 (2019).

Testing as an intervention in multi-pathogen epidemic models

Seba Contreras (Max-Planck-Institute for Dynamics and Self-Organisation)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Human behavior is one of the most unpredictable and uncertain factors in epidemiological modeling. Testing, which is crucial for understanding the spread of diseases, can also influence behavior. When people receive a positive or negative test result, they might change their behavior and alter the course of epidemics. On the one hand, including testing in epidemic models can expand the stable regime for values of the reproduction number that are greater than one. On the other hand, testing can also create conflicting results when driven by another disease, where measures to reduce the spread of one disease may artificially show the opposite effect. In this study, we explore the impact of testing as an intervention in multi-pathogen epidemic models. We apply this framework to study two examples: 1) SARS-CoV-2 and RSV subject to non-pharmaceutical interventions, studying the rebound waves that followed the relaxation of non-pharmaceutical interventions, and 2) HIV and curable STIs subject to HIV pre-exposure prophylaxis and behavioral self-regulation, studying the effect that a theoretical drug shortage might have on the endemic equilibrium. We analyze the possible dynamic regimes and their stability across parameter ranges, accounting for behavioral responses, compliance, and drug supply uncertainties.

Computing with nonlinear dynamical systems

Aida Todri-Sanial (Eindhoven University of Technology), Filip Sabo (TU/e University of Technology Eindhoven)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We discuss the computational capabilities of nonlinear coupled oscillatory networks. Oscillator neural networks (ONNs) consist of fully to sparsely coupled oscillator systems that exhibit rich dynamical states. Though, how to ensure that the ONN dynamics can effectively perform computation and under which conditions, is the focus of our research.

ONN computational model is based on oscillators acting as an activating neuron and coupling strengths between oscillators serve as the synaptic connections, taking inspiration from biological neural networks. We show that information can be encoded in the phase relations of oscillators that can further be exploited either for in-memory computing or solving combinatorial optimization problems.

Both oscillator behavior and coupling strengths between oscillators play an important role in the dynamics of the system by enabling oscillators to reach in-phase, out-of-phase or chaotic synchronization states. We investigate both linear and nonlinear oscillators to understand the difference in the complex dynamics that ONNs can achieve and their impact on computational tasks.

We present a detailed study of exploiting the behavior of nonlinear oscillators for performing different computational tasks. We study fully connected coupled oscillatory networks with nonlinear oscillator such as van der Pol model. Our results show that certain computational tasks such as pattern retrieval or solving optimization problems benefit from harnessing nonlinearity of oscillators that allows for higher accuracy and fast time to solution.

Scaling laws for deep neural network robustness

Rico Berner (Ambrosys GmbH), Thomas Seidler (Ambrosys GmbH), Markus Abel (Ambrosys GmbH)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Deep neural networks (DNNs) are the architectural basis for almost all modern uses of Machine Learning (ML). It is well-known how model performance scales with the number of parameters and amount of training data used (Neural Scaling Laws) in the limits of very large models and large amounts of training data. The usage of ML models for security and safety relevant applications necessitates a careful analysis of the models' capabilities in sub-optimal environments. In the context of the automotive projects Mannheim-Emdrive and Mannheim-CeCas, we develop test scenarios to benchmark particular AI components of systems for autonomous drive. Inference is especially affected by data corruptions, e.g. sensor deprivations, difficult weather conditions (snow, fog), or general, not only visual, noise in computer vision. We systematically study the generalization error of multiple state of the art image classification models on different datasets. We find a phase transition in model performance that depends on the intensity of noise in the data, and give the scaling laws for different (model, data) pairs, thus categorizing ML problems into universality classes with respect to their robustness to noise and their scaling behavior.

Self-excited and hidden attractors in a discontinuous Jerk system

Thoraya Alharthi (University of Bisha)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: This work aims to investigate the existence of self-excited and hidden attractors in discontinuous systems involving sliding mode. The hypersurface plays a critical role in allowing rapid transient responses to dynamic phenomena. To show the effect of a hypersurface on the dynamics of a discontinuous system, we present a simple discontinuous system composed of dissipative linear subsystems with no equilibria. Our results show that the periodic orbit can be completely established within a sliding region. We then proceed to a mathematical investigation into a path to self-excited and hidden attractors involving sliding mode in a discontinuous system composed of dissipative Jerk-like subsystems separated by a hypersurface. A Poincaré return map is developed to incorporate the presence of the hypersurface, providing the necessary conditions on the basis for sliding period-doubling orbits that lead to the sliding chaotic attractor. Our results

show that various types of attractors and their coexisting attractors, such as sliding period doubling and chaotic behaviour, are successfully captured. Using path-following techniques for discontinuous dynamical systems, the results are verified numerically.

Exploring Coupled Oscillator Networks with Highly-Configurable Integrated Circuit Designs

Salwa Yasmeen Neyaz (Forschungszentrum Juelich), Michael Schiek (Forschungszentrum Juelich), Arun Ashok (Forschungszentrum Juelich), Christian Grewing (Forschungszentrum Juelich), Pezhman Ebrahimzadeh (Forschungszentrum Juelich), Andre Zambanini (Forschungszentrum Juelich), Stefan van Waasen (University Duisburg-Essen, Duisburg, Germany)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: The analysis of the complex dynamics of coupled oscillator networks is crucial not only for the understanding of corresponding systems in biology (i.e., brain dynamics) but also for our technical world (e.g., the stability of power grids). Moreover, this knowledge paves the way to use coupled oscillator systems for bio-inspired computing. Both analytical methods and, in particular, numerical simulations have provided fundamental insights into the existence and coexistence of synchronization states and different symmetry breaking in completely symmetrical oscillator networks such as chimera states or solitary states (1). For example, the influence of the coupling strength and the phase shift in the coupling on the network dynamics was investigated in detail (2). However, despite the computing power of modern computers available today, there are limits to analyzing very large networks and their transient or adaptive dynamics over very long periods using numerical simulations. A way to overcome these restrictions is to perform experiments with physical implementations of large-scale coupled oscillator systems using state-of-the-art integrated circuit technology. Most of recently presented developments focus on bio-inspired computing applications and thus are rather restricted concerning the configurability of network topology and coupling terms (3). To mimic the response of real-world oscillatory networks like power grids to varying external and internal conditions, one needs to be able to change coupling topology and coupling terms of the physical implemented during the experiment, i.e., ‘on the fly’.

In our proposed integrated circuit system designed in a 28 nm CMOS technology, we employ a scalable architecture, organizing oscillators into clusters with adjustable all-to-all coupling within each cluster. A high level of configurability allows for programmable coupling terms (phase shift and coupling strength) within and between the clusters. The oscillators are realized by type 2 Phase Locked Loop (PLL) circuits of third order. The voltage controlled oscillators (VCOs) are implemented using ring oscillators, which are well known standard CMOS building blocks. The connectivity among the PLLs is studied with two alternative approaches, either by employing multiple phase and frequency detectors (PFD) and logic gates leading to the charge pump node or with multiple charge pumps directed towards a VCO node.

The eigenfrequency of the nodes can be configured individually, typically lying in the range of 10 MHz. External inputs can be fed into dedicated nodes by either modulating the frequency or initialization of phases of the controlled oscillators. The system dynamics are determined during operation in terms of phase and frequency synchronization within and between the clusters and this information is available in real-time, e.g., for control purposes. The proposed system is very well suited for exploring the complex long-term dynamics of large-scale oscillator networks.

[1] Maistrenko, Y., Penkovsky, B., & Rosenblum, M.; *Physical Review E* (2014).

[2] Ebrahimzadeh, P., Schiek, M., & Maistrenko, Y.; *CHAOS* (2022)

[3] Csaba, G., & Porod, W.; *Applied physics reviews* (2020)

Excitability and memory in a time-delayed optoelectronic neuron

Jonas Mayer Martins (University of Münster), Svetlana Gurevich (University of Münster), Julien Javaloyes (Universitat de les Illes Balears)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We study the dynamics of an optoelectronic circuit composed of an excitable nanoscale resonant-tunneling diode (RTD) driving a nanolaser diode (LD) coupled via time-delayed feedback. Using a com-

bination of numerical path-continuation methods and time simulations, we demonstrate that this RTD-LD system can serve as an artificial neuron, generating pulses in the form of temporal localized states (TLSs) that can be employed as memory for neuromorphic computing. In particular, our findings reveal that the prototypical delayed FitzHugh-Nagumo model previously employed to model the RTD-LD resembles our more realistic model only in the limit of a slow RTD. We show that the RTD time scale plays a critical role in memory capacity as it governs a shift in pulse interaction from repulsive to attractive, leading to a transition from stable to unstable multi-pulse TLSs. Our theoretical analysis uncovers novel features and challenges, including the multistability of TLSs and attractive interaction forces, stemming from the previously neglected intrinsic dynamics of the laser. These effects are crucial to consider since they define the memory properties of the RTD-LD.

Bifurcation of self-sustained activity in randomly coupled theta neurons

Ralf Tönjes (Universität Potsdam), Chunming Zheng, Wenping Cui

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Disordered networks of spiking neurons are known to sustain a regime of persistent irregular firing. When the average coupling strength is zero, common mean field theory can no longer predict the network state. Instead dynamic mean field theory is needed to describe the statistics of single neurons driven by dynamic noise, based self-consistently on the statistics of its neighbors and the coupling matrix. We analyze these self-consistent solutions with respect to changes in the pulse shape and for different random coupling matrix ensembles.

Phase Synchronization in a Sparse Network of Neurons with Random Connections Influenced by Poissonian Spike Inputs

Bruno Boaretto (Federal University of São Paulo (UNIFESP)), Elbert Macau, Cristina Masoller (Polytechnic University of Catalonia (UPC))

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: This poster investigates the emergence of phase synchronization in a network of randomly connected neurons by chemical synapses. The study uses the classic Hodgkin–Huxley model to simulate the neuronal dynamics under the action of a train of Poissonian spikes. In such a scenario, we observed the emergence of irregular spikes for a specific range of conductances and also that the phase synchronization of the neurons is reached when the external current is strong enough to induce spiking activity but without overcoming the coupling current. Conversely, if the external current assumes very high values, then an opposite effect is observed, i.e., the prevention of the network synchronization. We explain such behaviors considering different mechanisms involved in the system, such as incoherence, minimization of currents, and stochastic effects from the Poissonian spikes.

Quasi-potential landscape in bistable cloud system

Benjamin Hernandez (TU Delft/Faculty of Civil Engineering and Geosciences), Franziska Glassmeier (TU Delft/Faculty of Civil Engineering and Geosciences)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Clouds are an integral part of our climate system, as they play a fundamental role in the Earth's energy budget and water cycle. Among these, stratocumulus clouds stand out as the largest cloud type by coverage, characterized by their distinctive bistable cellular patterns. Despite their significance, current climate models struggle to accurately represent these clouds, leading to considerable uncertainties in our climate project. In this study, we employ a 2-dimensional dynamical system framework to investigate stratocumulus cloud fields. Our analysis reveals an S-shaped bifurcation in all major parameters, with two attractors characterized by substantially different relaxation times. By employing the large-deviation formalism, we map the Freidlin and Wentzell quasi-potential landscape. We aim to characterize the observed transitions between the two states and explain the observed frequency disparity in their occurrence. Fur-

thermore, we explore the system's resilience and provide insights into how all of these properties might change in future climates.

Temporal localized states in an injected Kerr-Gires-Tournois interferometer in the regime of anomalous dispersion

Tim Lohmann (Universität Münster), Thomas Seidel (Universität Münster), Julien Javaloyes (Universitat de les Illes Balears), Svetlana Gurevich (Universität Münster)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We are interested in the dynamics of temporal localized states (TLSs) in an injected Kerr-Gires-Tournois interferometer (KGTI) in the regime of anomalous dispersion. In this regime and in the uniform field limit, the first principle model based upon delay algebraic equations can be approximated by the Lugiato-Lefever partial differential equation, giving rise to the formation of TLSs. Using the combination of numerical simulations and path continuation techniques, we demonstrate that for parameter values far away from the Lugiato-Lefever regime a new form of short high-intensity TLSs emerges.

Oscillons: variational formalism and numerical continuation

Nora Alexeeva (University of Cape Town)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Oscillons are long-lived localised pulsating states in the nonlinear Klein-Gordon equations. We formulate a multiscale variational method for the analysis of oscillons that is free from singularities that marred all previously proposed variational techniques. An alternative approach treats oscillons as standing waves in a finite-size box; these are sought as solutions of a boundary-value problem on a two-dimensional domain. For the Kosevich-Kovalev model with a symmetric vacuum, a single-harmonic variational Ansatz provides an excellent agreement with the numerical results. For a model with broken symmetry (the ϕ^4 equation), the numerical analysis reveals that the standing wave's energy-frequency diagram is fragmented into disjoint segments with $\omega_{n+1} < \omega < \omega_n$. In the interval (ω_{n+1}, ω_n) , the structure's small-amplitude wings are formed by the n -th harmonic radiation ($n = 2, 3, \dots$). The variational approximation involving the first, zeroth and second harmonic components provides an accurate description of the oscillon with the frequency in (ω_3, ω_2) , but breaks down as ω falls out of that interval.

[1] I V Barashenkov and N V Alexeeva. Phys Rev D **108** 096022 (2023).

[2] N V Alexeeva, I V Barashenkov, A Dika and R De Sousa. Submitted for publication.

Population dynamics and games of variable size

Matheus Hansen (Center for Mathematics and Applications (NOVA Math), NOVA FCT, Universidade NOVA de Lisboa)

Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: This work introduces an initial concept of Variable Size Game Theory (VSGT), in which the number of players in a game is a strategic decision made by the players themselves. We start by discussing the main examples in game theory: dominance, coexistence, and coordination. We show that the same set of pay-offs can result in coordination-like or coexistence-like depending on the strategic decision of each player type. In the sequel, we consider a game involving prosocial and antisocial players, i.e., individuals who tend to play with large groups and small groups, respectively. In this game, a certain task should be performed, that will benefit one of the participants at the expense of the other players. We show that individuals able to gather large groups to perform the task may prevail, even if this task is costly, providing a possible scenario for the evolution of eusociality. In the last example, we show that different strategies regarding game size may lead to spontaneous separation of different types, i.e., a possible scenario for speciation without physical separation (sympatric speciation).

Simple Neuron Model with Complex Dynamics

Inese Bula (University of Latvia; and Institute of Mathematics and Computer Science of University of Latvia)
Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: We investigate the piecewise linear difference equation that is a prototype of one neuron model with the internal decay rate and the given signal function.

We study the periodic solutions of a discrete neuron model for period two or period three of the parameter (internal decay rate) ([1,2,3]). The existence of periodic points is different for parameter sequences with period two (even number) and three (odd number). In [5], the authors consider the situation when the parameter sequence is periodic with period 2 and show that at certain values of the parameter coefficients a chaotic attractor is formed. For certain values of the parameter coefficients with period 3 there exists an attracting interval for which the model is chaotic ([4]). On the other hand, if the initial value is chosen outside the mentioned attracting interval, then the solution of the difference equation either increases to positive infinity or decreases to negative infinity.

The properties of chaos (sensitivity to initial conditions) can be used in random number generation as well as in cryptography. An important role here is played the uniform distribution of elements of the solution. The investigation can be useful in the design of chaos-based neural networks architecture.

- [1] I. Bula, M.A. Radin, Periodic Orbits of a Neuron Model with Periodic Internal Decay Rate, Appl. Math. Comput., 266, 2015, 293 - 303.
- [2] I. Bula, M.A. Radin, N. Wilkins, Neuron Model with a Period Three Internal Decay Rate, Electron. J. Qual. Theory Differ. Equ., 46, 2017, 1 - 19.
- [3] I. Bula, M.A. Radin, Eventually periodic solutions of single neuron model, Nonlinear Anal. Model. Control, 25(6), 2020, 903 - 918.
- [4] I. Bula, M.A. Radin, New Type of Chaotic Attractors and Their Applications, Frontiers in Artificial Intelligence and Applications, 370, 2023, 142 - 148.
- [5] I. Bula, M.A. Radin, Chaotic Single Neuron Model with Periodic Coefficients with Period Two, Nonlinear Anal. Model. Control, 29(1), 2024, 111 - 123.

Using Freeman's curve encoding for the entropic analysis of trajectories

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Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Trajectories can have coexisting unpredictable and predictable nature, resulting from some underlying dynamical system. In this letter, we introduce the analysis of trajectories in two dimensions that allow us to distinguish between their stochastic portion and regular behavior. By discretizing the trajectory over a finite alphabet using chain coding, the resulting sequence is amiable to entropic analysis. The Hénon-Heiles model, often used as a test bench for complexity analysis, is then used to illustrate the approach. Kolmogorov-Sinai entropy, effective complexity, and informational distance are used to get a finer characterization of trajectories, including their classification according to their chaotic or regular nature.

Any continuous curve in a higher dimensional space can be considered a trajectory parameterized by a single variable, usually taken as time. It is well known that a continuous curve can have a fractional dimensionality, which can be estimated using already standard algorithms. However, characterizing a trajectory from an entropic perspective is far less developed. The search for such characterization leads us to use chain coding to discretize the description of a curve. Calculating the entropy density and entropy-related magnitudes from the resulting finite alphabet code becomes straightforward. In such a way, the entropy of a trajectory can be defined and used as an effective tool to assert creativity and pattern formation from a Shannon perspective. Applying the procedure to actual experimental physiological data and modeled trajectories of astronomical dynamics proved the robustness of the entropic characterization in a wealth of trajectories of different origins and the insight that can be gained from its use.

The procedure is extended to other cases involving real data. Such analysis is useful when analyzing real data with its unavoidable mixture of predictable and unpredictable components. Furthermore, the tools

presented go beyond the classification of regular and chaotic behavior to reveal more subtle behavior of trajectories.

We address here how such an entropic trajectory description can be done in practical terms, building from discrete trajectory coding. Our key results are as follows: (i) We describe a procedure to code arbitrary trajectories as a time series by discretization using the Freeman coding and show how usual characteristics, such as fractal dimension, can be computed from the resulting code. (ii) We define trajectory entropy and related quantities over the coding and discuss its interpretation regarding information generation and correlation at all scales. (iii) We show how the defined magnitudes can be used for classification purposes in three examples: in the known Hénon-Heiles model of motion to distinguish between regular and irregular trajectories; in the detection of Parkinson's disease in subjects from their gait; in the analysis of falling risk based on posture data.

The procedure is directly extensible to three-dimensional trajectories, where the Freeman alphabet has a cardinality of 26, yet for higher dimensions, the size of the coding alphabet could make it untractable.

Phase transition in collective behavior of particles with attractive-repulsive interaction

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Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Collective motion in complex systems has been the subject of theoretical and computational studies in various system length scales. To model this kind of collective behavior, the individuals' motions are usually modified by a local alignment rule. These Vicsek-type models which update the individuals' direction and speed considering their neighbors' velocities and distances, can explain phase transition in collective motion which developed the criticality hypothesis in living systems. In this study, we use distance-based Lennard-Jones interaction types in the presence of thermal noise and without the Vicsek-type alignment to simulate individuals' collective motion. The results show that noise disturbs the coherence of the whole collection and takes apart it into smaller clusters formed by aligned individuals while the distance interaction strength empowers the system coherency. The phase transition from the coherent to the clustered phase can be found by the statistics of the cluster sizes. We found that the range of particles' interactions can affect the collective behavior and the phase transition of the system.

Parametric perturbations as inducers of shrimp spirals in parameter planes and routes to chaos via bifurcations and orbit collisions

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Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: In this work, we study the effects of parametric perturbations in Lorenz-like systems, focusing on confined plasma models, such as the Rypdal model. We consider disturbances as a function of the dynamic variables, without dimensional increment and keeping the system autonomous. The resulting inclusion of saddle points and saddle-focus domains, verifying the occurrence of homoclinic connections, leads to the appearance of shrimp spirals in parameter planes of these systems that, without the disturbance, would not present it. Furthermore, within the shrimp-shaped periodic structures, routes to chaos are identified involving collisions of mirrored attractive orbits, with the merging of the respective attraction basins. The dynamics was analyzed in parameter planes using the Lyapunov spectrum together with isospikes counting. Given the symmetry of the studied systems, bistable regions with a pair of mirror-image attractors occur. These bands of bistability are interspersed, in the parameter plane, with the single attractors ones. We also observe multistability due to additional symmetries of the disturbed systems, with multiple chains of bifurcations and collisions of orbits. These collisions and mergers of attractors are governed by the saddle point manifolds. Complementing the analyzes, we express the period formation rules within shrimps. Such schemes vary depending on the disturbance considered.

Multi-dimensional Riemannian spaces to the Navier-Stokes system of equations

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Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Geometrical properties of the metrical tensor of $14D = 4D+4D+6D$ Riemannian space, associated with the Navier-Stokes systems of equations are studied. From this point of view the components of velocity and pressure of non-compressible and viscous liquid depend on conditions on the curvature tensor of the corresponding space. In the case of considered $14D$ space the Riemann tensor of curvature R_{ijkl} has big number components but the Ricci-tensor R_{ik} of the space has only four components that are equal exactly to zero on solutions of the NS-equations. Conditions of compatibility of the system of NS-equations are studied on base of special six-dim metric having 15 non-zero components depending on the functions of velocities and pressure $U(x, y, z, t)$, $V(x, y, z, t)$, $W(x, y, z, t)$, $P(x, y, z, t)$ of liquid. The behaviour of geodesics of considered metrics can be as regular wherine chaotic and may be of use to studies of the properties of liquid-flows.

Modelling phenotypic plasticity in dryland vegetation

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Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Due to global climate change, ecosystems are at increasing risk of irreversible tipping to dysfunctional states. Examples include rainforests becoming savannas, grasslands becoming deserts, and thawing of icy tundra. Understanding tipping phenomena and ways to evade them calls for the study of alternative pathways of ecosystem response that may culminate in functional ecosystem states. An essential response element is phenotypic changes at the individual plant level. Plants invest in adapting to stressful environments by changing their phenotypic traits, such as altering the shoot or root structure, changing leaf surface area, closing stomata, or growing thorns. Phenotypic changes in plants have been observed in response to light stress (phototropism), water stress (hydrotropism), grazing stress from herbivores and insects, etc. In this work, we construct a general dynamic model for phenotypic transitions under water stress. The model consists of vegetation-biomass and water variables, and a third variable (an order parameter) is introduced to quantify different phenotypes. Functional phenotype dependencies of the vegetation growth and mortality rates are introduced to account for the general tradeoff between tolerance to water stress and biomass production. Using this model, we study the bifurcation structure along the rainfall gradient and how phenotypic plasticity helps evade or delay tipping. As the precipitation rate decreases, we find that a solution branch describing stress-tolerant plants bifurcates from a solution describing fast-growing plants, and extends to significantly lower precipitation values. We further analyze the flow in phase space to understand the conditions for early tipping to the dysfunctional bare-soil state and whether such tipping can be evaded. Our results indicate that sudden severe droughts can cause early collapse to bare soil when the unstable solution representing the fast-growing phenotype no longer exists. However, at higher precipitation rates where this solution exists, its unstable manifold changes the phase-space flow and prevents the tipping to bare soil. The results further suggest that managing ecosystems so as to include a small portion of the stress-tolerant phenotype at high precipitation can evade tipping to bare soil following severe droughts by directing the flow in phase space toward the functional stress-tolerant vegetation state.

State estimation of high-dimensional nonlinear systems

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Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: In many physical systems, the equation of motion is available, but not all state variables can be measured. For such systems, state estimation methods are used to determine the unknown state vari-

ables. In this study, we focus on adoptODE, a modeling approach that combines measured data and expert knowledge of the underlying system. AdoptODE relies on automatic differentiation to formulate a method for continuous adjoint sensitivity analysis, enabling efficient estimation of the system states described by ordinary differential equations. To evaluate the performance of the state estimation, we applied adoptODE to the Lorenz-96 model and estimated the unmeasured variables.

Fractional Tumour-Immune Model with Drug Resistance

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Thursday, August 1, 16:00–18:30, Campus Center (IRC) Foyer

Abstract: Cancer is a group of diseases in which cells grow uncontrollably and can spread into other tissues. Various studies consider the interactions between cancer cells and the immune system as well as different types of treatment. Mathematical models have been used to study the growth of cancerous cells. We propose a mathematical model governed by differential equations of fractional order, namely non integer order differential equations. In this context, we study a fractional order model that describes some aspects of the interactions among host, effector immune, and cancer cells. A drug treatment is considered to analyse the cancerous cells proliferation. Due to the chemotherapy, sensitive cancer cells can suffer mutation and transform into resistant ones. We extend the tumour-immune model splitting the equation of the cancerous cells into two equations: an equation for the sensitive cells and another for the resistant ones. We show that not only the chemotherapy but also the drug resistance plays an important role in the growth rate of cancer cells. We compute the maximum number of cancerous cells (sensitive cells + resistant cells) in a time interval. In a continuous drug delivery, the maximum of cancerous cells values depend on the chemotherapy dose and the mutation rate. The efficiency of the treatment changes according to chemotherapy and mutation rate. We show that the order of the equation differential plays a crucial role in modelling a tumour-immune system with drug resistance. The dynamical behaviour is changed according to the order of the equation differential. We verify that the size of the parameter space region in which the cancer is suppressed depends on the order value of the equation differential.